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SOHA ICHIDA BUZILADIGAN SINGULYAR KOEFFITSIYENTLI GIPERBOLIK VA ARALASH TIPDAGI TENGLAMALAR UCHUN BITSADZE-SAMARSKIY MASALASI

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Uslubiy qo‘llanma

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"Soha ichida buziladigan singulyar koeffitsiyentli giperbolik va aralash tipdagi tenglamalar uchun Bitsadze-Samarskiy masalasi"

Oliy ta'lim muassasalarining magistrantlari uchun uslubiy qo'llanma.

Termiz. 2021. 52 b.

Ushbu uslubiy qo'llanma oliy ta'lim muassasalarining magistrantlari uchun mo'ljallangan bo'lib, unda soha ichida buziladigan giperbolik va aralash tipdagi tenglamalar uchun lokal va nolokal masalalar qamrab olingan.

O'quv qo'llanma matematika mutaxassisligi magistrantlari, doktorantlar va ilmiy xodimlarga mo'ljallangan.

O'quv metodik kengashning 2021-yil 30 oktyabr №3-sonli yig'ilish qaroriga asosan nashrga tavsiya qilingan.

Mas'ul muxarrir:

Fizika-matematika fanlari doktori, professor **Mirsaburov Miraxmat**

Taqrizchilar:

Xayrullayev I.N., Fizika-matematika fanlari nomzodi, dotsent.

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Kirish

Jahon miqyosida olib borilayotgan ko‘plab ilmiy amaliy tadqiqotlar aksariyat hollarda xususiy hosilali ikkinchi tartibli aralash tipdagi differensial tenglamalar uchun chegaraviy masalalarni tadqiq qilishga keltiriladi. Aralash tipdagi tenglamalar fizik va biologik jarayonlarni to‘liq tasvirlab beruvchi matematik model sifatida xizmat qiladi. Ushbu tenglamalar sinfiga qiziqishning ortishi, bu sohada olingan natijalarning katta nazariy ahamiyatga ega ekanligi bilan birga ularning gazlar dinamikasida, gidrodinamikada, cheksiz kichik bukiluvchi sirtlar nazariyasida, mexanikada, akustika va elektron sochilish nazariyasida va boshqa ko‘plab sohalarda qo‘llanilishi bilan izohlanadi. Shuning uchun aralash tipdagi differensial tenglamalar uchun chegaraviy masalalarning qo‘yilishi va ularning yechilishi zamonaviy differensial tenglamalar nazariyasining dolzarb masalalaridan biri bo‘lib hisoblanadi.

Hozirgi kunda buziluvchi giperbolik tenglamalar, shuningdek singulyar koeffitsiyentli aralash tipdagi tenglamalar uchun nolokal chegaraviy masalalarni tadqiq etish dunyo miqyosidagi dolzarb muammolardan biri sanaladi. Aralash tipdagi tenglamalar uchun o‘ziga xos, maxsus ulanish shartli chegaraviy masalalarning yechilishi matematik fizika tenglamalari nazariyasida muhim masalalardan hisoblanadi. Shundan kelib chiqib, aytish mumkinki, singulyar koeffitsiyentli aralash tipdagi tenglamalar uchun nolokal chegaraviy masalalar qo‘yish va ularni tadqiq etish matematik fizika tenglamalari sohasidagi ilmiy tadqiqotlarning ustuvor yo‘nalishlaridan biridir.

Mamlakatimizda fundamental fanlarning amaliy tadbiqqa ega bo‘lgan dolzarb yo‘nalishlariga e’tibor kuchaytirilmoqda. Ilm-fan oldiga fundamental tadqiqotlarni amaliyotga yaqinlashtirish masalasi muhim vazifa sifatida qo‘yilgan. Bu borada real obyektlardagi jarayonlarni modellashtiruvchi aralash tipdagi singulyar koeffitsiyentli tenglamalarni tadqiq etish masalalariga oid salmoqli natijalarga erishildi. Shuningdek matematik fanlarning ustuvor yo‘nalishlari bo‘yicha, ayniqsa, algebra va funksional analiz, differensial tenglamalar va matematik fizika, dinamik sistemalar nazariyasi, geometriya va topologiya, ehtimollar nazariyasi va matematik statistika, amaliy matematika va matematik modellashtirish bo‘yicha halqaro standartlar darajasidagi ilmiy tadqiqot ishlarini olib borish mamlakatimizdagi matematik ilmiy tadqiqotlarning asosiy vazifalari etib belgilangan[1]. Qaror ijrosini ta’minlashda xususiy hosilali tenglamalar nazariyasini, xususan buziluvchan giperbolik tenglamalar va singulyar koeffitsiyentli aralash turdagi tenglamalar nazariyasi bo‘yicha tadqiqotlarni rivojlantirish muhim ahamiyat kasb etadi.

Mazkur uslubiy qo‘llanma Universitetning matematika, mexanika-matematika fakultetlari uchun “matematik fizika tenglamalarining zamonaviy usullari” kursi dasturiga moslab yozilgan. Uslubiy qo‘llanmani yozishda O‘zbekiston va boshqa chet el olimlari tomonidan olib borilgan ilmiy izlanishlardan foydalanildi.

Mazkur o‘quv qo‘llanma ana shu nuqtai nazardan kelib chiqib, magistraturaning 705400100 – Matematika (differetsial tenglamalar va matematik fizika) mutaxassisligi ta’lim standarti asosida yaratilgan bo‘lib, undan differensial tenglamalar sohasi bo‘yicha o‘tgan asrning oxirgi 30-35 yili ichida olingan ilmiy natijalarning ba’zilari ham joy olgan. Shuning uchun undan magistrantlardan tashqari differensial tenglamalar sohasi bo‘yicha ilmiy-tadqiqot olibborayotgan ilmiy xodimlar va aspirantlar ham foydalanishi mumkin.

Ushbu o‘quv qo‘llanma 2 bobdan iborat bo‘lib, unda soha ichida buziladigan singulyar koeffitsiyentli giperbolik tipdagi tenglamalar uchun qo‘yiladigan asosiy boshlang‘ich va chegaraviy masalalar bilan birga bir qator yangi nostandart masalalar va ularni yechish usullari haqida ma’lumot berish maqsad qilib qo‘yilgan. Qo‘llanmaning I bobida singulyar koeffitsiyentli aralash turdagi Gellesrtedt tenglamasi uchun xarakteristikada lokal va nolokal shartli masalalar: Bitsadze-Samarskiy masalasi, chegaraviy xarakteristikada lokal va nolokal shartli masalalar, parametr $\beta_0 = -m/2$ bo‘lgan hol uchun masalalar keltirilgan.

Qo‘llanmaning II bobi “Soha ichida buziladigan giprebolik tenglamalar uchun nolokal chegaraviy masala” deb nomlangan bo‘lib, soha ichida buziladigan singulyar koeffitsiyentli giperbolik tipdagi tenglamalar uchun chegaraviy masalalarning qo‘yilishi va yechilishiga bag‘ishlangan. §1.1. paragrafda soha ichida buziladigan

[1] Ўзбекистон Республикаси Вазирлар Маҳкамасининг 2017 йил 18 майдаги №292 «Ўзбекистон республикаси фанлар академиясининг янгидан ташкил этилган илмий-тадқиқот муассасалари фаолиятини ташкил этиш чора-тадбирлари тўғрисида»ги қарори.

singulyar koeffitsiyentli Gellerstedt tenglamasi uchun Bitsadze-Samarskiy tipidagi nolokal chegaraviy masala tadqiq qilingan. §1.3. paragrafda parametr $\beta_0 = -m/2$ bo‘lgan holda soha ichida buziladigan singulyar koeffitsiyentli giperbolik tipdagi tenglamalar uchun umumiy ulanish shartiga ega bo‘lgan Bitsadze-Samarskiy shartli masala yechimining korrektligi keltirilgan. Masala yechimining korrekt ekanligini isbotlashda ketma-ket yaqinlashish usulidan foydalanilgan.

I-bob. XARAKTERISTIKADA LOKAL VA NOLOKAL SHARTLI MASALALAR.

1-§. Lokal va nolokal shartli masala.

Buziluvchan giperbolik va aralash tipdagi tenglamalar nazariyasining rivojlanish tarixi G. Darbu, F. Trikomi YE. Xolmgren va S.Gellerstedtlarning mos ravishda 1894, 1923, 1927 va 1935 yillarda chop etilgan fundamental ishlari bilan bog‘liq.

Aralash tipdagi tenglamalar uchun chegaraviy masalalar bo‘yicha dastlabki fundamental tadqiqotlar 1920 yili italyan matematigi Franchesko Trikomi tomonidan olib borilgan. Bu ishdan keyin aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasi asosan uchta yo‘nalish bo‘yicha rivojlana boshladi: birinchi yo‘nalish - Trikomi masalasini umumiyroq aralash tipdagi tenglamalar uchun o‘rganish bo‘lib, ularga S. Gellerstedt; A.V.Bitsadze; K.I.Babenko; L. I. Karol; S.P. Pulkin va boshqalarning ishlari bag‘ishlangan; ikkinchi yo‘nalish - Trikomi masalasining har xil modifikatsiyalariga bag‘ishlangan; uchinchi yo‘nalish esa aralash tipdagi tenglamalar uchun spektral masalalarni tadqiq etishdan iborat.

Aralash tipdagi tenglamalar uchun chegaraviy masalalarning rivojlanishida shved matematigi Sven Gellerstedt tomonidan ishlab chiqilgan potensiallar nazariyasi muhim o‘rin egallaydi. S. Gellerstedt yaratgan usul yordamida buziluvchan elliptik tipdagi tenglama uchun Dirixle va Xolmgren masalalarining yechimini qulay integral shaklda yozish mumkin va aralash tipdagi tenglama uchun chegaraviy masalani tadqiq etish juda qulay bo‘ladi. Shuningdek aralash tipdagi tenglama uchun chegaraviy masalalar nazariyasining rivojlanishiga A.V.Bitsadzening ekstremum prinsipi katta turtki bergan. Bu prinsip masala yechimining yagonaligini isbotlashda juda keng qo‘llaniladi. Aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasining rivojlanishida muhim o‘rin tutuvchi yana bir natijalardan biri bu S.G. Mixlin tomonidan ishlab chiqilgan Karlemanning Trikomi singulyar integral tenglamasini regulyarlashtirish usuli hisoblanadi va bu usul F.Trikomi integral tenglamasini yechishda qo‘llanilgan.

Quyidagi singulyar koeffitsiyentli buziluvchan giperbolik tipdagi tenglamani $z = x + iy$, $\operatorname{Im} z < 0$ kompleks yarim tekislikda o‘rganamiz

$$-(-y)^m u_{xx} + u_{yy} + \alpha_0 (-y)^{m/2-1} u_x + \beta_0 y^{-1} u_y = 0, \qquad (1.1)$$

bu yerda m, α_0 va β_0- haqiqiy sonlar hamda ular ushbu

$$-m/2 \le \beta_0 \le (m+4)/2, \; |\alpha_0| \le (m+2)/2,$$

shartlarni qanoatlantiradi D_0 soha $z = x + iy$ komplekis tekislikning bir bog‘lamli sohasi bo‘lib, u (1.1) tenglamaning

$$AC: \quad x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = -1,$$

$$BC: \quad x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

xarakteristikalari hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bir bog‘lamli sohasi bo‘lsin.

(1.1) tenglama shu narsa bilan e’tiborliki birinchidan bu tenglamaning kichik hadlari oldidagi koeffitsiyentlari singulyar maxsuslikka ega, ikkinchidan bu yerda

$$K(x,y)h(x,y)u_{xx} + u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = f(x,y) \qquad (1.2)$$

buziluvchan umumiy giperbolik tipdagi tenglama uchun Koshi masalasini normal yechilishining

$$\lim_{y \to -0} \frac{ya(x,y)}{\sqrt{-K(y)}} = 0, \qquad (1.3)$$

Protter sharti [38] buziladi, bu yerda $h(x,y) > 0$, $K(0) \equiv 0$, $K(y) < 0$, $y < 0$ da. (1.3) shart bajarilmasligiga qaramasdan, agar $|\alpha_0| \le m/2$, $\beta_0 = 0$ bo‘lsa (1) tenglama uchun Koshi masalasi korrekt qo‘yilgan [38].

Bundan (1.1) tenglama uchun Koshi masalasini normal yechilishida (1.3) shart zaruriy shart emasligi kelib chiqadi. Endi (1.1) tenglamada $\beta_0 = 0$, $\alpha_0 = -m/2$ bo‘lsin:

$$-(-y)^m u_{xx} + u_{yy} - (m/2)(-y)^{m/2-1} u_x = 0, \qquad (1.4)$$

(1.4) tenglama uchun Darbu masalasini ta’riflaymiz.

Darbuning ikkinchi masalasi: D_0 sohada (1.4) tenglamaning ushbu

$$u_y(x,0) = v(x) \ , \ x \in I \ : \ u|_{BC} = \psi(x) \ , \ x \in [0,1], \qquad (1.5)$$

shartlarni qanoatlantiruvchi regulyar $u(x,y) \in C(\overline{D}_0) \cap C^2(D_0)$ yechimi topilsin, bu yerda $v(x) \in C^2(I)$, $\psi(x) \in C^1(\bar{I}) \cap C^2(I)$, $I = (-1,1)$- $y = 0$ o‘qining intervali.

