O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

TOSHKENT TO`QIMACHILIK VA YENGIL SANOAT INSTITUTI

"Matematika" kafedrasi

Katta o`qituvchi: Atajanova M.A.
Katta o`qituvchi: Nalibayeva Z.A.

"Matematika" fanining

"Matematikadan misol va masalalar to`plami "

I- qism

(**C**hiziqli algebra,vektorlar algebrasi va tekislikdagi analitik geometriya" bo'limlari)_

(**Bakalavriatning barcha ta`lim yo`nalishlari uchun**)

Тоshкепт - 2016

O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

TOSHKENT TO'QIMACHILIK VA YENGIL SANOAT INSTITUTI

"Matematika" kafedrasi

Katta o`qituvchi: Atajanova M.A.
Katta o`qituvchi: Nalibayeva Z.A.

"Matematika" fanining

" Matematikadan misol va masalalar to`plami "

I- qism

(Bakalavriatning barcha ta`lim yo`nalishlari uchun)

UDK 517(075.8)
M 62

Mazkur o`quv uslubiy qo`llanma Oliy va o`rtamaxsus ta`lim vazirligi tomonidan tasdiqlangan “Matematika” o`quv dasturining chiziqli algebra elementlari, vektorlar algebrasi va tekislikdagi analitik geometriya bo`limiga muvofiq yozilgan bo`lib, bakalavriyat talabalari uchun mo`ljallangan. Unda qisqacha nazariy ma`lumotlar bilan birga, amaliy mashg`ulotlar bo`yicha xarakterli misollarni qo`yish va yechish uslubi keltirilgan.

Ushbu o`quv-uslubiy qo`llanma Toshkent to`qimachilik va yengil sanoat institutining ilmiy-uslubiy kengashi tomonidan nashrga tavsiya etilgan.

Tuzuvchilar: katta o’qituvchi M.A. Atajanova,
katta o’qituvchi Z.A. Nalibaeva.

Taqrizchilar: O’z.MU dotsent T.T. To’ychiyev
TTYeSI dotsent M.M. Caydamatov

Institut ilmiy-uslubiy
kengashida tasdiqlangan
“___”________2016 yil
_____№_Bayonnoma

TTYSI bosmaxonasida
“______” nusxada
ko’paytirilgan

So`z boshi

Respublikamizda yangi ta`lim tizimlari yangi ikki pog`onali bakalavr-magistr tizimi joriy etilishi bilan barcha fanlar qatorida “Matematika” fanining ham auditoriya soatlari hajmi qisman o`zgartirilib, mustaqil o`quv soatlari ko`paytirildi.

Shu bois , o`quv dasturlariga mos keladigan yangi pedogogik-texnologiyalar asosida, chet el dasturlarini e`tiborga olgan holda sodda va ravon tilda yozilgan o`quv- uslubiy(ko`rsatmlarni) yaratish dolzarb masala bo`lib qoldi

Mazkuro`quv- uslubiy ko`rsatmada Oliy va o`rta maxsus ta`lim vazirligi tomonidan tasduqlangan “Matematika” o`quv dasturining chiziqli algebra elementlari, vektorlar algebrasi va tekislikdagi analitik geometriya bo`limiga muvofiq yozilgan bo`lib, unda qisqacha nazariy ma`lumotlar bilan birga,ularga mos amaliy mashg`ulotlar uchun misollar keltirilgan

O`quv – uslubiy qo`llanmaning chiziqli algebra elementlari bo`limida matritsalar, teskari matritsa, matritsalar ustida chiziqli amallar, determinantlar va ularni hisoblash usullari, tenglamalar yechishning Gauss, Kramer va matritsa usulida t mumkinligi ko`rsatilgan.

Vektorlar algebrasi bo`limida, vektorlar tushunchasi va ular ustida amallar, skolyar, vektor va aralash ko`paytmalar haqida tushuncha berilgan.

Tekislikda analitik geometriya bo`limida esa tekislikda yotgan to`g`ri chiziqlarning turli ko`rinishdagi tenglamalari va ikkinchi tartibli chiziqlardan aylana, ellips, giperbola va parabola haqida tushuncha berilgan.

O`quv- uslubiy qo`llanma Toshkent to`qimachilik va yengil sanoat institutining barcha yo`nalishdagi talabalari uchun mo`ljallangan.

1-BOB

AMALIY MASHG`ULOT.

Mavzu: Matrisa. Matrisalar ustida amallar.

To`rtta sondan iborat

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

kvadrat jadval *ikkinchi tartibli kvadrat matritsa* deyiladi.

Sonlarning *m* ta satr va *n* ta ustundan iborat to`g`ri to`rtburchakli jadvalga $m \times n$ o`lchamli matritsa deyiladi. Bu matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko`rinishda yoziladi.

Agar *m=1* bo`lsa *satr matritsa*, *n=1* bo`lsa- *ustun matritsa*, *m=n* bo`lsa- *kvadrat matritsa* hosil bo`ladi. Kvadrat *A* matritsa uchun shu matritsaning elementlaridan tuzilgan *n* tartibli determinantni hisoblash mumkin. Bu determinant $\det A$ yoki $|A|$ orqali belgilaniladi:

$$\det A = |A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{mn} \end{vmatrix}$$

Agar $\det A = 0$ bo`lsa, u holda *A* matritsa *maxsus*, $\det A \neq 0$ bo`lsa, *maxsusmas* deyiladi.

Bosh diagonalida turgan elementlari birga, qolgan elementlari nolga teng bo`lgan kvadrat matritsa *birlik matritsa* deb ataladi va *E* bilan belgilanadi:

$$E = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots. & \dots. & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Ravshanki, $\det E = 1$

Bir xil $m\times n$ o`lchamli $A$ va $B$ matritsaning *yig`indisi* deb o`sha o`lchamli shunday $C=A+B$ matritsaga aytiladiki, uning har bir elementi A va B matritsalarning mos elementlari yig`indisidan iborat bo`ladi.

Masalan: $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ va $B=\begin{pmatrix} m & n \\ l & k \end{pmatrix}$ matritsalarning yig`indisi va ayirmasi quyidagicha topiladi:

$$\text{a) } C=A+B=\begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} m & n \\ l & k \end{pmatrix}=\begin{pmatrix} a+m & b+n \\ c+l & d+k \end{pmatrix}$$

$$\text{b) } A-B=\begin{pmatrix} a & b \\ c & d \end{pmatrix}-\begin{pmatrix} m & n \\ l & k \end{pmatrix}=\begin{pmatrix} a-m & b-n \\ c-l & d-k \end{pmatrix}$$

$m\times n$ o`lchamli $A$ matritsaning $\lambda$ songa ko`paytmasi deb, o`sha o`lchamdagi $B=\lambda\cdot A$ matritsaga aytiladiki, bu matritsa elementlari A matritsa elementlarini λ ga ko`paytirishdan hosil bo`ladi.

Masalan: $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ matritsani λ soniga ko`paytirish quyidagicha bo`ladi:

$$\lambda A=\begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{pmatrix}$$

$m\times k$ o`lchami $A$ matritsaning $k\times n$ o`lchamli *B matritsaga ko`paytmasi* deb, $m\times n$ o`lchamli shunday $C=A\cdot B$ matritsaga aytiladiki, uning c_{ij} elementi A matritsaning *i*- satr elementilarini B matritsaning *j*- ustunidagi mos elementlariga ko`paytmalari yig`indisiga teng, yani

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+...+a_{ik}b_{kj}$$

Agar *AB=BA* bo`lsa, u holda $A$ va $B$ matritsalar o`rin almashinadigan yoki kommutativ matritsalar deyiladi.

$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ va $B=\begin{pmatrix} m & n \\ l & k \end{pmatrix}$ ikkinch tartibli matritsalarning ko`paytmasi quyidagicha topiladi:

$$1.\ A\cdot B=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\cdot\begin{pmatrix} m & n \\ l & k \end{pmatrix}=\begin{pmatrix} am+bl & an+bk \\ cm+dl & cn+dk \end{pmatrix}$$

2. $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ va $B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ uchinchi tartibli matritsalarnuing ko`paytmasi quyidagicha topiladi:

$$A\cdot B=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}=$$

$$\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{22}b_{23}+a_{33}b_{33} \end{pmatrix}$$

3. $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$ va $B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{33} \end{pmatrix}$ matritsalarnuing ko`paytmasi quyidagicha topiladi:

$$A\cdot B=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} & a_{11}b_{13}+a_{12}b_{23} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} & a_{21}b_{13}+a_{22}b_{23} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} & a_{31}b_{13}+a_{32}b_{23} \end{pmatrix}$$ 4.

$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ va $B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$ matritsalarnuing ko`paytmasi quyidagicha topiladi:

$$A\cdot B=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} \end{pmatrix}$$

Misol. $A=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}; B=\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}$ matrisa berilgan:

1) *A+2B, 2) 3A-B, 3) A·B* lar topilsin.

1) $A+2B=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}+2\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}+\begin{pmatrix} 10 & 6 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} 12 & 9 \\ 6 & 9 \end{pmatrix}$

2). $3A-B=3\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}-\begin{pmatrix} 10 & 15 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} 6 & 9 \\ 12 & 3 \end{pmatrix}-\begin{pmatrix} 10 & 15 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} -4 & -6 \\ 10 & -5 \end{pmatrix}$

3)$A-B=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}-\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 3 & -3 \end{pmatrix}$

Misol: 1.

$A=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}; B=\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}$ matritsa berilgan: $A\cdot B$ ni topilsin.

Yechish. $A\cdot \mathbf{B}=\begin{pmatrix}2 & 3\\ 6 & 8\end{pmatrix}\cdot\begin{pmatrix}5 & 7\\ 1 & 4\end{pmatrix}=\begin{pmatrix}2\cdot 5+3\cdot 1 & 2\cdot 7+3\cdot 4\\ 6\cdot 5+8\cdot 1 & 6\cdot 7+8\cdot 4\end{pmatrix}=\begin{pmatrix}13 & 26\\ 38 & 74\end{pmatrix}$

2. $A=\begin{pmatrix}1 & 3 & 1\\ 2 & 0 & 4\\ 1 & 2 & 3\end{pmatrix}$ va $B=\begin{pmatrix}2 & 1 & 0\\ 1 & -1 & 2\\ 3 & 2 & 1\end{pmatrix}$ matritsalar berilgan: $A\cdot B$ va $B\cdot A$ ni toping.

Yechish. $A\cdot B=\begin{pmatrix}1\cdot 2+3\cdot 1+1\cdot 3 & 1\cdot 1+3\cdot(-1)+1\cdot 2 & 1\cdot 0+3\cdot 2+1\cdot 1\\ 2\cdot 2+0\cdot 1+4\cdot 3 & 2\cdot 1+0\cdot(-1)+4\cdot 2 & 2\cdot 0+0\cdot 2+4\cdot 1\\ 1\cdot 2+2\cdot 1+3\cdot 3 & 1\cdot 1+2\cdot(-1)+3\cdot 2 & 1\cdot 0+2\cdot 2+3\cdot 1\end{pmatrix}=\begin{pmatrix}8 & 0 & 7\\ 16 & 10 & 4\\ 13 & 5 & 7\end{pmatrix}=$

$B\cdot A=\begin{pmatrix}2\cdot 1+1\cdot 2+0\cdot 1 & 2\cdot 3+1\cdot 0+0\cdot 2 & 2\cdot 1+1\cdot 4+0\cdot 3\\ 1\cdot 1+1\cdot 2+0\cdot 1 & 1\cdot 3+1\cdot 0+2\cdot 2 & 1\cdot 1+-1\cdot 4+2\cdot 3\\ 3\cdot 1+2\cdot 2+1\mp\cdot 1 & 3\cdot 3+2\cdot 0+1\cdot 2 & 3\cdot 1+2\cdot 4+1\cdot 3\end{pmatrix}=\begin{pmatrix}4 & 6 & 6\\ 1 & 7 & 3\\ 8 & 11 & 14\end{pmatrix}$

3. $A=\begin{pmatrix}1 & 3 & 1\\ 2 & 0 & 4\\ 1 & 2 & 3\end{pmatrix}$ va $B=\begin{pmatrix}2 & 1 & 0\\ 1 & -1 & 2\\ 3 & 2 & 1\end{pmatrix}$ matritsalar berilgan: $A\cdot B$ va $B\cdot A$ ni toping.

Agar kvadrat matrirsa maxsusmas bo`lsa, u holda $AA^{-1}=A^{-1}A=E$ tenglikni qanoatlantiruvchi yagona $A^{-1}$ matritsa mavjud bo`ladi va u A matritsaga teskari matritsa deyiladi. A matritsaga A^{-1} teskar matritsa quyidagicha aniqlanadi:

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21} & \dots & A_{n1}\\ A_{12} & A_{22} & \dots & A_{n2}\\ \dots & \dots & \dots & \dots\\ A_{1n} & A_{2n} & \dots & A_{nn}\end{pmatrix}$$

$$A_{ik}=(-1)^{ij}M_{ij}$$

Bu erda A_{ik} A matritsa determinanti a_{ik} elementning *algebraik to`ldiruvchisi*, M_{ij} a_{ik} elementni minori deyiladi.

Misol. Berilgan $A=\begin{pmatrix}3 & 2 & 2\\ 1 & 3 & 1\\ 5 & 3 & 4\end{pmatrix}$ matritsaga teskari matritsani toping.

Yechish. Matritsaning determinantini hisoblaymiz:

$$\det A=\begin{vmatrix}3 & 2 & 2\\ 1 & 3 & 1\\ 5 & 3 & 4\end{vmatrix}=27+2-24=5$$

Demak, A matritsa maxsusmas matritsa ekan. Endi A_{ik} algebraik to`ldiruvchilarni hisoblaymiz:

$$A_{11}=\begin{vmatrix}3&1\\3&4\end{vmatrix}=9,\; A_{21}=-\begin{vmatrix}2&2\\3&4\end{vmatrix}=-2,\; A_{31}=\begin{vmatrix}2&2\\3&1\end{vmatrix}=-4,\; A_{12}=-\begin{vmatrix}1&1\\5&4\end{vmatrix}=1,\; A_{22}=\begin{vmatrix}3&2\\5&4\end{vmatrix}=2,$$

$$A_{32}=-\begin{vmatrix}3&2\\1&1\end{vmatrix}=-1,\; A_{13}=\begin{vmatrix}1&3\\5&3\end{vmatrix}=-12,\; A_{23}=-\begin{vmatrix}3&2`\\5&3\end{vmatrix}=1,\; A_{33}=\begin{vmatrix}3&2\\1&3\end{vmatrix}=7.$$

Teskari matritsa tuzamiz:

$$A^{-1}=\frac{1}{5}\begin{pmatrix}9&-2&-4\\1&2&-1\\-1`2&1&7\end{pmatrix}=\begin{pmatrix}\frac{9}{5}&-\frac{2}{5}&-\frac{4}{5}\\\frac{1}{5}&\frac{2}{5}&-\frac{1}{5}\\-\frac{12}{5}&\frac{1}{5}&\frac{7}{5}\end{pmatrix}.$$

$AA^{-1}=A^{-1}A=E$ ekanini tekshirish mumkin.

Msollar.

1. Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix}3&-2&4\\-5&3&8\\2&1&-4\end{pmatrix}\quad B=\begin{pmatrix}5&3&-7\\2&-4&5\\3&7&-8\end{pmatrix}\qquad 2A-5E+4B^2=?$$

2. Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix}2&-3&4\\5&1&-2\\4&-5&0\end{pmatrix}\qquad B=\begin{pmatrix}5&2&-3\\4&-5&2\\3&1&-4\end{pmatrix}\qquad 2B^2+3A-2E=?$$

3. Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix}5&-3&2\\4&1&-3\\7&-2&4\end{pmatrix}\qquad B=\begin{pmatrix}2&5&-4\\3&-2&1\\5&3&-7\end{pmatrix}\qquad (A+B)^2-3B=?$$

4. Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix}3&2&-4\\5&-7&-8\\2&1&0\end{pmatrix}\qquad B=\begin{pmatrix}5&-3&2\\4&5&-3\\1&3&7\end{pmatrix}\; 2B^2-3E+4A=?$$

5. Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix}1&3\\4&2\\0&-1\end{pmatrix}\qquad B=\begin{pmatrix}2&1&-1\\2&0&3\end{pmatrix}\qquad A\cdot B \text{ va } B\cdot A \text{ ni toping.}$$

6. $A=\begin{pmatrix} 2 & 3 \\ 4 & 2 \\ 1 & -1 \end{pmatrix}$ $\quad B=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 1 & 0 \end{pmatrix}$ $\quad A\cdot B$ va $B\cdot A$ ni toping.

7. $A=\begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 4 & -1 \end{pmatrix}$ $\quad B=\begin{pmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \end{pmatrix}$ $\quad A\cdot B$ va $B\cdot A$ ni toping.

8. $A=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{pmatrix}$ $\quad B=\begin{pmatrix} 1 & 1 \\ -2 & 2 \\ 0 & -1 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping.

9. $A=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \end{pmatrix}$ $\quad B=\begin{pmatrix} 1 & 5 \\ 4 & 2 \\ 0 & -1 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping.

10. $A=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 5 & 3 \end{pmatrix}$ $\quad B=\begin{pmatrix} 6 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping.

11. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

A= $\begin{pmatrix} 7 & -3. & 2 \\ 1 & 2 & -8 \\ 4 & -9 & 3 \end{pmatrix}$ $\qquad A^{-1}=?$

12. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

A= $\begin{pmatrix} 5. & 7 & -3 \\ 2 & -8 & 4 \\ 1 & 9 & -7 \end{pmatrix}$ $\qquad A^{-1}=?$

13. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

A= $\begin{pmatrix} 5 & -3 & 7 \\ 9 & 1 & -2 \\ 4 & -7 & 8 \end{pmatrix}$ $\qquad A^{-1}=?$

14. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

A= $\begin{pmatrix} 7 & -3 & 2 \\ 9 & 1 & -5 \\ 4 & -7 & 6 \end{pmatrix}$ $\qquad A^{-1}=?$

1-topshiriq .

1 Matritsalar ustida amallarni bajaring

A=$\begin{pmatrix} 3 & -5 & 2 \\ -4 & 3 & 7 \\ 8 & -2 & 1 \end{pmatrix}$ B=$\begin{pmatrix} 2 & 7 & -3 \\ 5 & 1 & -4 \\ 3 & -8 & 9 \end{pmatrix}$ $3A-5E=2B^2=?$

2 Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 3 & 5 & -7 \\ 2 & -4 & 8 \\ 7 & 3 & -5 \end{pmatrix}$ B=$\begin{pmatrix} 5 & -3 & 9 \\ 1 & 5 & -7 \\ 3 & -8 & 4 \end{pmatrix}$ $3B^2+4E+2A=?$

3.Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 4 & 5 & -3 \\ 1 & -2 & 4 \\ 3 & 4 & -7 \end{pmatrix}$ B=$\begin{pmatrix} 5 & -3 & 4 \\ -3 & 5 & 7 \\ 4 & -5 & 8 \end{pmatrix}$ $2A^2-4B+3E=?$

4 Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 5 & -3 & 4 \\ 1 & 2 & -7 \\ 3 & -4 & 6 \end{pmatrix}$ B=$\begin{pmatrix} 4 & 5 & -7 \\ 2 & -3 & 5 \\ 1 & 0 & -2 \end{pmatrix}$ $2A+B-3E^2=?$

5 Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 3 & -5 & 4 \\ 2 & 7 & -5 \\ 4 & -8 & 9 \end{pmatrix}$ B=$\begin{pmatrix} 2 & 4 & -7 \\ 3 & -5 & 7 \\ 3 & 2 & -6 \end{pmatrix}$ $3A-2B^2+4E=?$

6 Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 7 & -3 & 4 \end{pmatrix}$ B=$\begin{pmatrix} 5 & 2 & -3 \\ 4 & -3 & 5 \\ 2 & 4 & 7 \end{pmatrix}$ $2E+A-2B^2=?$

7 Matritsalar ustida amallarni bajaring.

A=$\begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -6 \\ 7 & -2 & 5 \end{pmatrix}$ B=$\begin{pmatrix} 5 & 2 & -1 \\ 4 & -3 & 2 \\ 5 & 7 & -3 \end{pmatrix}$ $(A+B)^2-2B=?$

8 Matritsalar ustida amallarni bajaring.

$$A=\begin{pmatrix} 3 & 2 & -5 \\ 4 & -3 & 7 \\ -8 & 2 & -1 \end{pmatrix} \qquad \begin{pmatrix} 5 & 4 & -3 \\ 2 & -5 & 8 \\ 3 & 4 & -6 \end{pmatrix} \qquad A^2-4E+3B=?$$

9 Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 4 & 5 & -7 \\ 8 & -9 & 1 \\ 3 & 4 & -6 \end{pmatrix} \qquad A^{-1}=?.$$

10.Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 3 & 5 & -2 \\ 4 & -4 & 5 \\ 2 & 6 & -1 \end{pmatrix} \qquad A^{-1}=?$$

12. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 5 & 3 & -2 \\ 4 & -3 & 6 \\ 1 & 2 & -4 \end{pmatrix} \qquad A^{-1}=?$$

13. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 4 & -5 & 3 \\ 2 & 1 & -4 \\ 7 & -2 & 5 \end{pmatrix} \qquad A^{-1}=?$$

14. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -1 & 0 \\ 4 & 5 & -7 \end{pmatrix} \qquad A^{-1}=?$$

15. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -1 & 7 \\ 6 & 3 & -5 \end{pmatrix} \qquad A^{-1}=?$$

16. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -1 & 7 \\ 6 & 3 & -5 \end{pmatrix} \qquad A^{-1}=?$$

17. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 5 & 2 & -4 \\ 3 & -1 & 7 \\ 8 & 5 & -3 \end{pmatrix} \qquad A^{-1}=?$$

18. Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

$$A=\begin{pmatrix} 4 & 5 & -7 \\ 1 & -2 & 3 \\ 7 & 8 & -9 \end{pmatrix} \qquad A^{-1}=?$$

Mavzu: Ikkinchi va uchinchi tartibli determinantlar. Determinantlarning asosiy xossalari. Yuqori tartibli determinantlar.

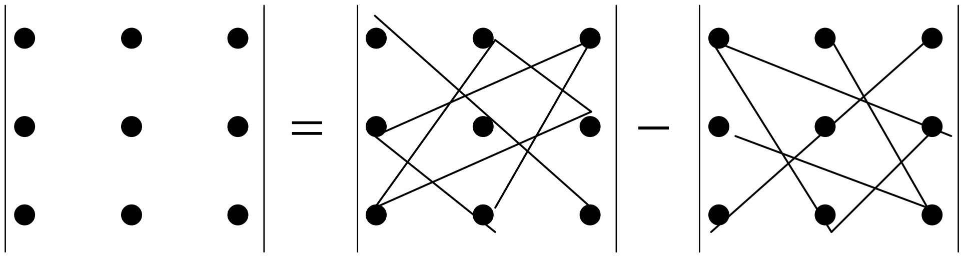
Ikkinchi tartibli kvadrat matritsaga mos keluvchi ikkinchi tartibli determinant deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\cdot a_{22} - a_{12}\cdot a_{21}$$

Uchinchi tartibli kvadrat matritsaga mos keluvchi uchinchi tartibli determinand deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{33} - a_{31}a_{22}a_{13} - a_{21}a_{12}a_{33} - a_{32}a_{23}a_{11}$$

Uchinchi tartibli determinantlarni hisoblash uchun ”*uchburchaklar qoidasi*” danfoydalanamiz.



Determinantdagining a_{ij} elementining M_{ij} minori deb, bu element turgan qator va ustunni o`chirish natijasida hosil bo`lgan determinantga aytiladi.

a_{ij} elementining algebraik to`ldiruvchisi deb, musbat yoki manfiy ishora bilan olingan minorga aytiladi va

$$A_{ij} = (-1)^{i+j} M_{ij}$$

munosabat bilan aniqlanadi.

Ixtiyoriy tartibli determinantni hisoblashning uchta usulini keltiramiz:

1.Determinant *tartibini pasaytirish usuli*- determinant biror qatori (ustun) elementlarining bittasidan boshqalarini oldindan nolga aylantirib olib, shu qator (ustun) bo`yicha yoyish usuli.

Masalan.

$$A = \begin{vmatrix} 3 & -1 & 12 & 8 \\ -5 & 3 & -34 & -23 \\ 1 & 1 & 3 & -7 \\ -9 & 2 & 8 & -15 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 12 & 8 \\ 4 & 0 & 2 & 1 \\ 4 & 0 & 15 & 1 \\ -3 & 0 & 32 & 1 \end{vmatrix} = -(-1)^3 \begin{vmatrix} 4 & 2 & 1 \\ 4 & 15 & 1 \\ -3 & 32 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 4 & 2 & 1 \\ 0 & 13 & 0 \\ -7 & 30 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 13 \\ -7 & 30 \end{vmatrix} = 91$$

2. Determinantni *uchburchak ko`rinishiga* keltirish usuli - determinantning bosh diagonalidan bir tomonida yotuvchi hamma elementlari nolga aylantiriladi va uchburchaksimon shaklga keltiriladi, masalan

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{vmatrix}$$

Ravshanki, uchburchak shaklidagi determinantning qiymati bosh diagonallari elementlari ko`paytmasiga teng:

$\Delta = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$

Masalan.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 8 \end{vmatrix} = 1 \cdot 2 \cdot 3 \cdot 8 = 48$$

Determinantni *satr yoki ustun bo`yicha yoyib hisoblash* quyidagicha bo`ladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Masalan.

$$\Delta = \begin{vmatrix} 1 & 7 & 3 & 0 \\ 0 & 10 & 2 & 3 \\ 0 & -14 & -8 & 2 \\ 0 & -8 & -6 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 10 & 2 & 3 \\ -14 & -8 & 2 \\ -8 & -6 & 1 \end{vmatrix} = 2 \cdot 2 \cdot 2 \cdot \begin{vmatrix} 5 & 1 & 3 \\ -7 & -4 & 2 \\ -4 & -3 & 1 \end{vmatrix} = 8 \cdot \begin{vmatrix} 17 & 10 & 0 \\ 1 & 2 & 0 \\ -4 & -3 & 1 \end{vmatrix} = 8 \begin{vmatrix} 17 & 10 \\ 1 & 2 \end{vmatrix} = 192$$

3. Sarrius usuli.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11}a_{12} \\ a_{21}a_{22} \\ a_{31}a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\begin{matrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

Masalan.

1. $$\begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} \begin{matrix} 3 & 2 \\ 1 & 3 \\ 5 & 3 \end{matrix} = 3 \cdot 3 \cdot 4 + 2 \cdot 1 \cdot 5 + 2 \cdot 1 \cdot 3 - 2 \cdot 3 \cdot 5 - 3 \cdot 1 \cdot 3 - 2 \cdot 1 \cdot 4 =$$

$$= 36 + 10 + 6 - 30 - 9 - 8 = 52 - 47 = 5$$

2. $$\begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 3 \cdot 3 \cdot 4 + 2 \cdot 1 \cdot 5 + 2 \cdot 1 \cdot 3 - 2 \cdot 3 \cdot 5 - 3 \cdot 1 \cdot 3 - 2 \cdot 1 \cdot 4 =$$

$$\begin{matrix} 3 & 2 & 2 \\ 1 & 3 & 1 \end{matrix}$$

$$= 36 + 10 + 6 - 30 - 9 - 8 = 52 - 47 = 5$$

Determinantlarning asosiy xossalari:

a) agar determinantning barcha satrlari mos ustunlari bilan almashtirilsa, uning qiymati o`zgarmaydi;

b) agar determinant nollardan iborat ustun yoki satrga ega bo`lsa , uning qiymati nolga teng bo`ladi;

v) agar determinant ikkita bir xil parallel satr yoki ustunga ega bo`lsa, uning qiymati nolga teng.