1-teorema. Darbuning ikkinchi masalasiga mos bir jinsli masala cheksiz ko‘p chiziqli bog‘liq bo‘lmagan yechimlarga ega, bir jinsli bo‘lmagan masala esa faqat va faqat,

$$v(2x-1) = ((m+2)/2)^{\beta} (1-x)^{\beta} \psi'(x) \ , \ x \in (0,1),$$

shart bo‘lgandagina yechimga ega bo‘ladi, bu yerda $\beta = m/(m+2)$.

Bir jinsli Darbuning ikkinchi masalasining barcha notrivial yechimlar

$$u(x,y) = \tau_0\left(x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \right) - \tau_0(1) \ ,$$

formula bilan beriladi, bu yerda $\tau_0(x) \in C(\bar{I}) \cap C^2(I)$ sinfdagi ixtiyoriy funksiya. Endi (1.4) tenglama uchun (1.5) Darbu shartlarini ushbu

$$u_y(x,0) = v(x) \ , \ x \in I \ ; \ u|_{AC} = \psi(x), \ x \in [-1,0] \qquad (1.6)$$

shaklda beramiz.

2-teorema. (1.4), (1.6) masala yagona yechimga ega.

1-teorema va 2-teoremalardan ushbu xulosa kelib chiqadi: qat’iy giperbolik tenglamalar uchun qo‘yilgan Koshi masalasining korrektligidan Darbu masalasining korrektligi kelib chiqadi, buziluvchan giperbolik tenglamalarda esa umuman olganda Koshi masalasi korrektligidan Darbu masalasining korrektligi kelib chiqmaydi. Buning ustiga (1.4) buziluvchan giperbolik tenglama uchun umuman olganda xarakteristikalar, chegaraviy shartlarning ularda qo‘yilishi ma’nosida teng huquqli emas.

(1.1) tenglamada $\alpha_0 = 0$ bo‘lsin:

$$-(-y)^m u_{xx} + u_{yy} + \left(\beta_0 / y\right) u_y = 0 \qquad (1.7)$$

bu tenglama juda ko‘p matematiklar tomonidan o‘rganilgan [6,12,20]. Umuman olganda, (1.7) tenglama uchun oddiy Koshi masalasi korrekt bo‘lmasligi mumkin. A. V. Bitsadze [6] (1.7) tenglama uchun boshlang‘ich shartlari bir jinsli bo‘lgan:

$$u(x,0) = 0,\ x \in \overline{I}\ ; \quad \lim_{y \to -0} \frac{\partial u}{\partial y} = 0,\ x \in I\ ;$$

Koshi masalasi $\beta_0 = -m/2$ bo‘lganda Ushbu

$$u_0(x,y) = \tau_0\left[x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right] - \tau_0\left[x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right],$$

ko‘rinishdagi notrival yechimlarga ega ekanligini ko‘rsatgan, bu yerda $\tau_0(x)$ ikki marta uzluksiz hosilaga ega bo‘lgan ixtiyoriy funksiY. Shu holatdan kelib chiqib A. V. Bitsadze boshlang‘ich shartlari

$$u(x,0) = \tau(x),\ x \in \overline{I}\ ; \quad \lim_{y \to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \nu(x),\ x \in I\ , \qquad (1.8)$$

ko‘rinishda bo‘lgan shakli o‘zgargan Koshi masalasini o‘rgangan va uni korrekt ekanligini ko‘rsatgan, bu yerda $-(m/2) \le \beta_0 < 1$.

Agar $\beta_0 \ge 1$ bo‘lsa, (1.7) tenglamaning yechimlari buzilish chizig‘i atrofida chegaralangan bo‘lmaydi. Haqiqatdan ham ushbu

$$u_0(x,y) = \begin{cases} (-y)^{1-\beta_0} & , \ \textit{агар} \ \ \beta_0 \ne 1 \ \ \textit{булса}, \\ \ln(-y) & , \ \textit{агар} \ \ \beta_0 = 1 \ \ \textit{булса} \end{cases}$$

xususiy yechimlar yuqoridagi fikrimizni tasdiqlaydi.

$\beta_0 > 1$ bo‘lganda Koshi masalasi korrekt bo‘lishi uchun boshlang‘ich shartlar

$$\lim_{y\to -0}(-y)^{\beta_0 -1}u(x,y)=\tau(x);\quad \lim_{y\to -0}(-y)^{2-\beta_0}\frac{\partial}{\partial y}\left((-y)^{\beta_0 -1}u(x,y)\right)$$

ko‘rinishda bo‘lishi kerak; $\beta_0 = 1$ bo‘lganda esa Koshi masalasi korrekt bo‘lishi uchun boshlang‘ich shartlar

$$\lim_{y\to -0}\frac{u(x,y)}{\ln(-y)^{(m+2)/2}}=\tau(x)\ ,$$

$$\lim_{y\to -0}(-y)\ln^2(-y)^{(m+2)/2}\frac{\partial}{\partial y}\left[\frac{u(x,y)-A(x,y)}{\ln(-y)^{(m+2)/2}}\right]=\nu(x)\ ,$$

ko‘rinishda bo‘lishi kerak, bu yerda $A(x,y)-$aniq ko‘rinishga ega bo‘lgan maxsus kiritilgan funksiY.

Shunday qilib, (1.1) tenglama yechiminning tuzilishi va differensial xossalari uning kichik hadlari oldidagi koeffitsiyentlar α_0 va β_0 ga bog‘liqdir. (1.1) tenglama uchun masalalar α_0 va β_0 parametrik tekislikda $P\,(\alpha_0,\beta_0)$ nuqtaning o‘zgarishiga qarab qo‘yiladi.

$y > 0$ yarim tekislikda

$$y^m u_{xx} + u_{yy} + (\beta_0 / y)u_y = 0 \qquad (1.9)$$

tenglamani o‘rganamiz.

(1.9) tenglama shu bilan xarakterliki uning uchun oddiy N masalasi korrekt emas. Haqiqatdan ham Ω_0- yuqori $y > 0$ yarim tekislikda yotuvchi va uchlar $A(-1,0)$, $B(1,0)$ nuqtada bo‘lgan (1.9) tenglamaning normal chizig‘i $\sigma_0 : x^2 + 4(m+2)^{-2}y^{m+2} = 1$ chizig‘i hamda $y = 0$ o‘qining AB kesmasi bilan chegaralangan bir bog‘lamli bo‘lsin. Ushbu masalani ta’riflaymiz.

N **masalasi**. Ω_0 sohada (1.9) tenglamaning ushbu

$$u\Big|_{\sigma_0}=\varphi_0(x,y)\ ,\ (x,y)\in\sigma_0\ ,$$

$$\frac{\partial u}{\partial y}\Big|_{y=0}=\nu(x)\ ,\ x\in I=(-1,1)\ ,$$

shartlarni qanoatlantiruvchi regulyar yechimi $u(x,y)\in C(\overline{\Omega}_0)\cap C^2(\Omega_0)$ topilsin. Bevosita tekshirish yordamida ko‘rsatish mumkinki ushbu

$$u(x,y)=\frac{1-x^2-\dfrac{4}{(m+2)^2}y^{m+2}}{\left(1-x+\dfrac{2}{m+2}y^{\frac{m+2}{2}}\right)^2+\left(1+x+\dfrac{2}{m+2}y^{\frac{m+2}{2}}\right)^2}$$

funksiya bir jinsli N masalaning notrivial yechimi bo‘ladi, ya’ni (1.9) tenglama uchun N masalasi korrekt emas. Shu munosabat bilan A.V.Bitsadze (1.9) tenglama uchun ushbu shakli o‘zgargan N masalasini o‘rgangan: Ω_0 sohada (1.9) tenglamaning ushbu

$$u\Big|_{\sigma_0}=\varphi_0(x,y)\ ,\quad (x,y)\in\sigma_0,$$

$$\lim_{y\to+0}y^{\beta_0}\frac{\partial u}{\partial y}=\nu(x),\quad x\in I=(-1,1)$$

shartlarni qanoatlantiruvchi regulyar yechimi topilsin.

Shakli o‘zgargan N masalasi korrekt qo‘yilgan. Ushbu qo‘llanmada asosan singulyar koeffitsiyentli

$$signy\,|y|^m u_{xx}+u_{yy}+\alpha_0|y|^{m/2-1}u_x+(\beta_0/y)u_y=0 \qquad (1.10)$$

tenglama ham o‘rganilgan. (1.10) tenglama $z=x+iy$, kompleks tekisligining $\operatorname{Im}z>0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan Γ: $y=f(x)$ chizig‘i bilan, $\operatorname{Im}z<0$ pastki yarim tekislikda esa (1.10) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganildi.

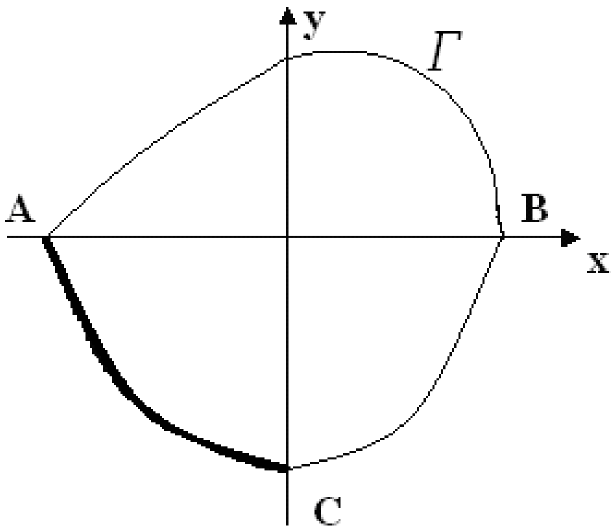
Asosiy e’tibor (1.6) tenglama uchun $D^{-}=D\bigcap\{y<0\}$ sohada shakli o‘zgargan Koshi masalasini o‘rganishga, $D^{+}=D\bigcap\{y>0\}$ sohada Dirixle va shakli o‘zgargan N masalasini, aralash D sohada esa Trikomi masalasini hamda Frankl turidagi nolokal masalalarni o‘rganishga qaratilgan.

Ushbu singulyar koeffitsiyentli

$$signy|y|^{m}u_{xx}+u_{yy}+(\beta_0/y)u_y=0 \tag{1.11}$$

tenglama qaraymiz, bu yerda $m>0, -\frac{m}{2}\le\beta_0\le 1$.

(1.11) tenglama $z=x+iy$, kompleks tekisligining $lmz>0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma: y=f(x)$ chizig‘i bilan, $lmz<0$ pastki yarim tekislikda esa (1.11) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganiladi.



(1.11) tenglama uchun F. Trikomi, V.I. Jegalov, A. M. Naxushev, masalalarini shartlarini barchasini o‘zida birlashtirib yaxlit bir masala sifatida ta’riflangan masalaning korrekt ekanligi isbotlash maqsad qilib qo‘yilgan. Masalaning ta’rifi β_0 parametrni o‘zgarishga qarab qo‘yiladi. D^{+} va D^{-} orqali D sohaning mos ravishda $y>0$ va $y<0$ yarim tekislikda yotuvchi qismlarini belgilaymiz, C_0 va C_1 orqali esa

$E(c,0)$ nuqtadan chiquvchi xarakteristikalarning mos ravishda AC va BC xarakteristikalar bilan kesishish nuqtasini belgilaymiz, bu yerda $c \in I = (-1,1) - y = 0$ o‘qining intervali.

1. $\beta_0 \in (-m/2,1)$. bo‘lgan xol.

F. Trikomi masalasi chegaraviy shartida AC va BC xarakteristikalar teng xuquqli ishtirok etmaydi, A. M. Naxushevning siljishli masalasida chegaraviy shartlarda xarakteristikalar teng xuquqli bo‘lib ularning barcha nuqtalarida chegaraviy shartlar berilgan.

Ushbu masalada AC va BC xarakteristikalarning mos ravishda C_0C va C_1C qismlari siljishli chegaraviy shartlardan ozod etiladi va bu yetishmaydigan A. M. Naxushev sharti AB xarakteristikada F.I.Frankl sharti bilan ekvivalent almashtirilgan masalaning korrekt ekanligi o‘rganiladi.

$БС$ **- masalasi**. D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y) \in C(\overline{D})$ funksiya topilsin.

1. $u(x,y) \in C^2(D^+)$ va bu sohada (1.11) tenglamani qanoatlantiradi.

2. D-sohada $u(x,y)$ funksiya (1.11) tenglamaning R_1 sinfga tegishli yechimi.

3. AB intervalda ushbu ulanish shartlari bajariladi

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{\beta_0}\frac{\partial u}{\partial y} \quad x \in I/\{c\},$$

shu bilan birga bu limitlar $x = \pm 1,\ x = c$ nuqtalarda $1-2\beta$ dan katta bo‘lmagan tartibda cheksizlikka aylanishi mumkin;

4. Ushbu chegaraviy shartlar bajariladi

$$u(x,y)\Big|_{\sigma_0} = \varphi(x), \quad -1 \le x \le 1,$$
$$u[\theta_0(x)] + \mu u[\theta_1(p(x))] = \psi(x), \quad -1 \le x \le c,$$
$$u(x,0) - \mu u(p(x),0) = f(x), \quad -1 \le x \le c,$$

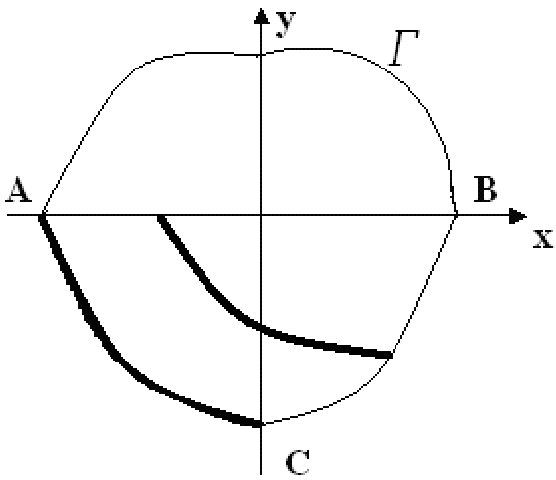
bu yerda $\theta(x_0)$ va $\theta_1(x_0)$ - mos ravishda AC va BC xarakteristikalarning $(x_0,0)$ $x_0 \in I$ nuqtadan chiqqan xarakteristikalar bilan kesishish nuqtasining affiksidir. $\varphi(x), \psi(x), f(x)$ - funksiyalar o‘zining berilish sohasining yopig‘ida uzluksiz funksiyalardir.