Misollar.

Determinantlarni hisoblang.

1. $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$ 2. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ 3. $\begin{vmatrix} 3 & 2 \\ 8 & 5 \end{vmatrix}$ 4. $\begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix}$ 5. $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$ 6. $\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$

7. $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$ 8. $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$ 9. $\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$ 10. $\begin{vmatrix} \frac{1-t^2}{1+t^2} & \frac{2t}{1+t^2} \\ \frac{-2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{vmatrix}$

11. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$ 12. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$ 13. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$ 14. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 2 & -7 & -5 \end{vmatrix}$

15. $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$ 16. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$ 17. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ 18. $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$

19. $\begin{vmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \end{vmatrix}$ 20. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix}$

Quyidagi determinantlarni ixtiyoriy ustun yoki satr elementlari bo`yicha yoyib hisoblang.

21. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ 22. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$ 23. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & -4 & 8 \end{vmatrix}$

24. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & b \end{vmatrix}$ 25. $\begin{vmatrix} 1 & 2 & 5 \\ 0 & 5 & 7 \\ 0 & -4 & 8 \end{vmatrix}$ 26. $\begin{vmatrix} 0 & 0 & 1 \\ 2 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

27. $\begin{vmatrix} 1 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & -1 & 8 \end{vmatrix}$ 28. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$ 29. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$

30. $\begin{vmatrix} 3 & -1 & -2 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{vmatrix}$ 31. $\begin{vmatrix} -1 & 2 & 5 \\ 2 & 0 & 6 \\ 4 & 0 & 7 \end{vmatrix}$ 32. $\begin{vmatrix} 1 & 7 & -1 \\ 2 & 6 & 2 \\ 1 & 1 & 4 \end{vmatrix}$

Determinantni tartibini pasaytirish usulidan foydalanib hisoblang:

33. $\begin{vmatrix} 1 & -4 & 0 & 3 \\ -4 & 3 & 2 & -3 \\ -2 & 3 & -1 & 4 \\ 3 & 2 & 5 & 0 \end{vmatrix}$ 34. $\begin{vmatrix} 2 & -1 & 0 & 5 \\ -1 & -3 & 2 & -4 \\ 4 & 2 & -1 & 3 \\ 3 & 0 & -4 & -2 \end{vmatrix}$

35. $\begin{vmatrix} 3 & -1 & 0 & 3 \\ 5 & 1 & 4 & -7 \\ 5 & -1 & 0 & 2 \\ 1 & -8 & 5 & 3 \end{vmatrix}$ 36. $\begin{vmatrix} 6 & -3 & 4 & 2 \\ -1 & 0 & 4 & 5 \\ 2 & 7 & 3 & 4 \\ 0 & -5 & -1 & 3 \end{vmatrix}$

Mavzu: Chiziqli tenglamalar sistemasini Gauss,Kramer va matritsalar usulida yechish

1. Ikki noma`lumli ikkita chiziqli tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$ shart bajarilganda

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Yechimga ega.

Masalan. Ushbu $\begin{cases} 3x + 2y = 7 \\ 4x - 5y = 40 \end{cases}$ chiziqli tenglamalar sistemasini yeching.

$$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & -5 \end{vmatrix} = 3 \cdot (-5) - 2 \cdot 4 = -15 - 8 = -23$$

$$x=\frac{\begin{vmatrix}7 & 2\\ 40 & -5\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 4 & -5\end{vmatrix}}=\frac{-35-80}{-15-8}=\frac{-115}{-23}=5$$

$$y=\frac{\begin{vmatrix}3 & 7\\ 4 & 40\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 4 & -5\end{vmatrix}}=\frac{120-28}{-15-8}=\frac{92}{-23}=-4$$

J: (5;-4)

2. Bir jinsli uch noma`lumli ikkita tenglamalar sistemasi

$$\begin{cases}a_1x+b_1y+c_1z=0\\ a_2x+b_2y+c_2z=0\end{cases}$$

ushbu

$$x=k\begin{vmatrix}b_1 & c_1\\ b_2 & c_2\end{vmatrix},\ y=-k\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix},\ z=k\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$$

Formula bilan aniqlanuvchi yechimlarga ega, bunda k- ixtiyoriy son.

Masalan: Ushbu $\begin{cases}2x-5y+2z=0\\ x+4y-3z=0\end{cases}$ tenglamalar sistemesini yeching.

$$x=k\begin{vmatrix}-5 & 2\\ 4 & -3\end{vmatrix}=k(15-8)=7k,\ y=k\begin{vmatrix}2 & 2\\ 1 & -3\end{vmatrix}=k(-6-2)=-8k,\ z=k\begin{vmatrix}2 & -5\\ 1 & 4\end{vmatrix}=k(8+5)=13k.$$

J: *x=7k;y=-8k;z=13k.*

3. Bir jinsli uch noma`lumli uchta tenglamalar sistemasi berilgan.

$$\begin{cases}a_1x+b_1y+c_1z=0\\ a_2x+b_2y+c_2z=0\\ a_3x+b_3y+c_3z=0\end{cases}$$

Uning determinanti $\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$ bo`lsa, tenglamalar sistemasi cheksiz ko`p yechimga ega.

Misol.Ushbu $\begin{cases}x+2y+3z=4\\ 2x+y-z=3\\ 3x+3y+2z=10\end{cases}$ tenglamalar sistemasini yeching.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & 3 & 2 \end{vmatrix} = 2-6+18-9+3-8 = 23-23 = 0$$

J: Sistema birgalikda emas.

4. Ikki noma`lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \\ a_3x + b_3y = c_3 \end{cases}$$

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ bo`lganda va uning hech qaysi ikkita tenglamasi o`zaro zid bo`lmasa, birgalikda bo`ladi.

Masalan. Ushbu $\begin{cases} 2x - 3y = 6 \\ 3x + y = 9 \\ x + 4y = 3 \end{cases}$ tenglamalar sistemasini yeching.

Yechish: $\Delta = \begin{vmatrix} 2 & -3 & 6 \\ 3 & 1 & 9 \\ 1 & 4 & 3 \end{vmatrix} = 6-27+72-6-72+27 = 0$

J:Tenglamalar sistemasi birgalikda.

5. Uch noma`lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1, \\ a_{21}x + a_{22}y + a_{23}z = b_2, \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

ning bosh determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

bo’lganda yagona yechimga ega bo’lib, bu yechim Kramer formulalari bilan hisoblanadi:

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta},$$

bunda

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Masalan: Ushbu

$$\begin{cases} x - 2y + z = -4, \\ 3x + 2y - z = 8, \\ 2x - 3y + 2z = -6 \end{cases}$$

chiziqli tenglamalar sistemasini yeching.

Yechilishi: **asosiy va yordamchi d**eterminantlarni topamiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix} = 1\cdot 2\cdot 2 + 3\cdot(-3)\cdot 1 + 2\cdot(-2)\cdot(-1) - [2\cdot 2\cdot 1 + 1\cdot(-3)\cdot(-1) + 2\cdot 3\cdot(-2)] = -1 - (-5) = 4.$$

Determinant $\Delta = 4 \neq 0$ bo'lgani uchun sistema yagona yechimga ega va Kramer formulasini qo'llab, uni topamiz:

$$\Delta_x = \begin{vmatrix} -4 & -2 & 1 \\ 8 & 2 & -1 \\ -6 & -3 & 2 \end{vmatrix} = -4\cdot 2\cdot 2 + 8\cdot(-3)\cdot 1 + (-6)\cdot(-2)\cdot(-1) - [(-6)\cdot 2\cdot 1 + (-4)\cdot(-3)\cdot(-1) + 2\cdot 8\cdot(-2)] =$$

$$= -52 - (-56) = 4;$$

$$\Delta_y = \begin{vmatrix} 1 & -4 & 1 \\ 3 & 8 & -1 \\ 2 & -6 & 2 \end{vmatrix} = 1\cdot 8\cdot 2 + 3\cdot(-6)\cdot 1 + 2\cdot(-4)\cdot(-1) - [2\cdot 8\cdot 1 + 1\cdot(-6)\cdot(-1) + 2\cdot 3\cdot(-4)] =$$

$$= 6 - (-2) = 8;$$

$$\Delta_z = \begin{vmatrix} 1 & -2 & -4 \\ 3 & 2 & 8 \\ 2 & -3 & -6 \end{vmatrix} = 1\cdot 2\cdot(-6) + 3\cdot(-3)\cdot(-4) + 2\cdot(-2)\cdot 8 - [2\cdot 2\cdot(-4) + 1\cdot(-3)\cdot 8 + (-6)\cdot 3\cdot(-2)] =$$

$$= -8 - (-4) = -4.$$

$$x = \frac{\Delta_x}{\Delta} = \frac{4}{4} = 1, \quad y = \frac{\Delta_y}{\Delta} = \frac{8}{4} = 2, \quad z = \frac{\Delta_z}{\Delta} = \frac{-4}{4} = -1$$

J: $x = 1, \quad y = 2, \quad z = -1.$

7. Gauss usuli bilan tenglamalar sistemasini yechish.

Masalan: Ushbu

$$\begin{cases} x+y+5z+2t=1, \\ x+y+3z+4t=-3, \\ 2x+3y+11z+5t=2, \\ 2x+y+3z+2t=-3 \end{cases}$$

chiziqli tenglamalar sistemasini Gauss usuli bilan yeching.

Yechish:Ikkinchi, uchinchi, to'rtinchi tenlamalardan x larni yo'qotamiz. Buning uchun birinchi tenglamani ketma-ket -1, -2, -2 ga ko'paytiramiz va mos ravishda ikkinchi, uchinchi, to'rtinchi tenglamalar bilan qo'shamiz. Natijada ushbu sistemaga ega bo'lamiz:

$$\begin{cases} x+y+5z+2t=1, \\ 2z-2t=4, \\ y+z+t=0, \\ -y-7z-2t=-5, \end{cases}$$

yoki

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ y+7z+2t=5, \\ z-t=2. \end{cases}$$

Uchinchi tenglamadan ikkinchi tenglamani ayiramiz:

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ 6z+t=5, \\ z-t=2, \end{cases}$$

so'ngra to'rtinchi tenglamani -6 ga ko'paytirib, uchinchi tenglamaga qo'shsak, uchburchakli sistema hosil bo'ladi:

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ z-t=2, \\ 7t=-7. \end{cases}$$

Bundan,

$$t = -1,$$
$$z = 2 + t = 1,$$
$$y = -z - t = 0,$$
$$x = 1 - y - 5z - 2t = -2.$$

J: $x = -2, \quad y = 0, \quad z = 1, \quad t = -1$.

6. n ta noma'lumli n ta chiziqli tenglamalar sistemasini

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$$

matritsa ko'rinishda

$$AX = B$$

kabi yozish mumkin, bunda

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix},$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}.$$

Agar A maxsusmas matritsa, ya'ni $\det A \neq 0$ bo'lsa, u holda bu sistemaning matritsa shaklidagi yechimi ushbu ko'rinishga ega bo'ladi:

$$X = A^{-1}B.$$

$AA^{-1} = A^{-1}A = E$ ekanini tekshirish mumkin.

Masalan: Tenglamalar sistemasini matrisa usuli yordamida yechini.

$$\begin{cases} 2x - 3y + z = -5 \\ x + 2y - 4z = -9 \\ 5x - 4y + 6z = 5 \end{cases}$$

<u>Yechish. Tenglamalar sistemasi yordamida *A* matritsani tuzamiz</u>

$$A=\begin{pmatrix}2 & -3 & 1\\ 1 & 2 & -4\\ 5 & -4 & 6\end{pmatrix},\ X=\begin{pmatrix}x\\ y\\ z\end{pmatrix},\ B=\begin{pmatrix}-5\\ -9\\ 5\end{pmatrix}$$

Ushbu matritsaning determinantini hisoblaymiz

$$\Delta=\begin{vmatrix}2 & -3 & 1\\ 1 & 2 & -4\\ 5 & -4 & 6\end{vmatrix}=24+60-4-10-32+18=56;$$

Endi matritsaning algebraik to`ldiruvchilarini topamiz

$$A_{11}=(-1)^2\begin{vmatrix}2 & -4\\ -4 & 6\end{vmatrix}=12-16=-4$$

$$A_{12}=(-1)^3\begin{vmatrix}1 & -4\\ 5 & 6\end{vmatrix}=-(6+20)=-26$$

$$A_{13}=(-1)^4\begin{vmatrix}1 & 2\\ 5 & -4\end{vmatrix}=-4-10=-14$$

$$A_{21}=(-1)^3\begin{vmatrix}-3 & 1\\ -4 & 6\end{vmatrix}=-(-18+4)=14$$

$$A_{22}=(-1)^4\begin{vmatrix}2 & 1\\ 5 & 6\end{vmatrix}=12-5=7$$

$$A_{23}=(-1)^5\begin{vmatrix}2 & -3\\ 5 & -4\end{vmatrix}=-(-8+15)=-7$$

$$A_{31}=(-1)^4\begin{vmatrix}-3 & 1\\ 2 & -4\end{vmatrix}=12-2=10$$

$$A_{32}=(-1)^5\begin{vmatrix}2 & 1\\ 1 & -4\end{vmatrix}=-(-8-1)=9$$

$$A_{33}=(-1)^6\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix}=4+3=7$$

Teskari matritsani tuzamiz

$$A^{-1}=\frac{1}{56}\begin{pmatrix}-4 & 14 & 10\\ -26 & 7 & 9\\ -14 & -7 & 7\end{pmatrix}$$