2-§. Bitsadze-Samarskiy masalasi.

Cingulyar koeffitsiyentli

$$signy|y|^m u_{xx} + u_{yy} + (\beta_0 / y) u_y = 0$$

aralash tipdagi tenglamani $z = x + iy$, kompleks tekisligining $Imz > 0$ yuqori yarim tekisligida uchlari $A(-1,0)$ va $B(1,0)$ nuqtalarda va yuqori yarim tekislikda joylashgan $\Gamma : y = f(x)$ chizig‘i bilan, $Imz < 0$ pastki yarim tekislikda esa (1.11) tenglamaning AC va BC xarakteristikalari bilan chegaralangan bir bog‘lamli D sohada o‘rganamiz.



(1.11) tenglama uchun Bitsadze-Samarskiy masalasining shartlarini parallel xarakteristikalardagi qiymatlarining kasr tartibli xosilalarini o‘zida birlashtirgan masalaning korrektligi o‘rganilgan. Ta’riflangan masalaning yagonaligi ekstremum prinsipi yordamida, mavjudligi isbotlashda singulyar integral tenglamalar, Viner-Xopf integral tenglamasi, Fredgolmning II-tur integral tenglamalar nazariyalaridan foydalanilgan.

D^+ va D^- orqali D sohaning mos ravishda $y > 0$ va $y < 0$ yarim tekislikda yotuvchi qismlarini belgilaymiz, C_0 va C_1 orqali esa $E(c,0)$ nuqtadan chiquvchi xarakteristikalarning mos ravishda AC va BC xarakteristikalar bilan kesishish nuqtasini belgilaymiz, bu yerda $c \in I = (-1,1) - y = 0$ o‘qining intervali.

$q(x)$ orqali $[c,1]$ kesmani $[-1,c]$ kesmaga akslantiruvchi funksiyani kiritamiz. Bu yerda $q'(x) < 0,\ q(1) = -1, q(c) = c$. Bu xossalarga ega bo‘lgan funksiya sifatida ushbu chiziqli funksiyani keltiramiz $q(x) = \rho - kx,$ bu yerda $k = (1+c)/(1-c), \rho = 2c/(1-c).$

1. $\beta_0 \in (-m/2,1)$. **bo‘lgan xol.**

$БC$ **masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$ funksiya topilsin:

1. $u(x,y) \in C(\overline{D})$;

2. $u(x,y) \in C^2(D^+)$ va bu sohada (1.11) tenglamani qanoatlantiradi;

3. $u(x,y)$ funksiya D^- sohada (1.11) tenglamaning D^- sohada R_1 sinfga tegishli yechimi;

4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{\beta_0}\frac{\partial u}{\partial y}\ , \quad x \in I \setminus \{c\}\ , \tag{1.12}$$

shu bilan birga bu limitlar $x = \pm 1, x = c$ nuqtalarda $1-2\beta$ kichik tartibdagi maxsuslikka ega bo‘lishi mumkin. Bu yerda $\beta = (m+2\beta_0)/2(m+2)$ ushbu shartlar bajariladi

5.
$$u(x,y)|_{\sigma} = \varphi(x), \quad -1 \le x \le 1 \tag{1.13}$$

$$a_0(x) D_{-1,x}^{1-\beta} u\left[\theta(q(x))\right] + b_0(x) D_{c,x}^{1-\beta} u\left[\theta^*(x)\right] = c_0(x), \ \ x \in [c,1]\ c \le x \le 1. \tag{1.14}$$

$$u(q(x),0) = \mu u(x,0) + f(x), \quad x \in [c,1] \tag{1.15}$$

Bu yerda $D_{c,x}^{1-\beta}$, $D_{-1,x}^{1-\beta}$ – kasr tartibli differensial operatorlar $\theta_0(x), \theta_1(x)$ AC va BC xarakteristikalarni $M(x_0,0),\ \ x_0 \in [c,1]$ nuqtadan chiquvchi xarakteristikalar bilan kesishish nuqtasining affiksi

$$\theta(\tilde{o}_0) = \frac{\tilde{o}_0 - 1}{2} - i\left(\frac{m+2}{4}(1+x_0)\right)^{\frac{2}{m+2}}, \qquad \theta^*(\tilde{o}_0) = \frac{\tilde{o}_0 - c}{2} - i\left(\frac{m+2}{4}(x_0+c)\right)^{\frac{2}{m+2}}. \tag{1.16}$$

$\varphi(x), \psi(x)$, $a_0(x)$, $b_0(x)$, $c_0(x)$ o‘zining aniqlanish sohasi yopig‘ida uzluksiz differensiallanuvchi funksiyalar bo‘lib ular uchun ushbu shartlar bajariladi

$$a_0^2(x)+b_0^2(x)\neq 0 \tag{1.17}$$

$\varphi(x)$ funksiya esa ushbu ko‘rinishda ifodalanadi

$$\varphi(x)=(1-x^2)\tilde{\varphi}(x) \tag{1.18}$$

bu yerda $\tilde{\varphi}(x)\in C^1(\bar{I})$, $\psi(-1)=0$.

Teorema. $БC$ masalasi ushbu $\mu(x)<0,\ 0<\mu_0<1$ shartlar bajarilganda bir qiymatli yechimga ega.

3-§. Chegaraviy xarakteristikalarda lokal va nolokal shartli masalalar

$БC$ **masalasining qo‘yilishi.**

Ushbu

$$sign y|y|^m u_{xx}+u_{yy}+(\beta_0/y)\,u_y=0\,, \tag{1.19}$$

tenglamani o‘rganamiz, bu yerda $m>0,\ \ -m/2<\beta_0<1.$ x , y erkli o‘zgaruvchilar tekisligida chekli bir bog‘lamli D soha bo‘lib u, $y>0$ yarim tekislikda uchlari $A=A(-1,0)$ va $B=B(1,0)$ nuqtalarda bo‘lgan $\sigma_0:\ x^2+4(m+2)^{-2}y^{m+2}=1$ normal chiziq bilan, $y<0$ yarim tekislikda esa (1.19) tenglamaning AC va BC xarakteristikalari bilan chegaralangan soha bo‘lsin.

D^+ va D^- orqali mos ravishda D sohaning, $y>0$ va $y<0$, yarim tekisliklarda yotgan qismini belgilaymiz. C_0 va C_1, orqali esa mos ravishda AC va BC xarakteristikalarning $E(c,0)$, nuqtadan chiquvchi xarakteristikalar bilanikesishish nuqtasini belgilaymiz, bu yerda $c\in I=(-1,1)$ - $y=0$ o‘qining intervali.

$БC$ **masalasi.** *D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$, funksiya topilsin:*

1. $u(x,y)\in C(\bar{D})$;

2. $u(x,y)\in C^2(D^+)$ va ushbu sohada (1.9) tenglamani qanoatlantiradi;

3. $u(x,y)$ funksiya D^- sohada umumlashgan yechim, ya’ni R_1 sinfga tegishli;

4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y\to -0}(-y)^{\beta_0}\frac{\partial u}{\partial y}=\lim_{y\to +0}y^{\beta_0}\frac{\partial u}{\partial y}\ ,\quad x\in I\setminus\{c\}\ , \tag{1.20}$$

shu bilan birga bu limitlar $x=\pm 1, x=c$ *nuqtalarda* $1-2\beta$ *dan kichik tartibli maxsuslikka ega bo'lishi mumkin, Bu yerda* $\beta=(m+2\beta_0)/2(m+2)$*;*

5. *Ushbu shartlar bajariladi*

$$u(x,y)|_{\sigma 0}=\varphi(x)\ ,\quad -1\le x\le 1\ , \tag{1.21}$$

$$u|_{AC_0}=\psi(x)\ ,\quad -1\le x\le (c-1)/2\ , \tag{1.22}$$

$$a_0(x)(1+x)^{\beta}D_{c,x}^{1-\beta}u\left[\theta_0(x)\right]+b_0(x)(1-x)^{\beta}D_{x,1}^{1-\beta}u\left[\theta_1(x)\right]=c_0(x)\,, \tag{1.23}$$

bu yerda $D_{c,x}^{1-\beta}$ *,* $D_{x,1}^{1-\beta}$ *–operatorlar* $1-\beta$ *kasr tartibli Liuvill ma'nosidagi hosilalardir.* $\theta_0(x)$ *va* $\theta_1(x)$ *esa mos ravishda* AC *va* BC *xarakteristikalarning* $M(x_0,0)$ *nuqtadan chiquvchi xarakteristikalar bilan kesishish nuqtalarining affikslaridir, bu yerda* $x_0\in[c,1]$*:*

$$\theta_0(x_0)=\frac{x_0-1}{2}-i\left(\frac{m+2}{4}(1+x_0)\right)^{\frac{2}{m+2}},\qquad \theta_1(x_0)=\frac{x_0+1}{2}-i\left(\frac{m+2}{4}(1-x_0)\right)^{\frac{2}{m+2}}. \tag{1.24}$$

Berilgan $\psi(x)$ *,* $a_0(x)$ *,* $b_0(x)$*,* $c_0(x)$ *funksiyalar o'zlarining aniqlanish sohasining yopig'ida uzluksiz differensiallanuvchi funksiyalardir, shu bilan birga*

$$a_0^2(x)+b_0^2(x)\ne 0,\ \ b_0(c)=b_0(1)=0$$

$$d(x)=a_0(x)+b_0(x)>0\quad,\ x\in(c,1) \tag{1.25}$$

$$d(c)+\lambda\pi\, ctg3\alpha\pi\,(a_0(c)-b_0(c)\ne 0$$

$\varphi(x)$ *funksiyani ushbu ko'rinishda tasvirlash mumkin.*

$$\varphi(x)=(1-x^2)^{1-2\beta}\tilde{\varphi}(x) \tag{1.26}$$

bunda $\tilde{\varphi}(x)\in C^1(\bar{I})$ *,* $\psi(-1)=0.$

Shuni ta'kidlab o'tamizki, *БС* masalasi F.Trikomi va A.M.Naxushev masalalarining umumlashmasidan iboratdir, ya'ni $c=-1$ *TH* masalasidan A.M.Naxushev masalasi, $c=1$ esa ushbu qo'shimcha shart bajarilganda:

$$a_0(x)(1+x)^{\beta}D_{c,x}^{1-\beta}\psi(0)\Big|_{\substack{x=1\\c=1}}+b_0(x)(1-x)^{\beta}D_{x,1}^{1-\beta}\varphi(1)\Big|_{x=1}=c_0(1)\,, \tag{1.27}$$

F.Trikomi masalasi kelib chiqadi.

Ushbu tenglikka asosan

$$D_{c,x}^{1-\beta}u[\theta_0(x)] = D_{-1,x}^{1-\beta}u[\theta_0(x)] - \frac{1}{\Gamma(\beta)}\frac{d}{dx}\int_{-1}^{c}\frac{u[\theta_0(t)]\,dt}{(x-t)^{1-\beta}}, \quad x\in(c,1)$$

(1.25) chegaraviy shartni ushbu ko‘rinishda yozib olamiz

$$a_0(x)(1+x)^{\beta}D_{-1,x}^{1-\beta}u\left[\theta_0(x)\right] + b_0(x)(1-x)^{\beta}D_{x,1}^{1-\beta}u\left[\theta_1(x)\right] = c_1(x), \quad x\in(c,1) \qquad (1.28)$$

$$c_1(x) = c_0(x) + \frac{\alpha_0(x)}{\Gamma(\beta)}\frac{d}{dx}\int_{-1}^{c}\frac{\psi((t-1)/2)\,dt}{(x-t)^{1-\beta}}.$$

Teorema. $БС$ masalasi bir qiymatli yechimga egadir.

$БС$ masalasi yechimini isbotlashda ekstremum prinsipidan foydalanilgan masla yechimini mavjudligini isbotlashda esa integral tenglamalar usuli qo‘llanilgan.

4-§. Parametr $\beta_0 = -m/2$ bo‘lgan xolda singulyar koeffitsiyentli Gellerstedt tenglamasini o‘rganish.

Singulyar koeffitsiyentli Gellerstedt tenglamasini $\beta_0 = -m/2$ bo‘lgan xolda o‘rganamiz

$$sign y\,|y|^m u_{xx} + u_{yy} - (m/2y)u_y = 0. \qquad (1.29)$$

A_0**- masalasi.** D sohada ushbu shartlarni qanoatlantiruvchi $u(x,y)$, funksiya topilsin:

1. $u(x,y)\in C(\overline{D})$;

2. $u(x,y)\in C^2(D^+)$ va bu sohada (1.29) tenglamani qanoatlantiradi.

3. D^- sohada $u(x,y)$ $R_1(\tau'(x),\nu(x)\in H)$ sinfga tegishli umumlashgan yechim.

4. I intervalda ushbu ulanish sharti bajariladi

$$\lim_{y\to-0}(-y)^{-m/2}\frac{\partial u}{\partial y} = \lim_{y\to+0}y^{-m/2}\frac{\partial u}{\partial y}, \quad x\in I\setminus\{c\}, \qquad (1.30)$$

shu bilan birga bu limitlar $x=\pm1, x=c$ nuktalarda birdan kichik maxsuslikka ega bo‘lishi mumkin.

5. Ushbu shartlar bajariladi

$$u(x,y) = \mu(x)u(x,0) + \rho(x), \quad -1\le x\le 1, \qquad (1.31)$$

$$u|_{AC_0} = \psi(x), \quad -1 \le x \le (c-1)/2 \ , \tag{1.32}$$

$$a_0(x)\frac{d}{dx}u[\theta_0(x)] - b_0(x)\frac{d}{dx}u[\theta_1(x)] = c_0(x), \ \ c \le x \le 1. \tag{1.33}$$

Bu masala xam F.Trikomi, A. M. Naxushev va Bitsadze-Samarskiy masalalarini o‘zida birlashtirgan masaladir.

Trikomi masalasida AC xarakteristikaning barcha nuqtalarida $u(x, y)$ funksiya qiymati beriladi $u(x, y)\big|_{AC} = \psi(x)$. Bu masalada esa xarakteristika ixtiyoriy ravishda ikkiga bo‘linib, bir qismida chegaraviy shart berilgan.

Qo‘yilgan masala yechimining yagonaligi va mavjudligi ham xuddi yuqoridagi masala yechimi kabi isbotlanadi.

II-bob. SOHA ICHIDA BUZILADIGAN SINGULYAR KOEFFITSIYENTLI GIPERBOLIK TIPDAGI TENGLAMA UCHUN NOLOKAL SHARTLI CHEGARAVIY MASALALAR.

§1.1. Soha ichida buziladigan sigulyar koeffitsiyentli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy tipidagi chegaraviy shartli masala

Ushbu soha ichida buziladigan sigulyar koeffitsiyentli giperbolik tipdagi tenglama uchun Bitsadze-Samarskiy tipidagi chegaraviy shartli masalani Γ orqali ifodalaymiz.

Γ masalaning qo‘yilishi.

Ushbu

$$-|y|^m u_{xx} + u_{yy} + \alpha_0 |y|^{m/2-1} u_x + (\beta_0 / y) u_y = 0 \qquad (2.1)$$

tenglamani qaraymiz.

(2.1) tenglamada $m>0$, α_0 va β_0 lar quyidagi shartlarni qanoatlantirsin: $-(m/2) \le \beta_0 < 1,\ 0 \le \alpha_0 < (m+2)/2$.

(2.1) tenglama uchun qo‘yilgan chegaraviy masalalarning korrektligi kichik hadlar oldidagi α_0 va β_0 parametrlarga bog‘liq [43,44,46].

$\alpha_0 o \beta_0$ parametrik tekislikda quyidagi to‘g‘ri chiziqlar bilan chegaralangan $S_0 B_0 C_0$ uchburchakni qaraymiz:

$$S_0B_0 : \beta_0 = 1, B_0C_0 : \beta_0 - \alpha_0 = -(m/2), C_0S_0 : \alpha_0 = 0.$$

Ω – bir bog‘lamli yopiq soha bo‘lib, XOY tekisligida (2.1) tenglamaning

$$\left.\begin{matrix} AC_1 \\ BC_1 \end{matrix}\right\}: \ x \mp \frac{2}{m+2} y^{\frac{m+2}{2}} = \mp 1, \quad y > 0,$$

$$\left.\begin{matrix} AC_2 \\ BC_2 \end{matrix}\right\}: \ x \mp \frac{2}{m+2} (-y)^{\frac{m+2}{2}} = \mp 1, \quad y < 0$$

xarakteristikalari bilan chegaralangan bo‘lsin, bunda $A(-1,0)$, $B(1,0)$.

$P(\alpha_0,\beta_0)\in\Delta S_0B_0C_0$ bo‘lsin va $\alpha\ge 0,\ \beta<1,\ 0\le\alpha+\beta<1,\ \beta<\alpha$ bo‘lib, bunda

$$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\}=\frac{m+2(\beta_0\pm\alpha_0)}{2(m+2)}.$$

Γ masalaning qo‘yilishi. Ω sohada (2.1) tenglamaning $C(\bar\Omega_1\cup\bar\Omega_2)\cap C^2(\Omega\setminus AB)$ sinfga tegishli

$$u(x,y)=\begin{cases}u_1(x,y),\ (x,\ y)\in\Omega_1=\Omega\cap\{y>0\},\\ u_2(x,\ y),\ (x,\ y)\in\Omega_2=\Omega\cap\{y<0\}.\end{cases}$$

va ushbu chegaraviy shartlarni qanoatlantiruvchi

$$u_j\left[\theta^{(j)}(x)\right]=\mu_1u_j\left[\theta^{(j)}_{k_1}(x)\right]+\mu_2u_j\left[\theta^{(j)}_{k_2}(x)\right]+\delta_j(x),\ \forall x\in I=AB,\ j=1,2 \qquad (2.2)$$

bu yerda mos ravishda $j=1$ Ω_1, soha $j=2$ – Ω_2 soha; hamda

$$\lim_{y\to+0}u_1(x,y)=c\lim_{y\to-0}u_2(x,y),\forall x\in[-1,1], \qquad (2.3)$$

$$\lim_{y\to+0}y^{\beta_0}\frac{\partial u_1}{\partial y}=\rho(x)\lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u_2}{\partial y}+\lambda(x),\ \forall x\in(-1,1), \qquad (2.4)$$

ulanish shartlarini qanoatlantiruvchi regulyar yechimi topilsin, bunda $\theta^{(j)}(x)(\theta^{(j)}_{k_1}(x),\ \theta^{(j)}_{k_2}(x))$ – $M(x_0,0)\in I$ nuqtadan chiquvchi va BC_j (Ω_j soha ichida yotuvchi va B nuqtadan chiquvchi $x+\left[2k_j/(m+2)\right]|y|^{(m+2)/2}=1$ chiziqlar) xarakteristikalar kesishish nuqtasi affiksi, $c=const\ne 0;$ $\mu_1,\mu_2=const;$ $\delta_j(x),\ \rho(x),\ \lambda(x)$ – berilgan funksiyalar $C^2(\bar I)\cap C^3(I)$ sinfga tegishli, shunga asosan $\rho(x)-c\ne 0,\ k_1>k_2>1,\ \delta^{(n)}_j(1)=0,\ \lambda^{(n)}(1)=0,\ n=0,1,2.$

$$\theta^{(j)}(x_0)=\frac{1+x_0}{2}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{4}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_1}(x_0)=\frac{1+k_1x_0}{1+k_1}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{2(k_1+1)}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_2}(x_0)=\frac{1+k_2x_0}{1+k_2}+(-1)^{j-1}i\left[\frac{(m+2)(1-x_0)}{2(k_2+1)}\right]^{\frac{2}{m+2}},$$

$c = const \neq 0;\quad \mu_1, \mu_2 = const;\quad \delta_j(x),\ \rho(x),\ \lambda(x)$ zadanniye funksii iz klassa $C^2(\bar{I}) \cap C^3(I)$, prichem $\rho(x) - c \neq 0,\ k_1 > k_2 > 1,\ \delta_j^{(n)}(1) = 0,\ \lambda^{(n)}(1) = 0,\ n = 0,1,2.$

Endi $L(a,b),\ a < b < \infty$ sinfga tegishli ixtiyoriy $f(x)$ ga nisbatan quyidagi kasr tartibli integro-differensial operatorlarni:

$$D_{a,x}^{l} f(x) = \begin{cases} \dfrac{1}{\Gamma(-l)} \displaystyle\int_a^x \dfrac{f(t)\,dt}{(x-t)^{1+l}}, \text{ агар } l < 0, \\ \dfrac{d^{n+1}}{dx^{n+1}} D_{a,x}^{l-(n+1)} f(x), \text{ агар } l > 0, \end{cases} \tag{2.5}$$

$$D_{x,b}^{l} f(x) = \begin{cases} \dfrac{1}{\Gamma(-l)} \displaystyle\int_x^b \dfrac{f(t)\,dt}{(t-x)^{1+l}}, \text{ агар } l < 0, \\ (-1)^{n+1} \dfrac{d^{n+1}}{dx^{n+1}} f(x), \text{ агар } l > 0, \end{cases} \tag{2.6}$$

ushbu ko‘rinishda yozib olamiz, bu yerda $n = [l]$- l ning butun qismi, $D_{a,x}^{l}$ va $D_{x,b}^{l}$ lar l kasr tartibli operator bo‘lib, agar $l < 0,$ bo‘lsa, Liuvill ma’nosida umumlashgan hosila, agar $l > 0$ bo‘lsa, $n = [l] -$ l ning butun qismi.

1. Ω_1 $(y > 0)$ soha uchun (2.2) chegaraviy shartni qaraymiz.

Ω_1 sohada (2.1) tenglama uchun

$$u_1(x,+0) = \tau_1(x),\ x \in \bar{I},\quad \lim_{y \to +0} y^{\beta_0} \frac{\partial u_1}{\partial y} = \nu_1(x), x \in I, \tag{2.7}$$

shakli o‘zgargan Koshi masalasining yechimini beruvchi Darbu formulasiga asosan:

$$\begin{aligned} u_1(x,y) = \gamma_1 \int_{-1}^{1} \tau_1\left[x + \frac{2t}{m+2} y^{\frac{m+2}{2}}\right](1+t)^{\beta-1}(1-t)^{\alpha-1}dt - \\ -\gamma_2 y^{1-\beta_0} \int_{-1}^{1} \nu_1\left[x + \frac{2t}{m+2} y^{\frac{m+2}{2}}\right](1+t)^{-\alpha}(1-t)^{-\beta}\,dt. \end{aligned} \tag{2.8}$$

bu yerda

$$\gamma_1 = \frac{2^{1-\alpha-\beta}\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\ \gamma_2 = -\frac{2^{\alpha+\beta-1}\Gamma(2-\alpha-\beta)}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}.$$

Bundan [23] yengil hisoblashlarga asosan,

$$u_1[\theta^{(1)}(x)] = \gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1 \frac{\tau_1(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}} + \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1 \frac{\nu_1(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}, \tag{2.9}$$

$$u_1[\theta_{k_1}^{(1)}(x)] = \gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\tau_1(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}} + \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\nu_1(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} \tag{2.10}$$

va

$$u_1[\theta_{k_2}^{(1)}(x)] = \gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\tau_1(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}} + \\ +\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\nu_1(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}}, \tag{2.11}$$

bu yerda $a_i = \dfrac{2}{k_i+1}$, $b_i = \dfrac{k_i-1}{k_i+1} = 1-a_i$, $i = 1,2$.

Endi (2.9)-(2.11) larni (2.2) chegaraviy shartga qo‘yib, quyidagilarni hosil qilamiz:

$$\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1 \frac{\tau_1(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}} + \gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1 \frac{\nu_1(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}} =$$

$$= \mu_1\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\tau_1(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\nu_1(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} +$$

$$+\mu_2\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\tau_1(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\nu_1(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}} + \delta_1(x) \tag{2.12}$$

Olingan natijalarga $D_{x,1}^{1-\alpha}$ kasr tartibli differensial operatorni qo‘llab, ushbu aytiyatga ko‘ra

$$D_{x,b}^{\alpha}\ (b-x)^{2\alpha-1}D_{x,b}^{\alpha-1}(b-x)^{-\alpha}\Phi(x)=(b-x)^{\alpha-1}D_{x,b}^{2\alpha-1}\Phi(x), \qquad (2.13)$$

$$D_{x,b}^{-\alpha}\ D_{x,b}^{\alpha}(b-x)^{-\alpha}\Phi(x)=(b-x)^{-\alpha}\Phi(x), \qquad (2.14)$$

quyidagini hosil qilamiz:

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}\tau_1(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\Gamma(1-\alpha)(1-x)^{-\beta}\nu_1(x)=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\tau_1(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\nu_1(s)ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\tau_1(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\nu_1(s)ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_1(x) \qquad (2.15)$$

(2.15) munosabat Ω_1. sohaning I intervalida noma’lum $\tau_1(x)$ va $\nu_1(x)$ funksiyalar orasidagi birinchi funksional munosabat deyiladi.

2. **Endi Ω_2 $(y<0)$.soha uchun (2.2) chegaraviy shartni qaraymiz.**

$$u_2(x,-0)=\tau_2(x), x\in\bar{I}, \quad \lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u_2}{\partial y}=\nu_2(x), x\in I, \qquad (2.16)$$

ко’rinishni o’zgartirgan Koshi shartlarni qanoatlantiruvchi Ω_2,sohadagi (2.1)

tenglamaning yechimi:

$$u_2(x,y)=\gamma_1\int_{-1}^{1}\tau_2\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1+t)^{\beta-1}(1-t)^{\alpha-1}dt-$$
$$-\gamma_2(-y)^{1-\beta_0}\int_{-1}^{1}\nu_2\left[x+\frac{2t}{m+2}(-y)^{\frac{m+2}{2}}\right](1+t)^{-\alpha}(1-t)^{-\beta}dt, \qquad (2.17)$$

Darbu formulasi bilan beriladi.