$X=A^{-1}B$ formulaga asosan noma`lumlarni topamiz

$$X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{56}\begin{pmatrix}-4 & 14 & 10\\ -26 & 7 & 9\\ -14 & -7 & 7\end{pmatrix}\begin{pmatrix}-5\\ -9\\ 5\end{pmatrix}=\begin{pmatrix}-4\cdot(-5)+14\cdot(-9)+10\cdot 5\\ -26\cdot(-5)+7\cdot(-9)+9\cdot 5\\ -14\cdot(-5)+(-7)\cdot(-9)+7\cdot 5\end{pmatrix}=$$

$$\begin{pmatrix}20-126+50\\ 130-63+45\\ 70+63+35\end{pmatrix}=\frac{1}{56}\begin{pmatrix}-56\\ 112\\ 168\end{pmatrix}=\begin{pmatrix}-\frac{56}{56}\\ \frac{112}{56}\\ \frac{168}{56}\end{pmatrix}=\begin{pmatrix}-1\\ 2\\ 3\end{pmatrix}, x=-1; y=2; z=3$$

J: (-1;2;3)

Misollar.Tenglamalar sistemasini yeching:

1. $\begin{cases} 2x+y=3 \\ 3x+2y=4 \end{cases}$ 2. $\begin{cases} 3x-y=2 \\ 6x-2x=1 \end{cases}$ 3. $\begin{cases} 2x+y=1 \\ 4x+2y=2 \end{cases}$ 4. $\begin{cases} ax-3y=1 \\ ax-2y=2 \end{cases}$

5. $\begin{cases} mx-ny=(m-n)^2 \\ 2x-y=n(m\neq 2n) \end{cases}$ 6. $\begin{cases} 3x+2y+2z=0 \\ 5x+2y+3z=0 \end{cases}$ 7. $\begin{cases} 2x-3y=6 \\ x+2y=4 \\ x-5y=5 \end{cases}$

8. $\begin{cases} 5x-y-z=0 \\ x+2y+3z=14 \\ 4x+3y+2z=16 \end{cases}$ 9. $\begin{cases} x+y-7z=0 \\ x-6y+z=0 \\ 5x-y-z=0 \end{cases}$ 10. $\begin{cases} 3x+4y-z=8 \\ 2x+y+z=2 \\ 3x-y+2z=0 \end{cases}$

Tenglamalar sistemasini Gauss usuli bilan yeching.

11. $\begin{cases} x+2y+z=8 \\ y+3z+t=15 \\ 4x+z+t=11 \\ x+y+5t=23 \end{cases}$ 12. $\begin{cases} x+y-3z+2t=6 \\ x-2y-t=-6 \\ y+z+3t=16 \\ 2x-3y+2z=6 \end{cases}$

Tenglamalar sistemasini matritsa usuli bilan yeching.

13. $\begin{cases} 3x-y+z=12 \\ x+2y+4z=6 \\ 5x+y+2z=3 \end{cases}$ 14. $\begin{cases} x+y+z=0 \\ 2x-3y+4z=0 \\ 4x-11y+10z=0 \end{cases}$

15. $\begin{cases} 2x-3y+z=2 \\ x+5y-4z+5=0 \\ 4x+y-3z+4=0 \end{cases}$ 17. $\begin{cases} 2x-4y+3z=1 \\ x-2y+4z=3 \\ 3x-y+5z=2 \end{cases}$ 18. $\begin{cases} 2x-y+z=2 \\ 3x+2y-2z=-2 \\ x-2y+z=1 \end{cases}$

20. $\begin{cases} x+2y+3z=5 \\ 2x-y-z=1 \\ x+3y+4z=6 \end{cases}$ 21. $\begin{cases} 3x+4y+2z=9 \\ x-y+4z=4 \\ 5x+2y+10z=17 \end{cases}$ 22. $\begin{cases} 2x-3y+z=0 \\ x+y+z=3 \\ 3x-2y+2z=3 \end{cases}$

1-topshiriq.

Berilgan tenglamalar sistemasini birgalikda ekanligini tekshiring, agar birgalikda bo’lsa, ularni:

A) Kramer qoidasidan foydalanib,

B) Gauss usuli bilan,

C) matritsa usuli bilan yeching.

1. $\begin{cases} 3x+2y+z=5, \\ 2x+3y+z=1, \\ 2x+y+3z=11. \end{cases}$

2. $\begin{cases} 4x-3y+2z=9, \\ 2x+5y-3z=4, \\ 5x+6y+2z=18. \end{cases}$

3. $\begin{cases} 2x-y-z=4, \\ 3x+4y-2z=11, \\ 3x-2y+4z=11. \end{cases}$

4. $\begin{cases} x+y-z=1, \\ 8x+3y-6z=2, \\ -4x-y+3z=-3. \end{cases}$

5. $\begin{cases} 7x-5y=31, \\ 4x+11z=-43, \\ 2x+3y+4z=-20. \end{cases}$

6. $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=20, \\ 3x-2y-5z=6. \end{cases}$

7. $\begin{cases} x+y+3z=-1, \\ 2x-y+2z=-4, \\ 4x+y+4z=-2. \end{cases}$

8. $\begin{cases} 3x+4y+2z=8, \\ 2x-y-3z=-1, \\ x+5y+z=-7. \end{cases}$

9. $\begin{cases} x-4y-2z=-7, \\ 3x+y-z=5, \\ -3x+5y+6z=7. \end{cases}$

10. $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=20, \\ 3x-y+z=0. \end{cases}$

11. $\begin{cases} x+5y+z=-2, \\ 2x-4y-3z=0, \\ 3x+4y+2z=3. \end{cases}$

12. $\begin{cases} 2x-3y+2z=-6, \\ 5x+8y-z=0, \\ x+2y+3z=6. \end{cases}$

13. $\begin{cases} x-4y-2z=0, \\ 3x-5y-6z=7, \\ 3x+y+z=6. \end{cases}$

14. $\begin{cases} 2x-y+5z=10, \\ 5x+2y-13z=21, \\ 3x-y+5z=12. \end{cases}$

15. $\begin{cases} 2x+y-5z=-1, \\ x+y-z=-2, \\ 4x-3y+z=13. \end{cases}$

16. $\begin{cases} 2x+3y+4z=-10, \\ 4x+11z=-29, \\ 7x-5y=7. \end{cases}$

17. $\begin{cases} 2x+7y-z=10, \\ 3x-5y+3z=-14, \\ x+2y+z=-1. \end{cases}$

18. $\begin{cases} 4x+y-3z=-6, \\ 8x+3y-6z=-15, \\ x+y-z=-4. \end{cases}$

19. $\begin{cases} 3x-2y-5z=-14, \\ x-2y+3z=0, \\ 2x+3y-4z=-10. \end{cases}$

20. $\begin{cases} 5x+6y-2z=-9, \\ 2x+5y-3z=-1, \\ 4x-3y+2z=-15. \end{cases}$

2-topshiriq.

A matritsa berilgan. A^{-1} teskari matrisani toping va $AA^{-1} = A^{-1}A = E$ ekanini tekshiring.

1. $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix}$ **2.** $\begin{pmatrix} 3 & 2 & -1 \\ 7 & 3 & 0 \\ 1 & 2 & 2 \end{pmatrix}$ **3.** $\begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

4. $\begin{pmatrix} -2 & 3 & 3 \\ 4 & 5 & 1 \\ -3 & 4 & 0 \end{pmatrix}$ **5.** $\begin{pmatrix} 0 & 1 & -3 \\ 1 & -5 & 4 \\ 2 & 3 & 2 \end{pmatrix}$ **.** $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

7. $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix}$ **8.** $\begin{pmatrix} -1 & 8 & 1 \\ -1 & 5 & 5 \\ 0 & -1 & 3 \end{pmatrix}$ **9.** $\begin{pmatrix} 4 & -2 & 1 \\ -3 & 4 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

10. $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix}$ **11.** $\begin{pmatrix} 3 & -3 & 4 \\ -1 & -5 & -7 \\ 0 & -1 & 5 \end{pmatrix}$ **12.** $\begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix}$

13. $\begin{pmatrix} 1 & -1 & 8 \\ 1 & -5 & 5 \\ -2 & 3 & 10 \end{pmatrix}$ **14.** $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 1 & 3 \\ -7 & 0 & 2 \end{pmatrix}$ **15.** $\begin{pmatrix} 5 & -1 & 3 \\ 4 & -2 & 0 \\ 2 & -4 & 5 \end{pmatrix}$

16. $\begin{pmatrix} 1 & -3 & -2 \\ -2 & 1 & 3 \\ -2 & 4 & 4 \end{pmatrix}$ **17.** $\begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix}$ **18.** $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix}$

19. $\begin{pmatrix} 4 & 1 & 2 \\ 3 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$ **20.** $\begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix}$ **21.** $\begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 1 & 4 & 8 \end{pmatrix}$

22. $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ **23.** $\begin{pmatrix} 8 & 7 & 3 \\ 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ **24.** $\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix}$

25. $\begin{pmatrix} 2 & 6 & 3 \\ 4 & 7 & 1 \\ -3 & -8 & -2 \end{pmatrix}$ **26.** $\begin{pmatrix} 4 & 1 & 7 \\ 9 & -1 & 1 \\ 6 & -1 & 10 \end{pmatrix}$

3-topshiriq.

Quyidagi matritsaviy tenglamani yeching:

1. $X\begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

2. $X\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

3. $X\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$

4. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}X = \begin{pmatrix} 5 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$

5. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}X = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \\ 0 & -1 & 1 \end{pmatrix}$

6. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}X = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$

7. $\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}X = \begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$

8. $\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix}X = \begin{pmatrix} 0 & 0 \\ 1 & 5 \end{pmatrix}$

9. $\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}X = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

10. $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

11. $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}X = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

12. $X\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

13. $\begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}X = \begin{pmatrix} 0 & -1 & 3 \\ 10 & 0 & 25 \\ -5 & 5 & 0 \end{pmatrix}$

14. $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}X = \begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

15. $X\begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

16. $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

17. $X \cdot \begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 0 \\ 3 & 4 & -5 \\ 1 & -1 & 2 \end{pmatrix}$

18. $X \cdot \begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 2 & -3 \\ -1 & 2 & 3 \\ 1 & 0 & -1 \end{pmatrix}$

19. $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & -3 \\ 6 & -5 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

20. $X \cdot \begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ 7 & -4 & 3 \\ -2 & 0 & 1 \end{pmatrix}$

21. $X \cdot \begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 6 \\ -2 & 5 & -3 \\ 1 & 8 & -1 \end{pmatrix}$

22. $X \cdot \begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 5 \\ -2 & 6 & 3 \\ 1 & -5 & 0 \end{pmatrix}$

23. $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{pmatrix}$ **24.** $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & -4 & 3 \\ 0 & 5 & 6 \\ 8 & 7 & -4 \end{pmatrix}$

25. $X \cdot \begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 7 \\ 3 & -5 & 9 \\ -2 & 4 & 3 \end{pmatrix}$ **26.** $\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 6 & 2 & 3 \\ 7 & -1 & -4 \\ -3 & 0 & 5 \end{pmatrix}$

2-BOB

Mavzu: Skalyar va vektorlar. Vektorlar ustida chiziqli amallar. Kollinear va komplanar vektorlar. Ba'zis vektorlar.Vektorni komponentlari bo'yicha yoyish. Vektorni o'qdagi proeksiyasi va yo'naltiruvchi kosinuslari.

Vektorlar ustida chiziqli amallar

Boshi A nuqtada, oxiri B nuqtada bo`lgan yo`naltirilgan kesma *vektor* deb ataladi va u $\vec{AB}$ yoki $\vec{a}$ kabi belgilaniladi.

$\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ vektorlarning *chiziqli kombinasiyasi* deb

$$\vec{a} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + ... + \lambda_n \vec{a}_n$$

formula bilan aniqlanuvchi $\vec{a}$ vektorga aytiladi, bunda $\lambda_1, \lambda_2, ..., \lambda_n$ - tayin sonlar

Agar $\vec{a}_1, ..., \vec{a}_n$ vektorlar sistemasi uchun kamida bittasi noldan farqli shunday $\lambda_1, ..., \lambda_n$ sonlar mavjud bo`lib, $\lambda_1 \vec{a}_1 + ... + \lambda_n \vec{a} = 0$ shart bajarilsa, u sistema *chiziqli bog`liq sistema* deyiladi.Agar yuqoridagi tenglik faqat $\lambda_1 = ... = \lambda_n = 0$ bo`lganda o`rinli bo`lsa, $\vec{a}_1, ..., \vec{a}_n$ vektorlar sistemasi *chiziqli erkli* deyiladi.

Ikkita kollinear vektor har doim chiziqli bog`liqdir. Shuningdek ,uchta komplanar vektor har doim chiziqli bog`liq. Fazodagi ixtiyoriy to`rtta yoki undan ortiq vektorlar har doim chiziqli bog`liq.

n ta chiziqli bog`liqmas vektorlar sistemasi $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ berilgan bo`lib, agar ixtiyoriy $\vec{a}$ vektorni ularning chiziqli kombinatsiyasi, y`ani

$$\vec{a} = \lambda_1 \vec{e}_1 + ... + \lambda_n \vec{e}_n$$

shaklida ifodalash mumkin bo`l`sa, u holda berilgan sistema *bazis* deyiladi.

Bu tenglik $\vec{a}$ vektorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazis boyicha *yoyilmasi* deyiladi.