Bunda

$$\gamma_1 = \frac{2^{1-\alpha-\beta}\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\ \gamma_2 = -\frac{2^{\alpha+\beta-1}\Gamma(2-\alpha-\beta)}{(1-\beta_0)\Gamma(1-\alpha)\Gamma(1-\beta)}.$$

bundan, osonlikcha hisoblash mumkin

$$u_2[\theta^{(2)}(x)] = \gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1 \frac{\tau_2(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}} +$$

$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1 \frac{\nu_2(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}}, \tag{2.18}$$

$$u_2[\theta_{k_1}^{(2)}(x)] = \gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\tau_2(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\nu_2(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} \tag{2.19}$$

va

$$u_2[\theta_{k_2}^{(2)}(x)] = \gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\tau_2(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\nu_2(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}} \tag{2.20}$$

Endi (2.18) – (2.20) ifodalarni (2.2), chegaraviy shartga qo’yamiz

$$\gamma_1 2^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}\int_x^1 \frac{\tau_2(t)dt}{(t-x)^{1-\beta}(1-t)^{1-\alpha}} + \gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^1 \frac{\nu_2(t)dt}{(t-x)^{\alpha}(1-t)^{\beta}} =$$

$$= \mu_1\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_1}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\tau_2(t)dt}{(a_1+b_1x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_1+b_1x} \frac{\nu_2(t)dt}{(a_1+b_1x-t)^{\beta}(t-x)^{\alpha}} +$$

$$+\mu_2\gamma_1(1-x)^{1-\alpha-\beta}\left(\frac{a_2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\tau_2(t)dt}{(a_2+b_2x-t)^{1-\alpha}(t-x)^{1-\beta}} +$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}\int_x^{a_2+b_2x} \frac{\nu_2(t)dt}{(a_2+b_2x-t)^{\beta}(t-x)^{\alpha}} + \delta_1(x) \tag{2.21}$$

Olingan natijaga $D_{x,1}^{1-\alpha}$ [32] kasr differensiallash operatorni qo'llaymiz va (2.13) va (2.14), ayniyatni inobatga olib,quyidagi natijaga erishamiz

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}\tau_2(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\Gamma(1-\alpha)(1-x)^{-\beta}\nu_2(x)=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{\nu_2(s)ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{\tau_2(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{\nu_2(s)ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_2(x) \qquad (2.22)$$

(2.22) munosabat Ω_2.sohadan I ga o'tkazilgan $\tau_2(x)$ va $\nu_2(x)$, nomalum funksiyalar orasidagi ikkinchi fundamental munosabatdir.

Endi (2.15) ifodani (2.3), (2.4) ulanish shartlariga ko'ra, ya'ni: $\tau_1(x)=c\tau_2(x),\ \nu_1(x)=\rho(x)\nu_2(x)+\lambda(x)$ inobatga olib quyidagi ko'rinishga olib kelamiz

$$\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}(1-x)^{-\beta}D_{x,1}^{-\beta}c\tau_2(x)+\gamma_2((m+2)/2)^{1-\alpha-\beta}\times$$
$$\times\Gamma(1-\alpha)(1-x)^{-\beta}(\rho(x)\nu_2(x)+\lambda(x))=$$

$$=\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{c\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_1+b_1x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+$$

$$+\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{c\tau_2(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+$$

$$+\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}D_{x,1}^{1-\alpha}\int\limits_x^{a_2+b_2x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+D_{x,1}^{1-\alpha}\delta_1(x) \qquad (2.23)$$

(2.22) va (2.23) dan $\tau_2(x)$ ni yo'qotsak $\nu_2(x)$ noma'lum funksiya orqali quyidagi integral tenglamani hosil qilamiz:

$$\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\nu_2(x)=\mu_1 D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{(\rho(s)-c)\nu_2(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}+$$
$$+\mu_2 D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{(\rho(s)-c)\nu_2(s)ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+f(x) \qquad (2.24)$$

bunda

$$f(x)=\frac{1}{\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}}D_{x,1}^{1-\alpha}(\delta_1(x)-c\delta_2(x))-\Gamma(1-\alpha)(1-x)^{-\beta}\lambda(x)+$$
$$+\mu_2 D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\lambda(s)ds}{(a_2+b_2x-s)^{\beta}(s-x)^{\alpha}}+\mu_1 D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\lambda(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}.$$

Endi (2.24) tenglamaning o'ng tomonidagi kasrli hosilalarni hisoblaymiz.

$$I_1(x)=D_{x,1}^{1-\alpha}\int_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}=\frac{d}{dx}D_{x,1}^{-\alpha}\int_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1+b_1x-s)^{\beta}(s-x)^{\alpha}}=$$
$$=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^1\frac{dt}{(t-x)^{1-\alpha}}\int_t^{a_1+b_1t}\frac{\nu(s)ds}{(a_1+b_1t-s)^{\beta}(s-t)^{\alpha}} \qquad (2.25)$$

$\nu(x)=(\rho(x)-c)\nu_2(x)$.

Bunda integrallash tartibini o'zgartiramiz

$$I_1(x)=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^1 dt\int_t^{a_1+b_1t}\frac{\nu(s)ds}{(a_1+b_1t-s)^{\beta}(t-x)^{1-\alpha}(s-t)^{\alpha}}=$$
$$=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\left[\int_x^{a_1+b_1x}\nu(s)ds\int_x^s\frac{dt}{(a_1+b_1t-s)^{\beta}(t-x)^{1-\alpha}(s-t)^{\alpha}}+\right.$$
$$\left.+\int_{a_1+b_1x}^1\nu(s)ds\int_{\frac{s-a_1}{b_1}}^s\frac{dt}{(a_1+b_1t-s)^{\beta}(t-x)^{1-\alpha}(s-t)^{\alpha}}\right]=I_1^*(x)+I_1^{**}(x). \qquad (2.26)$$

(2.26) dagi ichki integralni hisoblaymiz

$$I_1^*(x,s)=\int_x^s \frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}, \qquad (2.27)$$

$$I_1^{**}(x,s)=\int_{\frac{s-a_1}{b_1}}^{s} \frac{dt}{\left(a_1+b_1t-s\right)^{\beta}\left(t-x\right)^{1-\alpha}\left(s-t\right)^{\alpha}}. \qquad (2.28)$$

(2.27), integralda $t=s-(s-x)\sigma$, almashtirishni bajaramiz

$$I_1^*(x,s)=\int_0^1 \sigma^{-\alpha}(1-\sigma)^{\alpha-1}\left(1-\frac{b_1(s-x)}{a_1(1-s)}\right)^{-\beta}d\sigma. \qquad (2.29)$$

Bundan Gausning gipergiometrik funksiyaning integral ko'rinishi bilan foydalanamiz:

$$\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-xt)^{-b}dt=\frac{\Gamma(a)\Gamma(c-a)}{\Gamma(c)}F(a,b,c;x), \qquad (2.30)$$

bundan

$$I_1^*(x,s)=\left(a_1(1-s)\right)^{-\beta}\Gamma(1-\alpha)\Gamma(\alpha)F\left(1-\alpha,\beta,1;\frac{b_1(s-x)}{a_1(1-s)}\right). \qquad (2.31)$$

(2.28) integralda $t=s-\left(s-\frac{s-a_1}{b_1}\right)\sigma$, almashtirish bajaramiz va quyidagi natijaga erishamiz

$$I_1^{**}(x,s)=\left(a_1(1-s)\right)^{-\beta}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\int_0^1 \sigma^{-\alpha}(1-\sigma)^{-\beta}\left(1-\frac{a_1(1-s)}{b_1(s-x)}\right)^{\alpha-1}d\sigma \qquad (2.32)$$

(2.30) ni inobatga olib **Ошибка! Источник ссылки не найден.** dan

$$I_1^{**}(x,s)=\left(a_1(1-s)\right)^{-\beta}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)}\times$$
$$\times F\left(1-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right). \qquad (2.33)$$

(2.31) va (2.33) larga ko'ra (2.26) ni quyidagi ko'rinishda yozib olamiz

$$I_1(x)=\frac{1}{\Gamma(\alpha)}\frac{d}{dx}\int_x^{a_1+b_1x}\frac{\nu(s)}{(a_1(1-s))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(s-x)}{a_1(1-s)}\right)+$$

$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)}\frac{d}{dx}\int_{a_1+b_1x}^{1}\frac{\nu(s)}{(a_1(1-s))^{\beta}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\times$$

$$\times F\left(1-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right). \quad (2.34)$$

Endi

$$\frac{d}{dx}F(a,b,c;x)=\frac{ab}{c}F(a+1,b+1,c+1;x), \quad (2.35)$$

$$\frac{d}{dx}x^{a}F(a,b,c;x)=ax^{a-1}F(a+1,b,c;x), \quad (2.36)$$

formulaga ko’ra, (2.34) da hosilani hisoblaymiz

$$I_1(x)=\Gamma(1-\alpha)\left[\frac{\nu(a_1+b_1x)}{(a_1(1-a_1-b_1x))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(a_1+b_1x-x)}{a_1(1-a_1-b_1x)}\right)b_1-\right.$$

$$-\frac{\nu(x)}{(a_1(1-x))^{\beta}}F\left(1-\alpha,\beta,1;\frac{b_1(x-x)}{a_1(1-x)}\right)+$$

$$\left.+b_1\int_{x}^{a_1+b_1x}\frac{\nu(s)}{(a_1(1-s))^{\beta}}\frac{\beta(1-\alpha)}{1}F\left(2-\alpha,1+\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)\frac{ds}{a_1(1-s)}\right]+$$

$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)}\left[-\frac{\nu(a_1+b_1x)}{(a_1(1-a_1-b_1x))^{\beta}}\left(\frac{a_1(1-a_1-b_1x)}{b_1(a_1+b_1x-x)}\right)^{1-\alpha}\times\right.$$

$$\times F\left(1-\alpha,\beta,1;\frac{a_1(1-a_1-b_1x)}{b_1(a_1+b_1x-x)}\right)b_1+\int_{a_1+b_1x}^{1}\frac{(1-\alpha)\nu(s)}{(a_1(1-s))^{\beta}}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\alpha}\times$$

$$\left.\times F\left(2-\alpha,1-\alpha,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right)\frac{1}{s-x}\right].$$

Quyidagi

$$F(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad (2.38)$$

$$F(a,b,c;0)=1, \quad (2.39)$$

$$F(a,b,c;x)=(1-x)^{c-a-b}F(c-a,c-b,c;x), \quad (2.40)$$

formulalar yordamida (2.37) formulani quyidagi ko’rinishga keltiramiz

$$I_1(x) = -\Gamma(1-\alpha)(1-x)^{-\beta}\nu(x) -$$

$$-\mu_1\Gamma(1-\alpha)\beta(1-\alpha)\int_x^{a_1+b_1x}\frac{\nu(s)ds}{(a_1(1-s))^\alpha}\frac{b_1}{(a_1+b_1x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)+$$

$$+\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\beta(1-\alpha)\int_{a_1+b_1x}^{1}\frac{\nu(s)ds}{\left(a_1(1-s)\right)^\alpha}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\beta}\frac{1}{\left(s-a_1-b_1x\right)^{1+\beta-\alpha}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right) \qquad (2.41)$$

Shunga o’xshash, ravshanki

$$I_2(x) = D_{x,1}^{1-\alpha}\int_x^{a_2+b_2x}\frac{\nu(s)ds}{\left(a_2+b_2x-s\right)^\beta\left(s-x\right)^\alpha} = \Gamma\left(1-\alpha\right)\left(a_2\left(1-x\right)\right)^{-1}\nu\left(x\right)+$$

$$+\Gamma(1-\alpha)\beta(1-\alpha)\int_x^{a_2+b_2x}\frac{\nu(s)ds}{(a_2(1-s))^\alpha}\frac{b_2}{(a_2+b_2x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_2(s-x)}{a_2(1-s)}\right)-$$

$$-\frac{\Gamma(1-\alpha)\Gamma(1-\beta)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\beta(1-\alpha)\int_{a_2+b_2x}^{1}\frac{\nu(s)ds}{\left(a_2(1-s)\right)^\alpha}\left(\frac{a_2(1-s)}{b_2(s-x)}\right)^{1-\beta}\frac{1}{\left(s-a_2-b_2x\right)^{1+\beta-\alpha}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_2(1-s)}{b_2(s-x)}\right). \qquad (2.42)$$

Endi $I_1(x)$ va $I_2(x)$ lar uchun ,mos ravishda (2.41) va (2.42) dan (2.24) ga qo'ysak quyidagi natijaga erishamiz

$$\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\nu_2(x) = I(x)+f(x), \qquad (2.43)$$

bunda

$$I(x) = I_1(x)+I_2(x) =$$

$$= \Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\mu_1 a_1^{-\beta}\nu_2(x)+\mu_1\Gamma(1-\alpha)\beta b_1(1-\alpha)\times$$

$$\times\int_x^{a_1+b_1x}\frac{(\rho(s)-c)\nu_2(s)}{(a_1(1-s))^\alpha(a_1+b_1x-s)^{1-\alpha+\beta}}F\left(\alpha,1-\beta,2;\frac{b_1(s-x)}{a_1(1-s)}\right)ds-$$

$$-\mu_1\frac{\Gamma(1-\beta)\Gamma(1-\alpha)(1-\alpha)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\int_{a_1+b_1x}^{1}\frac{(\rho(s)-c)\nu_2(s)}{(a_1(1-s))^\alpha}\left(\frac{a_1(1-s)}{b_1(s-x)}\right)^{1-\beta}\frac{1}{(s-a_1-b_1x)^{1-\alpha+\beta}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_1(1-s)}{b_1(s-x)}\right)ds+\Gamma(1-\alpha)(\rho(x)-c)(1-x)^{-\beta}\mu_2 a_2^{-\beta}\nu_2(x)+$$

$$+\mu_2\Gamma(1-\alpha)\beta b_2(1-\alpha)\int_x^{a_2+b_2x}\frac{(\rho(s)-c)\nu_2(s)}{(a_2(1-s))^{\alpha}(a_2+b_2x-s)^{1-\alpha+\beta}}\times$$

$$\times F\left(\alpha,1-\beta,2;\frac{b_2(s-x)}{a_2(1-s)}\right)ds-\mu_2\frac{\Gamma(1-\beta)\Gamma(1-\alpha)(1-\alpha)}{\Gamma(2-\alpha-\beta)\Gamma(\alpha)}\times \tag{2.44}$$

$$\times\int_{a_2+b_2x}^{1}\frac{(\rho(s)-c)\nu_2(s)}{(a_2(1-s))^{\alpha}}\left(\frac{a_2(1-s)}{b_2(s-x)}\right)^{1-\beta}\frac{1}{(s-a_2-b_2x)^{1-\alpha+\beta}}\times$$

$$\times F\left(-\beta,1-\beta,2-\alpha-\beta;\frac{a_2(1-s)}{b_2(s-x)}\right)ds,$$

(2.44) ga asoslanib (2.43) tenglamani quyidagi ko'rinishda yozish mumkin

$$\nu_2(x)=\int_x^1\frac{K_1(x,s)\nu_2(s)ds}{|s-a_1-b_1x|^{\ell}}+\int_x^1\frac{K_2(x,s)\nu_2(s)ds}{|s-a_2-b_2x|^{\ell}}+f(x),\ \ \forall x\in I \tag{2.45}$$

bunda $\ell=1-\alpha+\beta<1$,

$$K_k(x,s)=\begin{cases}\dfrac{\mu_k\beta(1-\alpha)(1-x)^{\beta}b_k(\rho(s)-c)}{(a_k(1-s))^{\alpha}(\rho(x)-c)(1-a_k^{-\beta}\mu_1-a_k^{-\beta}\mu_2)}\times\\ \times F(\alpha,1-\beta,2;b_k(s-x)/a_k(1-s)),\quad x\le s\le a_k+b_kx,\\ -\dfrac{\mu_k\Gamma(1-\beta)(1-\alpha)(a_k(1-s))^{1-\beta-\alpha}(1-x)^{\beta}(\rho(s)-c)}{(b_k(s-x))^{1-\beta}\Gamma(2-\alpha-\beta)\Gamma(\alpha)(\rho(x)-c)(1-a_k^{-\beta}\mu_1-a_k^{-\beta}\mu_2)}\times\\ \times F(-\beta,1-\beta,2-\alpha-\beta;a_k(1-s)/b_k(s-x)),\\ a_k+b_kx\le s\le 1.\end{cases} \tag{2.46}$$

Shu bilan birgalikda

$$K_k(x,\,a_k+b_kx-0)=\frac{\mu_k(1-x)^{\beta-\alpha}\Gamma(1-\alpha+\beta)}{a_k^{\alpha}b_k^{\alpha}\Gamma(1-\alpha)\Gamma(\beta)},\quad k=1,2, \tag{2.47}$$

$$K_k(x,a_k+b_kx+0)=-\frac{\mu_k(1-x)^{\beta-\alpha}\Gamma(1-\beta)\Gamma(1-\alpha+\beta)}{a_k^{\alpha}b_k^{\alpha}\Gamma^2(1-\alpha)\Gamma(\alpha)},\quad k=1,2. \tag{2.48}$$

Quyidagi formula o'rinlidir.

Lemma. Funksiya $K_k(x,t)\in C^2(\bar{I})\setminus\gamma_{k,}$ $k=1,2$, bu funksiya $\gamma_k : t=a_k+b_k x,$ egri chiziqda birinchi tur uzilishga ega,(1,1) nuqtadan tashqari,bu nuqtada ular $\alpha-\beta$ tartibdagi maxsuslikga ega. Bu lemmaning isboti (2.47) dan darxol kelib chiqadi.

(2.45) integral tenglamani o'rganish. (2.45) tenglama shu bilan qiziqarliki,u a_1+b_1x va a_2+b_2x chiziqda kichik yadroga ega $x<a_1+b_1x<a_2+b_2x$, bunda $0<a_k$, $b_k<1$, $a_k+b_k=1$, $k=1,2$.

Qulaylik uchun $\nu_2(x)=\nu(x)$ deb olamiz.

(2.45) tenglamani yechish uchun ketma-ket yaqinlashish usuli bilan foydalanamiz [35]. Funksional ketma–ketlikning $\nu_0(x), \nu_1(x), \ldots, \nu_n(x), \ldots$ hadlarni topish uchun quyidagi rekurrent munosabatni tuzamiz

$$\nu_n(x)=\int_x^1\left(\frac{\mu_1K_1(x,t)}{|t-a_1-b_1x|^{\ell}}+\frac{\mu_2K_2(x,t)}{|t-a_2-b_2x|^{\ell}}\right)\nu_{n-1}(t)dt+f(x). \qquad (2.49)$$

Nol yaqinlashish sifatida $\vartheta_0(x)=f(x)$ ni olamiz. Bu holda birinchi yaqinlashish, quyidagi ko'rinishga ega

$$\nu_1(x)=\int_x^1\left(\frac{\mu_1K_1(x,t)}{|t-a_1-b_1x|^{\ell}}+\frac{\mu_2K_2(x,t)}{|t-a_2-b_2x|^{\ell}}\right)\nu_0(t)dt+f(x). \qquad (2.50)$$

Bundan osonlikcha quyidagi baholashni topamiz:

$$|\nu_1(x)-\nu_0(x)|\le|\mu_1|\int_x^1\frac{|K_1(x,t)||f(x)|dt}{|t-a_1-b_1x|^{\ell}}+|\mu_2|\int_x^1\frac{|K_2(x,t)||f(x)|dt}{|t-a_2-b_2x|^{\ell}}\le$$

$$\le|\mu_1|K_1M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{|t-a_1-b_1x|^{\ell}}+|\mu_2|K_2M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{|t-a_2-b_2x|^{\ell}}=$$

$$=|\mu_1|K_1M(1-x)^{\beta-\alpha}\left(\int_x^{a_1+b_1x}\frac{dt}{(a_1+b_1x-t)^{\ell}}+\int_{a_1+b_1x}^1\frac{dt}{(t-a_1-b_1x)^{\ell}}\right)+$$

$$+|\mu_2|K_2M(1-x)^{\beta-\alpha}\left(\int_x^{a_2+b_2x}\frac{dt}{(a_2+b_2x-t)^{\ell}}+\int_{a_2+b_2x}^1\frac{dt}{(t-a_2-b_2x)^{\ell}}\right)=$$

$$=|\mu_1|K_1M(1-x)^{\beta-\alpha}2^{1-\ell}\frac{a_1^{1-\ell}+b_1^{1-\ell}}{1-\ell}+|\mu_2|K_2M(1-x)^{\beta-\alpha}2^{1-\ell}\frac{a_2^{1-\ell}+b_2^{1-\ell}}{1-\ell},$$

где $\left|(1-x)^{\alpha-\beta}K_k(x,t)\right|\le K_k$, $|f(x)|\le M$.

Shunday qilib,

$$|\nu_1(x)-\nu_0(x)|\le M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right). \qquad (2.51)$$

Endi ikkinchi yaqinlashishni qo'llaymiz , (2.49) dan $n=2$ bo'lganida

$$\nu_2(x)=\int_x^1\left(\frac{\mu_1K_1(x,t)}{|t-a_1-b_1x|^{\ell}}+\frac{\mu_2K_2(x,t)}{|t-a_2-b_2x|^{\ell}}\right)\nu_1(t)dt+f(x) \qquad (2.52)$$

(2.49) dan (2.50) ni ayirsak

$$|\nu_2(x)-\nu_1(x)|\le\int_x^1\left(\frac{|\mu_1||K_1(x,t)|}{|t-a_1-b_1x|^{\ell}}+\frac{|\mu_2||K_2(x,t)|}{|t-a_2-b_2x|^{\ell}}\right)|\nu_1(x)-\nu_0(x)|dt.$$

Bundan, (2.51) ni inobatga olsak

$$|\nu_2(x)-\nu_1(x)|\le M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)\times$$

$$\times\left(|\mu_1|K_1M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{|t-a_1-b_1x|^{\ell}}+|\mu_2|K_2M(1-x)^{\beta-\alpha}\int_x^1\frac{dt}{|t-a_1-b_1x|^{\ell}}\right)=$$

$$=M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)^2 \qquad (2.53)$$

Bu amalni davom ettirsak

$$|\nu_n(x)-\nu_{n-1}(x)|\le M\left(\frac{|\mu_1|K_1\left(a_1^{1-\ell}+b_1^{1-\ell}\right)+|\mu_2|K_2\left(a_2^{1-\ell}+b_2^{1-\ell}\right)}{1-\ell}\right)^n. \qquad (2.54)$$

Funksional qator

$$\nu_0(x)+(\nu_1(x)-\nu_0(x))+(\nu_2(x)-\nu_1(x))+...+(\nu_n(x)-\nu_{n-1}(x))+... \qquad (2.55)$$

sonli qator bilan mojoranta bo'ladi.

$$M\sum_{k=0}^{\infty}\left(\frac{|\mu_1|K_1(a_1^{1-\ell}+b_1^{1-\ell})+|\mu_2|K_2(a_2^{1-\ell}+b_2^{1-\ell})}{1-\ell}\right)^n \qquad (2.56)$$

Shunday qilib,

$$\left|\frac{|\mu_1|K_1(a_1^{1-\ell}+b_1^{1-\ell})+|\mu_2|K_2(a_2^{1-\ell}+b_2^{1-\ell})}{1-\ell}\right|<1 \qquad (2.57)$$

shart bajarilganida tekis yaqinlashadi, va uning yig'indisi uzluksiz funksiyadir.

Aytib o'tish kerakki (2.45), bir jinsli integral tenglama notrivial yechimga ega. Haqiqatdan ham

$$\nu(x)=\sum_{k=1}^{2}\int_x^1\frac{K_k(x,t)\nu(t)dt}{|t-a_k-b_kx|^{1-\alpha+\beta}} \qquad (2.58)$$

Bir jinsli tenglamani qaraylik yoki

$$\nu(x)=\sum_{k=1}^{2}\int_x^{a_k+b_kx}\frac{\mu_k\beta(1-\alpha)(1-x)^{\beta}b_kF(\alpha,1-\beta,2;b_k(t-x)/a_k(1-t))\nu(t)dt}{a_k^{\alpha}(1-t)^{\alpha}(a_k+b_kx-t)^{1-\alpha+\beta}}+$$

$$+\sum_{k=1}^{2}\int_{a_k+b_kx}^{1}\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}(1-t)^{1-\alpha-\beta}(1-x)^{\beta}}{b_k^{1-\beta}(t-x)^{1-\beta}\Gamma(2-\alpha-\beta)\Gamma(\alpha)(t-a_k-b_k)^{1-\alpha+\beta}}\times \qquad (2.59)$$

$$\times F(-\beta,1-\beta,2-\alpha-\beta;a_k(1-t)/b_k(t-x))\nu(t)dt.$$

(2.59) ning o'ng tomonidagi birinchi yig'indida $t=x+(a_k+b_kx-x)\sigma=x+a_k(1-x)\sigma$, almashtirish,ikkinchi yig'indiga esa $t=a_k+b_kx+(1-a_k-b_kx)\sigma=a_k+b_kx+b_k(1-x)\sigma.$ almashtirish bajaramiz.

Bunday almashtirish yordamida (2.59) tenglamani quyidagi ko'rinishga olib kelamiz

$$\nu(x)=\sum_{k=1}^{2}\int_0^1\frac{\mu_k\beta(1-\alpha)F(\alpha,1-\beta,2;b_k\sigma/(1-a_k\sigma))}{a_k^{\beta}(1-a_k\sigma)^{\alpha}(1-\sigma)^{1-\alpha+\beta}}\nu\left[x+a_k(1-x)\sigma\right]d\sigma+$$

$$+\sum_{k=1}^{2}\int_0^1\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}b_k^{-\beta}\nu\left[a_k+b_kx+b_k(1-x)\sigma\right]d\sigma}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)\sigma^{1-\alpha+\beta}(a_k+b_k\sigma)^{1-\beta}}\times \qquad (2.60)$$

$$\times F(-\beta,1-\beta,2-\alpha-\beta;(1-\sigma)/(a_k+b_k\sigma))d\sigma.$$

(2.60) dan ko'rinib turibdiki, $\nu(x)=(1-x)^{\lambda}$, (bu yerda $\lambda>0$) μ_k larning yetarlicha kichik qiymatlarida (2.60) tenglamaning notrivial yechimidir.