Fazoda chiziqli bog`liq bo`lmagan har qanday uchta $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ vektor bazis tashkil qiladi, shu sababli fazodagi harqanday $\lambda_1, \lambda_2, ..., \lambda_n$ $\vec{a}$ vektor shu bzis bo`yicha yoyilishi mumkin:

$$\vec{a} = \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2 + \lambda_3 \vec{e}_3$$

$\lambda_1, \lambda_2, .\lambda_3$ sonlar $\vec{a}$ vektorning berilgan bazisdagi koordinatalari bo`lib, quyidagicha yoziladi:

$$\vec{a} = \{\lambda_1, \lambda_2, \lambda_3\}$$

Agar bazisning vektorlari o`zaro perpendikulyar va birlik uzunlikka ega bo`lsa, bu bazis ortonormallangan bazis deyilib, u ortlar deb ataluvchi $\vec{i}, \vec{j}, \vec{k}$ vektorlar orqali belgilanadi.

Agar $\vec{i}, \vec{j}, \vec{k}$ mos ravishda *OX,OY,OZ* o`qlari bo`yicha yo`naltirilgan ortlar bo`lsa, u holda ixtiyoriy $\vec{a}$ vektorning $\vec{i}, \vec{j}, \vec{k}$ bazisdagi yoyilmasi quyidagicha ifodalanadi:

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \text{ yoki } \vec{a} = \{a_x; a_y; a_z\},$$

Bunda $a_x; a_y; a_z$ - $\vec{a}$ vektorning koordinatalari.

Masalan. $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ vektorning koordinatalari $\{2;-3;4\}$ bo`ladi.

$\vec{a}$ vektorning uzunligi uning moduli deb ataladi , $|\vec{a}|$ kabi belgilaniladi va quyidagi formula bilan hisoblaniladi

$$|\vec{a}| = \sqrt{\vec{a}_x^2 + \vec{a}_y^2 + \vec{a}_z^2}$$

Masalan. $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ vektorning uzunligi quyidagicha topiladi.

$|\vec{a}| = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29}$ J: $\sqrt{29}$

Boshlang`ich va oxiri nuqtalari ustma-ust tushadigan vektor nol-vektor deyiladi va $\vec{0}$ ga teng.

Uzunligi birga teng vektor *birlik vektor* deyiladi. $\vec{a}$ vektorning birlik vektori $\vec{a}^{\,0}$ kabi belgilanadi

$$\vec{a}^{\,0} = \frac{a_x}{\left|\vec{a}\right|} i + \frac{a_y}{\left|\vec{a}\right|} j + \frac{a_z}{\left|\vec{a}\right|} k$$

Masalan. $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ berilgan bo`lsa, $\vec{a}^{\,0}$ vektor

$$\left|\vec{a}\right| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

$$\vec{a}^{\,0} = \frac{1}{3} i + \frac{2}{3} j + \frac{2}{3} k \text{ ga teng.}$$

Bir to`g`ri chiziqda yoki parallel to`g`ri chiziqlarda yotuvchi vektorlar *kollinear* vektorlar deyiladi.

Agar ikki vektor o`zaro kollinear, bir xil yo`nalgan va modullari teng bo`lsa, bu vektorlar *teng vektorlar* deyiladi.

Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlarni *komplanar* vektorlar deyiladi.

$\vec{a}$ vektorning yo`nalishi uning koordinata o`qlari bilan hosil qilgan α, β, γ burchaklari bilan aniqlanadi.

$\vec{a}$ vektorning yo`naltiruvchi kosinuslari

$$cos\alpha = \frac{a_x}{\left|\vec{a}\right|} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \ \cos\beta = \frac{a_y}{\left|\vec{a}\right|} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \ \cos\gamma = \frac{a_z}{\left|\vec{a}\right|} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

formula bilan aniqlanadi va ular

$$coa^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

munosabat bilan bog`langan.

Masalan. $\vec{a} = 20i + 30j + -60k$ bektornin yo`naltiruvchi kosinuslari topilsin.

$$\left|\vec{a}\right| = \sqrt{20^2 + 30^2 + (-60)^2} = \sqrt{4900} == 70$$

$$cos\alpha = \frac{a_x}{|\vec{a}|} = \frac{20}{70} = \frac{2}{7}, \ \cos\beta = \frac{a_y}{|\vec{a}|} = \frac{30}{70} = \frac{3}{7}, \cos\gamma = \frac{a_z}{|\vec{a}|} = -\frac{60}{70} = -\frac{6}{7}$$

$$coa^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \text{ ga ko`ra}$$

$$\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 = \frac{4}{49} + \frac{9}{49} + \frac{36}{49} = \frac{49}{49} = 1.$$

Vektorlar ustida amallar.

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$ va $\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$ vektorlar berilgan bo`lsin. U holda

$$\vec{a} \pm \vec{b} = (a_x \pm b_x)\vec{i} + (a_y \pm b_y)\vec{j} + (a_z \pm b_z)\vec{k}$$

$$\lambda\vec{a} = \lambda a_x\vec{i} + \lambda a_y\vec{j} + \lambda a_z\vec{k}$$

Agar vektorning bosh va oxirgi nuqtalarining koordinatalari $A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ be rilgan bo`lsa, u holda $\vec{AB}$ vektorning ortlar bo`yicha yoyilmasi

$$\vec{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

ko`rinishds bo`ladi.

Masalan. $A(1;3;2)$ va $B(5;8;-1)$ nuqtalar berilgan. $\vec{AB} = u$ vektor uning koordinatalari aniqlansin.

Yechish:

$\vec{AB} = (5-1)\vec{i} + (8-3)\vec{j} + (-1-2)\vec{k} = 4\vec{i} + 5\vec{j} - 3\vec{k},$ J: $\{4;5;-3\}$

A va *B* nuqtalar orasidagi masofa yoki $\vec{AB}$ vektorning uzunligi

$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

formula bilan hisoblaniladi.

Masalan: $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ vektorning uzunligi topilsin.

Yechish: $|\vec{AB}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

J: 7 ga teng.

AB kesmani berilgan λ nisbatda bo`luvchi *M(x;y)* nuqtaning koordinatalari quyidagicha aniqlanadi;

$$x=\frac{x_1+\lambda x_2}{1+\lambda};\ y=\frac{y_1+\lambda y_2}{1+\lambda};\ z=\frac{z_1+\lambda z_2}{1+\lambda}.$$

Xususan, agar $\lambda=1$ bo`lsa, *M* nuqta *AB* kesmaning o`rtasida yotadi va uning koordinatalari

$$x=\frac{x_1+x_2}{2};\ y=\frac{y_1+y_2}{2};\ z=\frac{z_1+z_2}{2}.$$

munosabatlardan topiladi.

Masalan. $A(-2;1)$ va $B(3;6)$ nuqtalar berilgan. AB kesmani $AM:MB=3:2$ nisbatda bo`luvchi *M(x;y)* nuqta topilsin.

Yechish: $\lambda=AM:MB=\frac{3}{2}$ bo`lganligi uchun formulaga asosan

$$x=\frac{-2+1{,}5\cdot 3}{1+1{,}5}=\frac{2{,}5}{2{,}5}=1;\ y=\frac{1+1{,}5\cdot 6}{1+1{,}5}=\frac{10}{2{,}5}=4$$ j: *M(1;4)*

$\vec{a}=\vec{AB}$ vektorning *l* o`q bo`yicha tashkil etuvchi (komponenti) deb, shu vektor boshi va oxirining proeksiyalarini birlashtiruvchi $\vec{A_1B_1}$ vektorga aytiladi.

Fazoda nuqtaning hamda vektorning to`g`ri burchakli koordinatalari

$\vec{a}=\vec{AB}$ vektorning *l* o`q yo`nalishi bilan bir xil yoki bir xil emasligiga qarab, ”+” yoki ”-” ishora bilan olinadigan tashkil etuvchisining uzunligiga aytiladi.

$$pr_l\,\vec{AB}=\pm\left|\vec{A_1B_1}\right|$$

$\vec{a}$ vektorning *l* o`qqa proeksiyasi a_1 deb belgilanadi, yani:

$$pr_l\,\vec{a}=a_1$$

$A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ berilgan bo`lsa, u holda $\vec{AB}$ vektorning koordinata o`qlaridagi proeksiyalari quyidagilardan iborat:

$$\left.\begin{aligned} pr_x\,\vec{AB}&=X=x_2-x_1\\ pr_y\,\vec{AB}&=Y=y_2-y_1\\ pr_z\,\vec{AB}&=Z=z_2-z_1\end{aligned}\right\}$$

Proeksiyalarning asosiy xossalari:

a) $pr_l\,\vec{a}=\left|\vec{a}\right|\cos\varphi$ yoki $a_l=\left|\vec{a}\right|\cos\varphi$

Bunda $\varphi - \vec{a}$ vektor bilan o`q orasidagi burchak;

b) $pr_l\left(\vec{a}+\vec{b}\right) = pr_l\,\vec{a} + pr_l\,\vec{b}$ yoki $pr_l\left(\vec{a}+\vec{b}\right) = a_l + b_l$;

v) $pr_l\lambda\,\vec{a} = \lambda pr_l\,\vec{a}$ yoki $pr_l\lambda\,\vec{a} = \lambda a_l$.

Masalan. $\vec{a} = \{3;2;-5\}$ va $\vec{b} = \{2;-3;1\}$ vektorlar berilgan. $2\vec{a}-\vec{b}$ vektorning koordinata o`qlaridagi proeksiyalari topilsin.

Yechish: $2\vec{a}-\vec{b} = \{2\cdot 3 - 2; 2\cdot 2 - (-3); 2\cdot(-5) - 1\} = \{4;7;-11\}$.

Fazodagi nuqtaning radius-vektori.

$\vec{OM} = \vec{r}$ radius-vektorning moduli yoki uzunligi ushbu:

$$\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$$

formula bilan aniqlanadi.

Masalan. *M(5;-3;4)* nuqtaning radius-vektorining uzunligi topilsin.

Yechish: $\vec{OM} = \vec{r} = \sqrt{5^2 + (-3)^2 + 4^2} = \sqrt{50} = 5\sqrt{2}.$

Mavzu: Ikkita vektorning skalyar ko'paytmasi va uning xossalari. Ikki vektorlar orasidagi burchak.

Ikkita $\vec{a}$ va $\vec{b}$ vektorning *skalyar ko`paytmasi* deb, $\vec{a}\cdot\vec{b}$ ko`rinishda belgilanuvchi va shu vektorlar uzunliklari va ular orasidagi burchak kosinusi bilan ko`paytmasiga teng bo`lgan songa aytiladi:

$$\vec{a}\cdot\vec{b} = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot\cos\varphi$$

Skalyar ko`paytmaning asosiy xossalari:

a) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (o`rin almashtirish qonuni);

b) $\vec{a}\cdot\left(\vec{b}+\vec{c}\right) = \vec{a}\cdot\vec{b} + \vec{a}\,\vec{c}$ (taqsimot qonuni);

v) $(\lambda a)\cdot\vec{b} = \vec{a} = \vec{a}\left(\lambda\,\vec{b}\right) = \lambda\left(\vec{a}\cdot\vec{b}\right)$ (guruhlash qonuni);

g) agar $\vec{a} = 0$, yoki $\vec{b} = 0$, yoki $\vec{a}\perp\vec{b}$ bo`lsa, $\vec{a}\cdot\vec{b} = 0$ bo`ladi (vektorlarning ortogonallik sharti);

d) $\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$ yoki $\vec{a}^{\,2}=\left|\vec{a}\right|^2$;

e) $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot pr_{\vec{a}}\vec{b}=\left|\vec{b}\, pr_{\vec{b}}\vec{a}\right|$;

Koordinata o`qlari ortlarining skalyar ko`paytmasi:

$\vec{i}^{\,2}=1, \vec{j}^{\,2}=1, \vec{k}^{\,2}=1, \vec{i}\cdot\vec{j}=0, \vec{i}\cdot\vec{k}=0, \vec{j}\cdot\vec{k}=0,$

$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}\; va\; \vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$

bektorlar berilgan bo`lsin. U holda:

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z;$$

$$\vec{a}^{\,2}=\left|\vec{a}\right|^2=a_x^2+a_y^2+a_z^2$$

$\vec{a}$ va $\vec{b}$ vektorlar orasidagi φ burchak ushbu formula bo`yicha hisoblanadi:

$$\cos\varphi=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|}=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}.$$

$\vec{a}$ va $\vec{b}$ vektorlarning perpendikulyarlik sharti:

$$\vec{a}\cdot\vec{b}=0 \text{ yoki } a_xb_y+a_yb_y+a_zb_z=0.$$

$\vec{F}$ kuch jisimni l vector yo`nalishida $\vec{BC}$ masofaga ko`chirish natijasida bajargan ish ushbu formula bilan hisoblanadi:

$$A=\vec{F}\cdot\vec{BC}=\left|\vec{F}\right|\cdot\left|\vec{BC}\right|\cdot\cos\varphi,$$

bunda φ- ko`chish yo`nalishi $\vec{l}\; va\, \vec{F}$ kuchning ta`sir etuvchi orasidagi burchak.

Masalan.: $\vec{a}=i+3j+3k\; va\; \vec{b}=i+k$ vektorlarning skalyar ko`paytmasi topilsin.

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_za_z=1\cdot1+3\cdot0+3\cdot1=1+3=4$$

Masalan: $\vec{a}=-\vec{i}+\vec{j}\quad ва\; \vec{b}=\vec{i}-2\vec{j}+2\vec{k}$ vektorlar orasidagi burchak aniqlansin.

$$\cos\alpha=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|}=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a^2{}_x+a^2{}_y+a^2{}_z}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}=\frac{-1-2}{\sqrt{2}\cdot\sqrt{9}}=\frac{-3}{3\sqrt{2}}=-\frac{1}{\sqrt{2}}$$

$$\cos\alpha = -\frac{-1}{\sqrt{2}} \qquad \alpha = 135^0$$

Masalan. M ning qanday qiymatida $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$ ва $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ vektorlar perpendikulyar bo`ladi.