Haqiqatdan ham

$$\nu\left[x+a_k(1-x)\sigma\right]=(1-x-a_k(1-x)\sigma)^{\lambda}=(1-x)^{\lambda}(1-a_k\sigma)^{\lambda}$$

$$\nu\left[a_k+b_kx+b_k(1-x)\sigma\right]=(b_k(1-x)(1-\sigma))^{\lambda}.$$

Bu holda (2.60) dan

$$\begin{aligned}(1-x)^{\lambda}=\sum_{k=1}^{2}\int_0^1\frac{\mu_k\beta(1-\alpha)F(\alpha,1-\beta,2;b_k\sigma/(1-a_k\sigma))}{a_k^{\beta}(1-\alpha_k\sigma)^{\alpha}(1-\sigma)^{1-\alpha+\beta}}\times\\ \times(1-x)^{\lambda}(1-a_k\sigma)^{\lambda}d\sigma+\\ +\sum_{k=1}^{2}\int_0^1\frac{\mu_k\Gamma(1-\beta)(1-\alpha)a_k^{1-\alpha-\beta}b_k^{-\beta}b_k^{\lambda}(1-x)^{\lambda}(1-\sigma)^{\lambda}}{\Gamma(\alpha)\Gamma(2-\alpha-\beta)\sigma^{1-\alpha+\beta}(a_k+b_k\sigma)^{1-\beta}}\times\\ \times F(-\beta,1-\beta,2-\alpha-\beta;(1-\sigma)/(a_k+b_k\sigma))d\sigma.\end{aligned}\quad(2.61)$$

Shunday qilib, ixtiyoriy $\lambda>0$ da (2.60) tenglik bajarilishi uchun μ_1 va μ_2 tenglash mumkin.

Bundan esa, (2.61) yechilishi uchun (2.57) shart juda muhim ekan.

$\nu_2(x)$ nomalum funksiya topilganidan so'ng, (2.23) tenglamani $\tau_2(x)$ noma'lum funksiyaga nisbatan ushbu integral tenglamaga keltiramiz

$$\begin{aligned}c\gamma_1\Gamma(\beta)2^{\alpha+\beta-1}\tau_2(x)=\\ =\mu_1\gamma_1\left(\frac{a_1}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha}D_{x,1}^{1-\alpha+\beta}\int_x^{a_1+b_1x}\frac{c\tau_2(s)ds}{\left(a_1+b_1x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+\\ +\mu_2\gamma_1\left(\frac{a_2}{2}\right)^{\alpha+\beta-1}(1-x)^{1-\alpha}D_{x,1}^{1-\alpha+\beta}\int_x^{a_2+b_2x}\frac{c\tau_2(s)ds}{\left(a_2+b_2x-s\right)^{1-\alpha}\left(s-x\right)^{1-\beta}}+F(x),\end{aligned}\quad(2.62)$$

Bunda

$$\begin{aligned}F(x)=(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\delta_1(x)-\gamma_2((m+2)/2)^{1-\alpha-\beta}\times\\ \times\Gamma(1-\alpha)D_{x,1}^{\beta}(\rho(x)\nu_2(x)+\lambda(x))+\\ +\mu_1\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\int_x^{a_1+b_1x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_1+b_1x-s\right)^{\beta}\left(s-x\right)^{\alpha}}+\\ +\mu_2\gamma_2\left(\frac{m+2}{2}\right)^{1-\alpha-\beta}(1-x)^{\beta}D_{x,1}^{1-\alpha+\beta}\int_x^{a_2+b_2x}\frac{(\rho(s)\nu_2(s)+\lambda(s))ds}{\left(a_2+b_2x-s\right)^{\beta}\left(s-x\right)^{\alpha}},\end{aligned}\quad(2.63)$$

bu yerda $0<1-\alpha+\beta<1.$

(2.62) tenglamani o'rganish (2.45) tenglamani o'rganish bilan bir xildir.

Teorema 1.1.1. Γ masala yetarlicha kichik μ_1 *va* μ_2 lar uchun (2.57) shart bajarilsa, yechimga ega bo'ladi.

2-§. Soha ichida buziladigan singulyar koeffitsentli giperbolik tipdagi tenglamalar uchun soha ichida yotuvchi 2 ta maxsus chiziqdagi yechimning xarakteristikadagi qiymati bilan bog'laydigan Bitsadze-Samarskiy shartli masala.

Endi soha ichida buziladigan singulyar koeffitsentli giperbolik tipdagi tenglamalar uchun soha ichida yotuvchi 2 ta maxsus chiziqdagi yechimning xarakteristikadagi qiymati bilan bog'laydigan Bitsadze-Samarskiy shartli masalani parametr bo'lgan hol uchun o'rganamiz.

Quyidagi

$$-|y|^m u_{xx} + u_{yy} - \frac{m}{2y} u_y = 0, \quad m > 0. \qquad (2.64)$$

tenglamani o'rganamiz. Ω soha kompleks x, y o'zgaruvchili tekislikning bir bog'lamli yopiq sohasi bo'lib, (2.64) tenglamaning

$$\left.\begin{matrix} AC_1 \\ BC_1 \end{matrix}\right\}: \quad x \mp \frac{2}{m+2} y^{\frac{m+2}{2}} = \mp 1, \quad y > 0,$$

$$\left.\begin{matrix} BC_1 \\ BC_2 \end{matrix}\right\}: x \mp \frac{2}{m+2} (-y)^{\frac{m+2}{2}} = \mp 1, \; y < 0,$$

xarakteristikalari bilan chegaralangan bo'lsin.

Γ **masalaning qo'yilishi.** Ω sohada (2.64) tenglamaning $C(\bar{\Omega}_1 \cup \bar{\Omega}_2) \cap C^2(\Omega \setminus AB)$ sinfga tegishli va quyidagi

$$u_j[\theta^{(j)}(x)] = \mu_1 u_j[\theta_{k_1}^{(j)}(x)] + \mu_2 u_j[\theta_{k_2}^{(j)}(x)] +$$

$$+ \frac{1}{2}\mu_1 u_j(p_1(x), 0) - \frac{1}{2}\mu_2 u_j(p_2(x), 0) + \delta_j(x), \; \forall x \in I = AB \qquad (2.65)$$

bu yerda $p_1(x)=a_1+b_1x,\ p_2(x)=a_2+b_2x,$ bunda $a_i=\dfrac{2}{k_i+1},\ b_i=\dfrac{k_i-1}{k_i+1},\ i=1,2$

chegaraviy shartlarni qanoatlantiruvchi

$$u(x,y)=\begin{cases} u_1(x,y),\ (x,\ y)\in\Omega_1=\Omega\cap\{y>0\}, \\ u_2(x,\ y),\ (x,\ y)\in\Omega_2=\Omega\cap\{y<0\}, \end{cases}$$

bunda $j=1$ Ω_1, sohaga tegishli, a $j=2$ Ω_2 sohaga tegishli,

regulyar yechimi topilsin va [16]

$$\lim_{y\to+0}u_1(x,y)=c\lim_{y\to-0}u_2(x,y),\forall x\in\bar{I}, \tag{2.66}$$

ulanish shartlari

$$\lim_{y\to+0}y^{\beta_0}\frac{\partial u_1}{\partial y}=\rho(x)\lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u_2}{\partial y}+\lambda(x),\ \forall x\in I, \tag{2.67}$$

bunda $\theta^{(j)}(x)-\ M(x_0,0)\in I$ nuqtadan chiqqan va BC_j xarakteristikalar kesishgan nuqtaning affiksi, $\theta^{(j)}_{k_1}(x),\theta^{(j)}_{k_2}(x)-$lar $x+\left[2k_j/(m+2)\right]|y|^{(m+2)/2}=1$ egri chiziq bilan $M(x_0,0)\in I$ nuqtadan chiquvchi xarakteristikalar kesishgan nuqtaning affikslari

$$\lim_{y\to+0}u_1(x,y)=c\lim_{y\to-0}u_2(x,y),\ \forall x\in\bar{I} \tag{2.68}$$

$$\lim_{y\to+0}y^{-\frac{m}{2}}\frac{\partial u_1}{\partial y}=\rho(x)\lim_{y\to-0}(-y)^{-\frac{m}{2}}\frac{\partial u_2}{\partial y}+\lambda(x),\quad \forall x\in I \tag{2.69}$$

bu yerda $\theta^{(j)}(x)(\theta^{(j)}_{k_1}(x),\ \theta^{(j)}_{k_2}(x))$

$$\theta^{(j)}(x_0)=\frac{1+x_0}{2}+(-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{4}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_1}(x_0)=\frac{1+k_1x_0}{1+k_1}+(-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{2(k_1+1)}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_2}(x_0)=\frac{1+k_2x_0}{1+k_2}+(-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{2(k_2+1)}\right]^{\frac{2}{m+2}}$$

lar BC_j *xarakteristika bilan* $M(x_0,0)\in I;$ *nuqtafan chiquvchi xarakteristikalar kesishish nuqtalari hamda* Ω_j *sohada yotuvchi* $x+\left[2k_j/(m+2)\right]|y|^{(m+2)/2}=1,$ *maxsus chiziq bilan kesishuvchi nuqtalar affukslari, hamda* $c=const;$ $\mu_1,\mu_2=const;$ $\delta_j(x),\ \rho(x),\ \lambda(x)$ *berilgan funksiyalar ushbu* $C^2(\bar{I})\cap C^3(I),$ *tegishli, bunda*

$$\tau(1)=\tau'(1)=\tau''(1)=0 \tag{2.70}$$

$\rho(x)-c\neq 0,\ k_1>k_2>1,\ \delta_j^{(n)}(1)=0,\ \lambda^{(n)}(1)=0,\ n=0,1,2.$

1. Γ masala yechimini izlash.

Теорема. *Agar*

$$\lambda_1+\lambda_2<1, \tag{2.71}$$

shart bajarilsa, u holda Γ masala yechimi birqiymatli bajariladi, bunda

$\lambda_k=\dfrac{b_k\mu_k}{b_1\mu_1+b_2\mu_2-1}>0,\ k=1,2.$

Isbot. 1. (2.65) chegaraviy shartlarni qaraymiz. Ω_1 sohada shakli o'zgargan

$$u_1(x,+0)=\tau_1(x),\ x\in\bar{I};\quad \lim_{y\to+0} y^{-\frac{m}{2}}\frac{\partial u_1}{\partial y}=\nu_1(x),\ x\in I \tag{2.72}$$

Koshi masalasi yechimini beruvchi

$$u(x,y)=\frac{\tau\left(x-\frac{2}{m+2}(-y)^{(m+2)/2}\right)+\tau\left(x+\frac{2}{m+2}(-y)^{(m+2)/2}\right)}{2}-$$
$$-\frac{(-y)^{(m+2)/2}}{m+2}\int_{-1}^{1}\nu\left[x+\frac{2t}{m+2}(-y)^{(m+2)/2}\right]dt, \tag{2.73}$$

Dalamber formulasidan foydalanamiz.

Bundan esa sodda hisoblashlarni bajarib, [23]

$$u_1\left[\theta^{(1)}(x)\right]=\frac{1}{2}\left[\tau_1(x)+\tau_1(1)\right]-\frac{1}{2}\int_{-1}^{x}\nu_1(t)dt\,, \tag{2.74}$$

$$u_1\left[\theta_{k_1}^{(1)}(x)\right]=\frac{1}{2}\left[\tau_1(a_1+b_1x)+\tau_1(x)\right]-\frac{1}{2}\int_{x}^{a_1+b_1x}\nu_1(t)dt\,, \tag{2.75}$$

$$u_1\left[\theta_{k_2}^{(1)}(x)\right]=\frac{1}{2}\left[\tau_1(a_2+b_2x)+\tau_1(x)\right]-\frac{1}{2}\int\limits_{x}^{a_2+b_2x}\nu_1(t)dt\,. \tag{2.76}$$

munosabatlarni olamiz

Endi olingan (2.74)-(2.76) munosabatlarni (2.65) chegaraviy shartga qo'yib, quyidagi

$$\tau_1(x)-\int\limits_{x}^{1}\nu_1(t)dt=\mu_1\tau_1(x)+\mu_2\tau_1(x)-\mu_1\int\limits_{x}^{a_1+b_1x}\nu_1(t)dt+\mu_2\int\limits_{x}^{a_2+b_2x}\nu_1(t)dt+2\delta_1(x)\,. \tag{2.77}$$

munosabatlarga ega bo'lamiz.

(2.77) munosabat Ω_1 sohaning $y=0$ buzilish chizig'i kesmasida noma'lum $\tau_1(x)$ va $\nu_1(x)$ funksiyalar orasidagi birinchi funsional munosabat deyiladi.

2. Endi Ω_1 sohada shakli o'zgargan

$$u_2(x,-0)=\tau_2(x),\ x\in\bar{I};\quad \lim_{y\to-0}(-y)^{-\frac{m}{2}}\frac{\partial u_2}{\partial y}=\nu_2(x),\ x\in I \tag{2.78}$$

Koshi masalasi yechimini beruvchi Dalamber formulasidan foydalanib, ayrim sodda hisoblashlarga ko'ra

$$u_2\left[\theta^{(2)}(x)\right]=\frac{1}{2}\left[\tau_2(x)+\tau_2(1)\right]-\frac{1}{2}\int\limits_{-1}^{x}\nu_2(t)dt\,, \tag{2.79}$$

$$u_2\left[\theta_{k_1}^{(2)}(x)\right]=\frac{1}{2}\left[\tau_2(a_1+b_1x)+\tau_2(x)\right]-\frac{1}{2}\int\limits_{x}^{a_1+b_1x}\nu_2(t)dt\,, \tag{2.80}$$

$$u_2\left[\theta_{k_2}^{(2)}(x)\right]=\frac{1}{2}\left[\tau_2(a_2+b_2x)+\tau_2(x)\right]-\frac{1}{2}\int\limits_{x}^{a_2+b_2x}\nu_2(t)dt\,. \tag{2.81}$$

munosabatlarni hosil qilamiz.

(2.79) - (2.81) ifodalarni chegaraviy shartlarga (2.65) qo'yib, biz olamiz

$$\tau_2(x)-\int\limits_{x}^{1}\nu_2(t)dt=\mu_1\tau_2(x)+\mu_2\tau_2(x)-\mu_1\int\limits_{x}^{a_1+b_1x}\nu_2(t)dt+\mu_2\int\limits_{x}^{a_2+b_2x}\nu_2(t)dt+2\delta_2(x)\,. \tag{2.82}$$

(2.82) munosabat Ω_2 sohaning $y=0$ o'qidagi noma'lum $\tau_2(x)$ va $\nu_2(x)$ funktsiyalar orasidagi ikkinchi funktsional munosabatlar deyiladi.

(2.77) munosabatdagi $\tau_2(x)$ ni $\tau_1(x) = c\tau_2(x)$, $\nu_1(x) = \rho(x)\nu_2(x) + \lambda(x)$ ayniyatga ko'ra bog'lasak, hamda (2.77) va (2.81) munosabatlardan mos ravishda (2.66), (2.67) chegaraviy shartlarga asosan $\nu_2(x)$ noma'lum funksiyaga bog'lik navbatdagi integral tenglamani hosil qilamiz:

$$\int_x^1 (\rho(t) - c)\nu_2(t)dt = \mu_1 \int_x^{a_1+b_1x} (\rho(t) - c)\nu_2(t)dt + \mu_2 \int_x^{a_2+b_2} (\rho(t) - c)\nu_2(t)dt + f(t) \qquad (2.83)$$

bu yerda

$$f(x) = 2\delta_1(x) - 2c\delta_2(x) + \int_x^1 \lambda(t)dt + \mu_1 \int_x^{a_1+b_1x} \lambda(t)dt + \mu_2 \int_x^{a_2+b_2x} \lambda(t)dt .$$

(2.83) munosabatni x bo'yicha differensiallab, quyidagini hosil qilamiz:

$$\nu(x) = \lambda_1(x)\nu(a_1 + b_1x) + \lambda_2(x)\nu(a_2 + b_2x) + f_1(x) \qquad (2.84)$$

bunda

$$\nu(x) = (\rho(x) - c)\nu_2(x),$$

$$\lambda_1(x) = \frac{b_1\mu_1}{b_1\mu_1 + b_2\mu_2 - 1}, \quad \lambda_2(x) = \frac{b_2\mu_2}{b_1\mu_1 + b_2\mu_2 - 1}, \quad f_1(x) = \frac{1}{b_1\mu_1 + b_2\mu_2 - 1}\frac{d}{dx} f(x) .$$

(2.84) munosabat funksional tenglama deyiladi.

(2.84) funktsional tenglamaning yechimi $x = 1$. bir nuqtada chegaralangan funktsiyalar sinfida qidiriladi. Agar bu talabdan voz kechilsa, unga mos keladigan bir hil (2.84) funktsional tenglama

$$\nu(x) = \lambda_1(x)\nu(a_1 + b_1x) + \lambda_2(x)\nu(a_2 + b_2x) \qquad (2.85)$$

aynan noldan farqli yechim bo'lishi mumkin.

Masalan. $p_1(x) = a + bx,\ p_2(x) = p_1(p_1(x)) = b^2x + ba + a$ bo'lsin, bunda $a - b = c_1,\ c_1b + a = c_2$, u holda ishonch hosil qilish qiyin emaski,

$$\nu(x) = (1 - x)^\delta, \quad \text{где} \quad \delta = \log_b \frac{\sqrt{\lambda_1^2 + 4\lambda_2} - \lambda_1}{2\lambda_2} \qquad (2.86)$$

funksiya (2.86) bir jinsli tenglamaning notrivial yechimi iborat, haqiqatan ham:

$$\nu(p_1(x)) = (1 - p_1(x))^\delta = b^\delta(1 - x)^\delta,$$

$$\nu(p_2(x)) = (1 - p_2(x))^\delta = b^{2\delta}(1 - x)^\delta,$$

bo'ladi. U holda bu qiymatlarni (2.86) munosabatga qo'yib, navbatdagi kvadrat tenglamani hosil qilamiz:

$$\lambda_2 b^{2\delta} + \lambda_1 b^{\delta} - 1 = 0\,.$$

Bundan esa

$$\delta = \log_b \frac{\sqrt{\lambda_1^2 + 4\lambda_2} - \lambda_1}{2\lambda_2}$$

va (2.69) shart $\delta < 0$ bo'lganda, bir jinsli (2.87) funksional tenglamaning yechimi (2.86) uchun $x = 1.$ da chegaralanmagan ekanligini ko'rish oson.

U holda, (2.84) funksional tenglamaning yechimi izlanayotgan sinfda mavjud.

(2.84) tenglamning yechimi uchun kombirilangan usuli – ketma-ket yaqinlashishi va iteratsiya usullarini qo'llaymiz.

$v_0(x)$, $v_1(x),..., v_n(x),...$ ketma-ketlikning hadlarini qurish uchun ushbu

$$v_n(x) = \lambda_1 v_n(\alpha(x)) + \lambda_1 v_{n-1}(\beta(x)) + f_1(x) \qquad (2.87)$$

rekkurent munosabatdan foydalanamiz,

bunda $\alpha(x) = p_1(x),\ \beta(x) = p_2(x)$.

$v_0(x)$ nolinchi yaqinlashish sifatida ushbu

$$v_0(x) = \lambda_1 v_0(\alpha(x)) + f_1(x)\,. \qquad (2.88)$$

funksional tenglama yechimini misol sifatida keltiramiz.

(2.88) funksional tenglamani yechishda iteratsiya usulini qo'llaymiz.

(2.88) da x ni $\alpha(x)$ bilan almashtiramiz:

$$v_0(\alpha(x)) = \lambda_1 v_0(\alpha(\alpha(x))) + f_1(\alpha(x))\,. \qquad (2.89)$$

Endi (2.89) tenglikni (2.88) ga qo'yamiz va

$$v_0(x) = \lambda_1^2 v_0(\alpha_2(x)) + \lambda_1 f_1(\alpha(x))\,, \qquad (2.90)$$

munisabatni hosil qilamiz,

bunda $\alpha_1(x) = \alpha(x),\ \alpha_2(x) = \alpha_1(\alpha_1(x))$.

(2.90) munosabat (2.87) tenglamaning birinchi iteratsiyasi deyiladi.

Bu jarayonni davom ettrib, n- chi iteratsiya uchun quyidagi munosabatni olamiz:

$$\nu_0(x)=\lambda_1^{n+1}\nu_0(\alpha_{n+1}(x))+\sum_{k=0}^{n}\lambda_1^k f_1(\alpha_k(x)) \qquad (2.91)$$

bunda $\alpha_0(x)=x,\ \alpha_{n+1}(x)=\alpha_n(\alpha_1(x))=\alpha_1(\alpha_n(x)).$

Eslatib o'tamizki, (2.69) shartga ko'ra

$$\lim_{n\to\infty}\lambda_1^{n+1}=0. \qquad (2.92)$$

bo'ladi.

Xuddi shuningdek, $\alpha(x)>x,\ \alpha(1)=1,$ u holda $\alpha_n(x)=\alpha_{n-1}(\alpha_1(x))=\alpha_1(\alpha_{n-1}(x))>\alpha_{n-1}(x),$ va h.k. $\{\alpha_n(x)\}$ funksional ketma-ketlik monoton o'suvchi va yuqoridan bir bilan chegaralangan, ya'ni $\alpha_n(x)\le 1,\ \forall x\in\bar{I}.$ Haqiqatan ham, monoton o'suvchi chegaralangan ketma-ketliklar uchun limitlar haqidagi teoremaga ko'ra

$$\lim_{n\to\infty}\alpha_n(x)=\alpha^0(x),\ x\in\bar{I}. \qquad (2.93)$$

ea ega bo'lamiz.

Endi $\alpha_n(x)=\alpha_1(\alpha_{n-1}(x))$ tenglikka asosan, $n\to\infty$ da limitga o'tsak, $\alpha^0(x)=\alpha_1(\alpha^0(x)),$ ni hosil qilamiz, bundan esa $\alpha^0(x)\equiv 1,\ \forall x\in\bar{I},$ xulosaga kelamiz, xuddi shuningdek, $\alpha(x)$ uchun ham $x=1$ nuqta yagona siljimaydigan nuqta bo'ladi.

Shunday holda,

$$\lim_{n\to\infty}\alpha_n(x)=1,\ \forall x\in\bar{I}. \qquad (2.94)$$

bo'ladi.

Endi (2.91) tenglikda $n\to\infty$ da limitga o'tamiz, (2.92) munosabatga ko'ra $\nu_0(x)$ ning

$$\nu_0(x)=\sum_{k=0}^{n}\lambda_1^k f_1(\alpha_k(x)) \qquad (2.95)$$

qiymatini olamiz. (2.95) yechim (2.94) funksional tenglamaning yechimi deyiladi.

Yuqoridagilardan esa

$$|\nu_0(x)|\le\sum_{k=0}^{n}\lambda_1^k\,|f_1(\alpha_k(x))|\le\sum_{k=0}^{n}M\lambda_1^k=\frac{M}{1-\lambda_1}, \qquad (2.96)$$

ga ega bo'lish mumkin, bunda $\max_{x\in I}|f_1(x)|=M$. Haqiqatan ham, (2.96) funksional qatorning o'ng qismi tekis yaqinlashadi va $\bar{I}$ kesmada $f_1(x)$ ning uzluksizligidan ushbu xulosaga kelamiz - $\nu_0(x)\in C(\bar{I})$. Bundan esa $\nu_0(x)\in C^2(\bar{I})$. ekanligiga ham ishonch hosil qilish qiyin emas.

Endi (2.87) munosabatda $n=1$ bo'lsin, u holda $\nu_1(x)$ ga nisbatan ushbu funksional tenglamani hosil qilamiz:

$$\nu_1(x)=\lambda_1\nu_1(\alpha(x))+\lambda_2\nu_0(\beta(x))+f_1(x), \tag{2.97}$$

bunda $\nu_0(x)$ - (2.95) tenglik bilan aniqlanuvchi ma'lum funksiya.

Funksional tenglamaga iteratsiya usulini qo'llab, xuddi yuqoridagi kabi (2.97) tenglamaning yechimini ushbu

$$\nu_1(x)=\sum_{k=0}^{\infty}\lambda_1^k\lambda_2\nu_0(\beta(\alpha_k(x)))+\sum_{k=0}^{\infty}\lambda_1^k f_1(\alpha_k(x)). \tag{2.98}$$

ko'rinishda hosil qilamiz.

Ishonch hosil qilish qiyin emaski, (2.98) funksional qatorning o'ng qismi tekis yaqinlashadi va $\nu_1(x)\in C^2(\bar{I})$. bo'ladi.

Bu jarayonni davom ettirib, ushbu

$$\nu_n(x)=\sum_{k=0}^{\infty}\lambda_1^k\lambda_2\nu_{n-1}(\beta(\alpha_k(x)))+\sum_{k=0}^{\infty}\lambda_1^k f_1(\alpha_k(x)). \tag{2.99}$$

munosabatni hosil qilamiz va $\nu_n(x)\in C^2(\bar{I})$. bo'ladi.

Bunday holda, bia quyidagi

$$\nu_0(x)\,,\,\nu_1(x),...,\,\nu_n(x),...\ . \tag{2.100}$$

funksional ketma-ketliklarni qurdik.

Endi (2.100) funksional ketma-ketlikning yaqinlashuvchi ekanligini isbotlaymiz. Buning uchun ushbu

$$v_0(x)+|v_1(x)-v_0(x)|+|v_2(x)-v_1(x)|+...+|v_n(x)-v_{n-1}(x)|+... \quad . \qquad (2.101)$$

funksional qatorni qaraymiz.

(2.100) funksional qator uchun mojorant qatorni topamiz.

(2.101) dan (2.93) ni hisobga olsak, hamda (2.96) ga asosan,

$$|v_1(x)-v_0(x)|\le\sum_{k=0}^{n}\lambda_1^k\lambda_2 v_0(\beta(\alpha_k(x)))\le\frac{\lambda_2}{1-\lambda_1}\frac{M}{1-\lambda_1}=\frac{M}{\lambda_2}\left(\frac{\lambda_2}{1-\lambda_1}\right)^2. \qquad (2.102)$$

ni hosil qilamiz.

(2.102) ga asoslanib, bu jarayonni davom ettrib, quyidagini

$$|v_n(x)-v_{n-1}(x)|\le\sum_{k=0}^{n}\lambda_1^k\lambda_2|v_n(\beta(\alpha_k(x)))-v_{n-1}(\beta(\alpha_k(x)))|\le$$

$$\le\frac{\lambda_2}{1-\lambda_1}\frac{M}{\lambda_2}\left(\frac{\lambda_2}{1-\lambda_1}\right)^2=\frac{M}{\lambda_2}\left(\frac{\lambda_2}{1-\lambda_1}\right)^{n+1}. \qquad (2.103)$$

hosil qilamiz.

(2.101) funksional qator $\sum_{n=0}^{\infty}\frac{M}{\lambda_2}\left(\frac{\lambda_2}{1-\lambda_1}\right)^{n+1}$ sonli qator bilan mojoranta bo'ladi va u $\bar{I}$ kesmada tekis yaqinlashadi.

Shunday qilib, $\{v_n(x)\}$ funksional ketma-ketlik tekis yaqinlashadi, ya'ni mavjud

$$\lim_{n\to\infty}v_n(x)=v(x),\ \forall x\in\bar{I}.$$

va $v(x)\in C^2\left(\bar{I}\right)$.

Teorema isbotlandi.

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Mundarija

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