Yechish. Vektorlarning skalyar ko`paytmasini topamiz:

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z a_z = m\cdot 4 + 3\cdot m - 28 = 7m - 28;$$

$\vec{a} \perp \vec{b}$ bo`lsa, $\vec{a}\cdot\vec{b} = 0$ tengligidan *7m-28=0, m=4.* j: 4.

Masalan.

Agar $|\vec{a}| = 2, |\vec{b}| = 3, \vec{a} \perp \vec{b}$ bo`lsa, $\left(5\vec{a} + 3\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right)$ ni hisoblang.

Yechish.

$$\left(5\vec{a} + 3\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right) = 10\vec{a}^2 - 5\vec{a}\vec{b} + 6\vec{a}\vec{b} - 3\vec{b}^2 = 10\vec{a}^2 - 3\vec{b}^2 = 40 - 27 = 13.$$

Mavzu: Ikki vektorning vektor ko'paytmasi va uning xossalari. Uchlarining koordinatalari berilgan uchburchakning yuzi.

$\vec{a}$ vektorning $\vec{b}$ vektorga vektor ko`paytmasi deb $\vec{c} = \vec{a}\times\vec{b}$ ko`rinishda belgilanuvchi va quyidagi shartlarni qanoatlantiruvhi $\vec{c}$ vektorga aytiladi:

a) $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga *perpendikulyar*:

b) $\vec{c}$ vektor uchidan qaralganda $\vec{a}$ vektordan $\vec{b}$ vektorga eng qisqa burilish soat mili yo`nalishiga teskari yo`nalishda kyzatiladi($\vec{a}$, $\vec{b}$, $\vec{c}$ vektorning bunday joylashuvining *o`ng uchlik* deyiladi);

v) $\vec{c}$ vektorning moduli $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramning S yuziga teng, yani $|\vec{c}| = S = |\vec{a}||\vec{b}|\sin\varphi$ ($\varphi - \vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak).

Vektor ko`paytmaning asosiy xossalari:

a) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$;

b) $\left(\lambda\vec{a}\right)\times\vec{b} = \vec{a}\times\left(\lambda\vec{b}\right) = \lambda\left(\vec{a}\times\vec{b}\right)$;

v) $\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$;

g) Agar $\vec{a}=\vec{0}$, yoki $\vec{b}=\vec{0}$, yoki $\vec{a}//\vec{b}$ bo`lsa, u holda $\vec{a}\times\vec{b}=\vec{0}$. Xususan $\vec{a}\times\vec{a}=\vec{0}$.

Koordinata o`qlari *ortlauirning* vektor ko`paytmasi:

$$\vec{i}\times\vec{i}=0,\ \vec{j}\times\vec{j}=0,\vec{k}\times\vec{k}=0.$$

$$\vec{i}\times\vec{j}=\vec{k},\ \vec{j}\times\vec{k}=\vec{i},\vec{k}\times\vec{i}=\vec{j}.$$

Agar

$$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$$

$$\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$$

Vektorlar koordinatalari bilan berilgan bo`lsa, u holda vektorlar ko`paytma quyidagicha topiladi:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

Masalan: $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko`paytmasini toping.

$$\vec{a}=2j+k \quad va\ \vec{b}=\vec{i}+2\vec{k}$$

$$\vec{a}\,x\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 2 & 1\\ 1 & 0 & 2\end{vmatrix}=4\vec{i}+\vec{j}-2\vec{k}.$$

Agar $\vec{a}$ va $\vec{b}$ vektorlar *kollinear* bo`lsa, u holda

$$\frac{a_x}{b_x}=\frac{a_y}{b_y}=\frac{a_z}{b_z}.$$

$\vec{a}$ va $\vec{b}$ vektorlardan yasalgan *parallelogramning yuzi*:

$$S=\left|\vec{a}\times\vec{b}\right|,$$

shu vektorlarda yasalgan *uchburchakning yuzi*:

$$S_\Delta=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

Jism *A* nuqtasiga qo`yilgan $\vec{F}$ kuchning *O* nuqtaga nisbatan $\vec{M}$ *momenti*

$$\vec{M} = \vec{OA} \times \vec{F}$$

formula bilan hisoblanadi.

Masalan. $\vec{a} = 2\vec{i} - 3\vec{j}$ va $\vec{b} = 3\vec{i} + 4\vec{j}$ vektorlarga qurilgan parallelogramning yuzini toping.

Yechish: $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramning *S* yuzi shu vektorlar vektor ko`paytmasining moduliga teng: $s = \left|\vec{a} \times \vec{b}\right|$.

$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko`paytmasini toping.

$$\vec{a}\,x\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 3 & 0 & 4 \end{vmatrix} = -12\vec{i} - 8\vec{j} + 9\vec{k}\,.$$

Demak, $S = \sqrt{(-12)^2 + (-8)^2 + 9^2} = \sqrt{144 + 64 + 81} = 17kv.$ birlik.

Masalan. $\vec{a} = 2j + k \quad va\, \vec{b} = \vec{i} + 2\vec{k}$ vektorlardan yasalgan uchburchak yuzi topilsin.

Yechish:

$$\vec{a}\,x\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix} = 4\vec{i} + \vec{j} - 2\vec{k} == 4\vec{i} + \vec{j} - 2\vec{k}$$

Demak ,uchburchak yuzi

$$\text{S}=\frac{\left|\vec{a}x\vec{b}\right|}{2} = \frac{\sqrt{16+1+4}}{2} = \frac{\sqrt{21}}{2}\text{j: S}=\frac{\sqrt{21}}{2}kv.birlik.$$

Masalan. Uchlari $A(1;1;1), B(2;3;4)\ va\ C(4;3;2)$ nuqtalarda bo`lgan uchburchak yuzasi hisoblansin.

Yechish. $\vec{AB}\ va\ \vec{AC}$ vektorlarni topamiz:

$$\vec{AB} = (2-1)\vec{i} + (3-1)\vec{j} + (4-1)\vec{k} = \vec{i} + 2\vec{j} + 3\vec{k},$$
$$\vec{AC} = (4-1)\vec{i} + (3-1)\vec{j} + (2-1)\vec{k} = 3\vec{i} + 2\vec{j} + \vec{k}$$

$\vec{AB}\ va\ \vec{AC}$ vektorlardan yasalgan parallelogramning yuzini yarmi uchburchakning yuziga teng, shuning uchun $\vec{AB}\ va\ \vec{AC}$ vektorlarning vektor ko`paytmasini topamiz;

$$\vec{AB}\times\vec{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=-4\vec{i}+8\vec{j}-4\vec{k}.$$

Bundan

$$S_{ABC}=\frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|=\frac{1}{2}\sqrt{16+64+16}=\sqrt{24}(kv.bir.)\quad \text{j:}\sqrt{24}kv.bir.$$

Masalan. $\vec{a}+3\vec{b}$ *va* $3\vec{a}+\vec{b}$ vektorlardan yasalgan parallelogramning yuzini hisoblang, agar $\left|\vec{a}\right|=\left|\vec{b}\right|=1, \left(\vec{a},\hat{\ }\vec{b}\right)=30^0$ ga teng bo`lsa.

Yechish.

$$\left(\vec{a}+3\vec{b}\right)\times\left(3\vec{a}+\vec{b}\right)=3\vec{a}\times\vec{b}+\vec{a}\times\vec{b}+9\vec{b}\times\vec{a}+3\vec{b}\times\vec{b}=$$

$$=3\cdot 0+\vec{a}\times\vec{b}-9\vec{a}\times\vec{b}+3\cdot 0=-8\vec{a}\times\vec{b}$$

($\vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0,\ \vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$ ekanligidan) . Demak,

$$S=8\left|\vec{a}\times\vec{b}\right|=8\cdot 1\cdot 1\cdot \sin 30^0=4(kv.bir.)\text{j: 4 kv.birlik.}$$

Mavzu: Uchta vektorning aralash ko'paytmasi va uning geometrik ma'nosi. Uchta vektorning komplanarlik sharti.

Ta`rif. $\vec{a}$ ,$\vec{b}$ va $\vec{c}$ vektorlarning aralash ko`paytmasi deb $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$ ko`rinishdagi ifodaga aytiladi.

Agar $\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlar o`zlarining koordinatalari bilan berilgan bo`lsa, u holda aralash ko`paytma quyidagicha ifodalanadi:

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\begin{vmatrix}a_ч & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}.$$

Aralash ko`paytma xossalari.

a) $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=-\left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=-\left(\vec{c}\times\vec{b}\right)\cdot\vec{a}$;

b) $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{a}\,\vec{b}\,\vec{c}$;

c) $\vec{a}\,\vec{b}\,\vec{c}=\vec{b}\,\vec{c}\,\vec{a}=\vec{c}\,\vec{a}\,\vec{b}$;

d) agar vektorlardan aqalli bittasi *nol vektor* yoki $\vec{a}$,$\vec{b}$, $\vec{c}$ vektorlar *komplanar* bo`lsa, y holda $\vec{a}\,\vec{b}\,\vec{c} = 0$ bo`ladi.

Agar $\vec{a},\vec{b},\vec{c}$ vektorlar *komplanar* bo`lsa , u holda

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0 .$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlardan yasalgan *parallelepipedning hajmi*:

$$V = \pm \vec{a}\,\vec{b}\,\vec{c} \begin{cases} + \text{ vektorlar o`ng bog`lam tashkil etadi,} \\ - \text{ vektorlar chap bog`lam tashkil etadi.} \end{cases}$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlardan yasalgan *piramidaning* hajmi:

$$V_{pir.} = \pm\frac{1}{6}\vec{a}\,\vec{b}\,\vec{c}$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlarda yasalgan *tetraedrning* hajmi:

$$V_{tetroed..} = \pm\frac{1}{3}\vec{a}\,\vec{b}\,\vec{c}$$

Masalan. Uchta vektorning aralash ko`paytmasini toping.

$$\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}; \qquad \vec{b} = \vec{i} + 4\vec{j} - 5\vec{k} \text{ va } \vec{c} = 3\vec{i} - 2\vec{j} + 6\vec{k} .$$

Yechish:

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 4 & -5 \\ 3 & -2 & 6 \end{vmatrix} = 48 - 8 + 45 - 48 - 20 + 18 = 35 . \qquad \text{J:35.}$$

Masalan. $a = -i + 3j + 2k, b = 2i - 3j - 4k, c = -3i + 12j + 6k$ vektorlarning o`zaro komplanar ekani ko`rsatilsin.

Yechish:

$$abc = \begin{vmatrix} -1 & 3 & 2 \\ 2 & -3 & -4 \\ -3 & 12 & 6 \end{vmatrix} = 18 + 48 + 36 - 18 - 48 - 36 = 0;$$

Masalan. Uchlari $A(1;2;0), B(-1;2;1), C(0;-3;2)$ *va* $D(1;0;1)$ nuqtalarda bo`lgan piramidaning hajmini hisoblang.

Yechish. Piramidaning A uchidan chiqqan qirralariga mos keluvchi vektorlarni topamiz:

$$\vec{AB}=\{-2;0;1\},\vec{AC}=\{-1;-5;2\},\vec{AD}=\{0;-2;1\}.$$

Piramidaning hajmi shu vektorlarga qurilgan parallelepiped hajmining $\frac{1}{6}$ qismiga teng bo`lganligi sababli

$$V=\pm\frac{1}{6}\begin{vmatrix}-2 & 0 & 1\\ -1 & -5 & 2\\ 0 & -2 & 1\end{vmatrix}=\frac{1}{6}\cdot 4=\frac{2}{3}kub.birlik.$$

MISOLLAR.

1. $\overline{a}(3,2)$, $\vec{b}(5;1)$, $\vec{c}(-1,3)$ vektorlar berilgan $2\vec{a}+3\vec{b}-\vec{c}$, $16\overline{a}+5\vec{b}-9\vec{c}$ vektorlarning koordinatalarini toping.

2. $\overline{a}(3,0,-2)$, $\vec{b}(1,2,-5)$, $c(-1,1,1)$, $\vec{d}(8,4,1)$ vektorlar berilgan $-5\overline{a}+\vec{b}-6\vec{c}+\vec{d}$, $3\overline{a}-\vec{b}-\vec{c}-\vec{d}$ vektorlarning koordinatalarini toping.

3. A(2;2;0) va B(0;-2;5) nuqtalar berilgan. $\vec{AB}=u$ vektor yasalsin hamda uning uzunligi va yo`naltiruvchi kosinuslari aniqlansin.

4. a) $\vec{a}=\{12;-15;-16\}$ vektorning yo'naltiruvchi kosinuslarini toping.

b) $\vec{a}=\{3;-2;6\}$ va $\vec{b}=\{-2;1;0\}$ vektorlar berilgan 1) $\vec{a}+\vec{b}$ 2) $\vec{a}-\vec{b}$ 3) $2\vec{a}$ vektorlarning koordinatalarini toping.

5. α va β ning qanday qiymatlarida $\vec{a}=-2i+3j+\beta k$ va $\vec{b}=\alpha i-6j+2k$ vektorlar kollinear bo'ladi.

6. Uchburchakning *A(-1;-2;4), B(-4;-2;0) va C(3;-2;1)* uchlari berilgan. Uning *B* uchidagi ichki burchagini toping.

7. To'rtburchakning *A(1;-2;2), B(1;4;0), C(-4;1;1)* va *D(-5;5;3)* uchlari bo'lsa, *AC* va *BD* diagonallarining perpendikulyarligini isbotlang.

8. $\vec{a}=\{2;-4;4\}$ $\vec{b}=\{-3;2;6\}$ vektorlar orasidagi burchakning kosinusini toping.

9. $\vec{a}=\{5;2;5\}$ va $\vec{b}=\{2;-1;2\}$ vektorlar berilgan. $pr_{\vec{b}}\vec{a}$ *va* $pr_{\vec{a}}\vec{b}$ lar aniqlansin.

10. Ushbuamallarni bajaring.

1. $\vec{i}\times\left(\vec{j}+\vec{k}\right)-\vec{j}\times\left(\vec{i}+\vec{k}\right)+\vec{k}\times\left(\vec{i}\times\vec{j}+\vec{k}\right)=$

2. $\left(\vec{a}+\vec{b}+\vec{c}\right)\times\vec{c}+\left(\vec{a}+\vec{b}+\vec{c}\right)\times\vec{b}+\left(\vec{b}-\vec{c}\right)\times\vec{a}=$

3. $\left(2\vec{a}+\vec{b}\right)\times\left(\vec{c}-\vec{a}\right)+\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}\right)=$

4. $2\vec{i}\cdot\left(\vec{j}\times\vec{k}\right)+3\vec{j}\cdot\left(\vec{i}\times\vec{k}\right)+4\vec{k}\left(\vec{i}\times\vec{j}\right)=$

11. *A(1;2;0), B(3;0;-3), C(5;2;6)* nuqtalar berilgan *ABC* uchburchakning yuzini toping. Ma`nosi

12. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\vec{a}\times\vec{b}$ ekani isbotlansin.

11. Uchburchakning uchlari *A(1;-1;2), B(5;-6;2), C(1;3;-1)* bo'lsin. Uning *B* uchidan *AC* tomonga tushirilgan balandligini hisoblang.

12. $\vec{a}=\{2;-2;1\}$, $\vec{b}=\{2;3;6\}$ vektorlar orasidagi burchakning sinusini toping..

13. $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar, $\widehat{\vec{a}\vec{b}}=30^0$, $|a|=6$, $|b|=3$, $|c|=3$ bo'lsa, $\vec{a}\vec{b}\vec{c}$ aralash ko'paytmani hisoblang.

14. $\vec{a}=\{2;3;-1\}$ $\vec{b}=\{1;-1;3\}$ $\vec{c}=\{1;9;-1\}$ vektorlarni komplanarligini tekshiring.

15. *A(1;2;-1), B(0;1;5), C(-1;2;1)* va *D(2;1;3)* nuqtalar bir tekislikda yotishini isbotlang.

16. Uchlari *A(2;-1;1), B(5;5;4), C(-1;2;1)* va *D(2;1;3)* nuqtalarda bo'lgan tetraedrning hajmini hisoblang.

17. Tetraedrning uchlari *A(2;3;1), B(4;1;-2), C(6;3;7)* va *D(-5;-4;8)* bo'lsin. Uning *D* uchidan tushirilgan balandligini hisoblang.

18. Tetraedrning hajmi *V=5* uning uchta uchlari *A(2;1;-1), B(3;0;1)* va *C(2;-1;3)*. Agar uning to'rtinchi *D* uchi *OY* o'qida yotsa *D* ning koordinatalarini toping.

19. $\vec{a}$ *={3;-1;-2}* va $\vec{b}$ *={1;2;-1}* vektorlar berilgan. 1) $\vec{a}\times\vec{b}$ 2) $\left(2\vec{a}+\vec{b}\right)\times\vec{b}$ vektor ko'paytmaning koordinatalarini toping.

20. $\left(\vec{a}+\vec{b}\right)^2+\left(\vec{a}-\vec{b}\right)^2=2\left(\vec{a}^2+\vec{b}^2\right)$ ayniyatni isbotlang va uning geometric ma'nosini ifodalang.

21. $\vec{a}=\{2;-1;3\}$ $\vec{b}=\{-6;3;-9\}$ vektorlarning kollinearligini isbotlang va ularning uzunliklarini va yo'nalishlarini taqqoslang.

22. *A(3;-1;2), B(1;2;-1) C(-1;1;-3)* va *D(3;-5;3)* nuqtalar trapetsiyaning uchlari ekanini ko'rsating.

23. *A(-1;5;-10)*, *B(5;-7;8)*, *C(2;2;-7)* va *D(5;-4;2)* nuqtalar berilgan. $A\overline{B}$ va $C\overline{D}$ vektorlar kollinear ekanini isbotlang.

24. A(2;2;0) va B(0;-2;5) nuqtalar berilgan. $\vec{AB}=u$ vektor yasalsin hamda uning uzunligi va yo`nalishi aniqlansin.

1-topshiriq.

ABCD piramidaning uchlari berilgan.

a) Piramidani berilgan qirralari orasidagi burchak kosinusini toping;

b) piramidaning berilgan yog`i yuzini toping:

1. $A(6;-4;1),\ B(6;3;-1),\ C(2;5;7),\ D(-4;-2;3);$
$a)\ AB\ va\ AC;\ \ b)\ ACD$

2. $A(6;4;-7),\ B(-5;-4;2),\ C(5;7;-4),\ D(4;2;3);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

3. $A(-2;87),\ B(6;-2;-3),\ C(8;2;-3),\ D(3;5;3);$
$a)\ CA\ va\ CD;\ \ b)\ BAD$

4. $A(4;4;3),\ B(2;-4;5),\ C(-1;3;-4),\ D(4;-7;-9);$
$a)\ DA\ va\ DB;\ \ b)\ DAC$

5. $A(-5;-3;2),\ B(4;-2;-4),\ C(5;7;2),\ D(1;3;4);$
$a)\ AB\ va\ AD;\ \ b)\ CBD$

6. $A(-5;6;4),\ B(-6;2;4),\ C(9;-5;3),\ D(7;2;-8);$
$a)\ BC\ va\ BA;\ \ b)\ DAC$

7. $A(1;-9;7),\ B(3;-5;1),\ C(-9;3;-5),\ D(2;4;7);$
$a)\ CB\ va\ CD;\ b)\ ABD$

8. $A(4;-2;9),\ B(3;5;-1),\ C(5;1;7),\ D(-6;-3;5);$
$a)\ DA\ va\ DC;\ \ b)\ ABC$

10. $A(2;-5;1),\ B(3;-6;-7),\ C(-9;-6;7),\ D(7;2;5);$
$a)\ BD\ va\ BA;\ \ b)\ CAD$

11. $A(2;-5;-3),\ B(9;7;3),\ C(8;7;1),\ D(-2;-1;7);$
$a)\ CA\ va\ CB;\ \ b)\ ABD$

12. $A(67;4;3),\ B(0;-4;8),\ C(-3;1;5),\ D(-5;-6;-7);$
$a)\ DB\ va\ DC;\ \ b)\ ABC$

13. $A(-9;2;6),\ B(-7;2;3),\ C(5;-6;-4),\ D(4;-4;5);$
$a)\ AB\ va\ AC;\ \ b)\ DBC$

14. $A(-3;0;4),\ B(8;-6;5),\ C(4;-4;-3),\ D(6;3;5);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

15. $A(-3;8;2),\ B(-8;2;4),\ C(3;-7;5),\ D(5;4;-6);$
$a)\ CA\ va\ CD;\ \ b)\ BCD$

16. $A(5;-3;9),\ B(8;-5;1),\ C(-7;5;-3),\ D(4;2;5);$
$a)\ DA\ va\ DC;\ \ b)\ BAC$

17. $A(5;-1;6),\ B(-6;7;5),\ C(2;5;7),\ D(2;1;3);$
$a)\ AC\ va\ AD;\ \ b)\ BCD$

18. $A(1;2;3),\ B(3;-3;2),\ C(7;-5;4),\ D(-3;-7;-4);$
$a)\ BD\ va\ BA;\ \ b)\ CAD$

19. $A(4;-3;1),\ B(0;-3;5),\ C(-3;-2;1),\ D(9;4;7);$
$a)\ CA\ va\ CB;\ \ b)\ ABD$

20. $A(5;-4;-2),\ B(7;5;1),\ C(3;2;-4),\ D(-2;-5;3);$
$a)\ DB\ va\ DC;\ \ b)\ ABC$

21. $A(-7;2;3),\ B(0;-2;6),\ C(-1;3;7),\ D(-3;-4;-5);$
$a)\ AB\ va\ AD;\ \ b)\ CBD$

22. $A(-7;6;4),\ B(-4;1;1),\ C(3;-2;6),\ D(6;-2;3);$
$a)\ BC\ va\ BA;\ \ b)\ ACD$

23. $A(4;1;5),\ B(5;-3;2),\ C(3;-5;-4),\ D(8;5;7);$
$a)\ DA\ va\ DC;\ \ b)\ ABD$

24. $A(-5;4;2),\ B(-4;6;2),\ C(1;-5;3),\ D(3;6;-4);$
$a)\ DB\ va\ DC;\ \ b)\ BAC$

25. $A(3;-5;6;),\ B(6;-3;4),\ C(-5;3;-2),\ D(2;4;3);$
$a)\ AB\ va\ AC;\ \ b)\ DBC$

26. $A(4;-2;8),\ B(-2;2;3),\ C(6;4;1),\ D(-4;;-3;-5);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

27. $A(-3;2;4;),\ B(-2;5;3),\ C(6;4;1),\ D(4;;-2;-3);$
$a)\ CA\ va\ CD;\ \ b)\ BAD$

28. $A(-4;4;3),\ B(4;-3;-2),\ C(6;4;-1),\ D(1;3;1);$
$a)\ DA\ va\ DB;\ \ b)\ CAB$

29. $A(2;2;1),\ B(4;-2;3),\ C(-3;5;-2),\ D(6;;5;-7);$
$a)\ AC\ va\ AD;\ b)\ BCD$

30. $A(-3;-6;3),\ B(6;-3;-2),\ C(1;2;1),\ D(5;4;-3);$
$a)\ BC\ va\ BD;\ b)\ ACD$

2-topshiriq.

A,B va C nuqtalarning koordinatalari berilgan.

a) $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak kosinusini;

b) $\alpha\vec{a}+\beta\vec{b}$ vektorning $\vec{a}$ vektor yo`nalishidagi proeksiyasini toping:

1. $A(9;10;1),\ B(7;6;-1),\ C(4;0;-4),$
$\vec{a}=2\vec{AB}-3\vec{AC},\vec{b}=4\vec{BC}+\vec{AC};\alpha=1,\beta=2.$

2. $A(0;2;1),\ B(1;2;0),\ C(0;3;-1),$
$\vec{a}=3\vec{AC}-3\vec{BC},\vec{b}=2\vec{AB}+5\vec{BC};\alpha=-1,\beta=2.$

3. $A(0;4;8),\ B(-5;4;-2),\ C(-1;4;1),$
$\vec{a}=\vec{AB}-4\vec{AC},\vec{b}=4\vec{AC}+\vec{AB};\alpha=-2,\beta=3.$

4. $A(3;0;1),\ B(-2;3;2),\ C(1;1;-2),$
$\vec{a}=\vec{BC}-3\vec{AB},\vec{b}=6\vec{BC}+5\vec{AC};\alpha=2,\beta=-3.$

5. $A(4;1;-3),\ B(5;1;-2),\ C(-1;3;3),$
$\vec{a}=4\vec{AC}-2\vec{CB},\vec{b}=7\vec{AB}+5\vec{BC};\alpha=\beta=3.$

6. $A(4;1;1),\ B(3;1;2),\ C(0;1;-2),$
$\vec{a}=3\vec{BC}-4\vec{CA},\vec{b}=6\vec{BA}-\vec{AC};\alpha=3,\beta=2.$

7. $A(-3;4;-5),\ B(0;1;-2),\ C(-1;2;3),$
$\vec{a}=4\vec{AB}-3\vec{BC},\vec{b}=25\vec{CA}-2\vec{BA};\ \alpha=-2,\beta=5.$

8. $A(7;5;-2),\ B(6;0;0),\ C(7;2;2),$
$\vec{a}=\vec{AB}-3\vec{BC},\vec{b}=2\vec{CB}+5\vec{AC};\alpha=-4,\beta=2.$

9. $A(-3;-7;-3),\ B(-1;-3;-1),\ C(2;3;2),$
$\vec{a}=2\vec{BC}-5\vec{AB},\vec{b}=5\vec{AC}-5\vec{CB};\alpha=-3,\beta=1.$

10. $A(2;-1;8),\ B(3;1;7),\ C(2;0;7),$
$\vec{a}=\vec{AB}-3\vec{BC},\vec{b}=2\vec{CB}-2\vec{AC};\alpha=5,\beta=6.$

11. $A(-1;-1;8),\ B(4;-1;-2),\ C(0;-1;1),$
$\vec{a}=6\vec{BC}-32\vec{AB},\vec{b}=2\vec{AC}+5\vec{AB};\alpha=-4,\beta=3.$

12. $A(-2;4;-2),\ B(3;1;0),\ C(0;3;-4),$
$\vec{a}=3\vec{AB}-4\vec{AC}, \vec{b}=2\vec{BC}+5\vec{CA}; \alpha=3, \beta=-6.$

13. $A(1;1;4),\ B(-2;1;5),\ C(-1;3;3),$
$\vec{a}=4\vec{AC}-2\vec{BC}, \vec{b}=2\vec{AC}+3\vec{AB}; \alpha=-5, \beta=3.$

14. $A(4;2;6),\ B(2;2;8),\ C(-4;2;0),$
$\vec{a}=35\vec{AB}-7\vec{AC}, \vec{b}=2\vec{BC}+3\vec{BA}; \alpha=9, \beta=12.$

15. $A(15;-12;0),\ B(6;-3;0),\ C(9;-6;3),$
$\vec{a}=\vec{AC}-6\vec{BC}, \vec{b}=\vec{AB}+3\vec{BC}; \alpha=-7, \beta=6.$

16. $A(-1;-5;-2),\ B(0;-6;4),\ C(-1;-8;2),$
$\vec{a}=3\vec{BC}+5\vec{AB}, \vec{b}=\vec{AC}-+3\vec{AB}; \alpha=-3, \beta=4.$

17. $A(-1;-10;-5),\ B(1;-6;-3),\ C(0;0;4),$
$\vec{a}=2\vec{BC}-3\vec{AC}, \vec{b}=4\vec{AB}+3\vec{BC}; \alpha=4, \beta=-6.$

18. $A(-3;3;7),\ B(-2;3;6),\ C(-3;2;6),$
$\vec{a}=4\vec{AB}-\vec{AC}, \vec{b}=2\vec{BC}-3\vec{BA}; \alpha=-3, \beta=8.$

19. $A(2;-2;-8),\ B(5;-2;-4),\ C(1;-2;-1),$
$\vec{a}=5\vec{AB}-3\vec{BC}, \vec{b}=4\vec{CA}+\vec{AB}; \alpha=-4, \beta=1.$

20. $A(1;2;4),\ B(-4;1;-6),\ C(-1;1;2),$
$\vec{a}=3\vec{CA}-2\vec{BC}, \vec{b}=2\vec{AB}+4\vec{BC}; \alpha=3, \beta=-5.$

21. $A(1;1;4),\ B(-2;5;1),\ C(-1;3;3),$
$\vec{a}=3\vec{AB}+\vec{AC}, \vec{b}=2\vec{BC}+-3\vec{AB}; \alpha=3, \beta=-4.$

22. $A(0;1;-2),\ B(3;1;2),\ C(4;1;1),$
$\vec{a}=2\vec{AC}-3\vec{BA}, \vec{b}=3\vec{BC}-4\vec{AB}; \alpha=-2, \beta=6.$

23. $A(6;-8;10),\ B(0;-2;4),\ C(2;-4;6),$
$\vec{a}=3\vec{AB}+6\vec{BC}, \vec{b}=2\vec{AC}-5\vec{BC}; \alpha=2, \beta=8.$

24. $A(0;3;2),\ B(-2;-1;0),\ C(-5;-7;-3),$
$\vec{a}=5\vec{BC}-2\vec{CA}, \vec{b}=6\vec{AB}+4\vec{AC}; \alpha=-2, \beta=5.$

25. $A(-1;4;6),\ B(0;2;5),\ C(-1;3;5),$
$\vec{a}=8\vec{AC}-4\vec{AB}, \vec{b}=2\vec{BC}-6\vec{AB}; \alpha=-3, \beta=-4.$

26. $A(1;-2;3),\ B(4;-2;-1),\ C(0;-2;4),$
$\vec{a}=32\vec{AC}-3\vec{BC}, \vec{b}=3\vec{AB}-4\vec{BC}; \alpha=2, \beta=1.$

27. $A(-1;1;1),\ B(-6;4;3),\ C(-3;2;-1),$
$\vec{a} = \vec{AB} - 3\vec{BC},\ \vec{b} = \vec{AC} + \vec{BC};\ \alpha = 4,\ \beta = -6.$

28. $A(1;1;4),\ B(-2;5;5),\ C(-1;3;3),$
$\vec{a} = 2\vec{AC} - 3\vec{BC},\ \vec{b} = 2\vec{AB} + \vec{BC};\ \alpha = -2,\ \beta = 6.$

29. $A(-3;-2;-1),\ B(-1;-2;0),\ C(0;-1;-1),$
$\vec{a} = 3\vec{BC} - 4\vec{BC},\ \vec{b} = 2\vec{AC} + 3\vec{BC};\ \alpha = -6,\ \beta = 4.$

30. $A(5;-4;3),\ B(2;-1;0),\ C(3;-2;1),$
$\vec{a} = \vec{BC} + \vec{AC},\ \vec{b} = 2\vec{AB} - 3\vec{CA};\ \alpha = -5,\ \beta = 3.$

3-topshiriq.

Piramidaning uchlari A,B,C,D berilgan.

a) Ko`rsatilgan yoq yuzini;b) piramidani *l* qirrasi va berilgan ikkita uchidan o`tuvchi kesim yuzini; v) piramidani hajmini hisoblang:

1. $A(5;-4;3),\ B(2;-1;0),\ C(3;-2;1),\ D(0;2;1)$
$a)\ ABC;\ b)\ l = AD,\ B\ va\ C.$

2. $A(0;1;2),\ B(1;-2;2),\ C(-1;2;1),\ D(2;0;1)$
$a)\ BCD;\ b)\ l = BA,\ D\ va\ C.$

3. $A(-5;-4;3),\ B(6;-1;2),\ C(1;0;1),\ D(0;2;1)$
$a)\ ACD;\ b)\ l = CB,\ A\ va\ D.$

4. $A(2;-1;1),\ B(-3;0;-6),\ C(-5;3;-2),\ D(-1;10;3)$
$a)\ ABD;\ b)\ l = CD,\ A\ va\ B.$

5. $A(1;-3;7),\ B(-1;0;3),\ C(-4;-2;1),\ D(4;2;-1)$
$a)\ ABD;\ b)\ l = BD,\ A\ va\ C.$

6. $A(-4;1;3),\ B(5;-1;2),\ C(2;1;-4),\ D(1;-3;0)$
$a)\ BCD;\ b)\ l = AC,\ A\ va\ C.$

7. $A(5;3;-4),\ B(1;0;3),\ C(2;-1;4),\ D(0;3;1)$
$a)\ ACD;\ b)\ l = AB,\ C\ va\ D.$

8. $A(3;7;-4),\ B-4(-4;1;3),\ C(2;3;0),\ D(-1;-1;-2)$
$a)\ ABD;\ b)\ l = BC,\ A\ va\ D.$

9. $A(-8;2;-5),\ B(-1;-3;0),\ C(-4;1;2),\ D(6;-5;-3)$
$a)\ ABC;\ b)\ l = BC,\ C\ va\ D.$

10. $A(7;-10;-3),\ B(3;-3;-1),\ C(0;-6;5),\ D(-3;-4;2)$
$a)\ BCD;\ b)\ l = AD,\ B\ va\ C.$

11. $A(-3;6;-4),\ B(1;0;-1),\ C(1;2;2),\ D(6;3;1)$
$a)\ ACD;\ b)\ l = BD, A\ va\ C.$

12. $A(-4;2;-5),\ B(8;5;-10),\ C(0;-3;2),\ D(6;2;-4)$
$a)\ ABD;\ b)\ l = AC, B\ va\ D.$

13. $A(1;2;-4),\ B(1;1;3),\ C(-2;-1;7),\ D(4;2;7)$
$a)\ ABC;\ b)\ l = AD, B\ va\ C.$

14. $A(6;-3;-6),\ B(2;-3;-7),\ C(2;5;-1),\ D(4;1;2)$
$a)\ BCD;\ b)\ l = AB, C\ va\ D.$

15. $A(7;6;-10),\ B(-3;6;3),\ C(-3;0;-6),\ D(2;-5;-1)$
$a)\ ACD;\ b)\ l = CB, A\ va\ D.$

16. $A(3;-6;-1),\ B(-9;-5;1),\ C(5;3;-2),\ D(-1;-1;0)$
$a)\ ABD;\ b)\ l = CD, A\ va\ B.$

17. $A(1;1;-1),\ B(4;2;1), C(0;5;2),\ D(0;2;5)$
$a)\ ABC;\ b)\ l = BD, A\ va\ C.$

18. $A(-7;9;-10),\ B(-6;0;5),\ C(1;2;1),\ D(-2;-1;2)$
$a)\ BCD;\ b)\ l = AC, B\ va\ D.$

19. $A(6;-4;1),\ B(-4;-8;4),\ C(1;7;-1),\ D(-4;0;-2)$
$a)\ ACD;\ b)\ l = AB, C\ va\ D.$

20. $A(-1;2;-2),\ B(-3;-6;-2),\ C(2;-3;-5),\ D(5;4;14)$
$a)\ ABD;\ b)\ l = CB,\ A\ va\ D.$

21. $A(-9;4;8),\ B(6;2;5),\ C(-3;0;3),\ D(0;2;1)$
$a)\ ABC;\ b)\ l = CD, A\ va\ B.$

22. $A(5;2;-4),\ B(1;2;3),\ C(-1;2;1),\ D(2;-1;2)$
$a)\ BCD;\ b)\ l = AD, B\ va\ C.$

23. $A(-2;0;-1),\ B(4;-2;2),\ C(3;1;-1),\ D(2;1;1)$
$a)\ ACD;\ b)\ l = BD, A\ va\ C.$

24. $A(-3;5;7),\ B(7;3;6;),\ C(-2;1;4),\ D(1;3;2)$
$a)\ ABD;\ b)\ l = AC, B\ va\ D.$

25. $A(-8;9;5),\ B(1;2;3),\ C(2;3;1),\ D(-1;1;1)$
$a)\ ABD;\ b)\ l = AD, B\ va\ C.$

26. $A(-12;8;-4),\ B(3;7;-2),\ C(3;6;-3),\ D(-7;5;1)$
$a)\ BCD;\ b)\ l = AB, C\ va\ D.$

27. $A(4;5;2),\ B(0;-2;-5),\ C(-4;5;1),\ D(-7;4;-3)$
$a)\ ACD;\ b)\ l = CB, A\ va\ D.$

28. $A(5;4;3),\ B(-2;1;2),\ C(0;-1;4),\ D(-3;2;-1)$
$a)\ ABD;\ b)\ l = CD, A\ va\ D.$

29. $A(-6;2;8),\ B(1;-5;0),\ C(0;1;-2),\ D(3;-1;4)$
$a)\ ABC;\ b)\ l = BD, A\ va\ C.$

30. $A(-4;-2;2),\ B(-1;1;2),\ C(3;0;-2),\ D(1;-1;1)$
$a)\ BCD;\ b)\ l = AC, B\ va\ D.$

TESTLAR

1. $\left|\vec{a}\right| = 4,\ \left|\vec{b}\right| = 3,\ (\vec{a} \wedge \vec{b}) = 60^0$. λ ning qanday qiymatida $(\vec{a} + \lambda\vec{b}) \perp \vec{a}$ bo'ladi?

A) $2\frac{2}{3}$ B) $-2\frac{2}{3}$ C) $1\frac{2}{3}$ D) 1 E) –1

2. $\vec{a} = \{2;\ 3;\ 4\}, \vec{b} = \{-2; 5; -3\}$ vektorlar berilgan . $\vec{a} + \vec{b}$ =?

A) {0;8;1} B) {0;7;1} C) {0;8;-1} D) {1;8;1} E) {0;-8;1}

3. $\vec{a} = \{2;\ 3;\ 4\}$ va $\vec{b} = \{-2; 5; -3\}$ vektorlarni skalyar ko'paytiring.

A) 0 B) 1 C) -1 D) 2 E) –2

4. $\vec{a} = \{-3; 4; -2\}$ vektorni 3 ga ko'paytiring va uzunligini toping.

A) $\sqrt{29}$ B) $\sqrt{260}$ C) $\sqrt{226}$ D) $\sqrt{261}$ E) $\sqrt{262}$

5. B(4;2;0) nuqta $\vec{a} = \{-2;\ 3; -1\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (6;-1;-1) B) (6;1;-1) C) (-6;-1;1) D) (-6;-1;-1) E) (6;-1;1)

6. $\vec{a} = \{0;1\}$ va $\vec{b} = \{2;1\}$ vektorlar berilgan. x ning qanday qiymatlarida $\vec{b} + x\vec{a}$ vektor $\vec{b}$ vektorga perpendikulyar bo'ladi?

A) 2 B) -2 C) 5 D) –5 E) 0

7. $\vec{a} = \{1; 2; 2\}$ vektorning birlik vektori toping.

A) (6;-1;-1) B) (6;1;-1) C) (-6;-1;1) D) (-6;-1;-1) E) (6;-1;1)

8. $\vec{a} = \{2;-3;1\},\ \vec{b} = \{1;2;-4\},\ \vec{c} = \{5;-4;6\}$ vektorlarga qurilgan parallelepipedning hajmini toping.

A) 53 B) 54 C) 55 D) 56 E) 58

9. $\left|\vec{a}\right| = 4,\ \left|\vec{b}\right| = 3,\ (\vec{a} \wedge \vec{b}) = 60^0$. λ ning qanday qiymatida $(2\vec{a} - \lambda\vec{b}) \perp \vec{b}$ bo'ladi ?

A) $\frac{4}{3}$ B) $\frac{3}{4}$ C) $-\frac{3}{4}$ D) 1. E) -1

10. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}| = |\vec{b}| = 3$ bo'lsa, $(\vec{a}+\vec{b})$ bilan $(\vec{a}-\vec{b})$ vektorlar orasidagi burchakni toping.

A) 30^o B) 45^o C) 60^o D) 90^o E) 120^o

11. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}| = |\vec{b}| = 2$ bo'lsa, $(\vec{a}-\vec{b})$ bilan $(\vec{a}+\vec{b})$ vektorlar qanday burchak tashkil etadi?

A) 30^o B) 45^o C) 60^o D) 90^o E) 120^o

12. $\vec{b} = \{0;-2\}$ va $\vec{c} = \{-3;4\}$ vektorlar berilgan. $\vec{a} = 3\vec{b} - 2\vec{c}$ vektorning koordinatalarini toping.

A) (6;-11) B) (6;-1) C) (-6;-14) D) (-6;14) E) (6;-14)

13. $\vec{a} = \{2;-3\}$ va $\vec{b} = \{-2;-3\}$ vektorlar berilgan. $\vec{m} = \vec{a} - 2\vec{b}$ vektorning koordinatalarini toping.

A) (6;3) B) (6;-3) C) (-6;3) D) (-6;-3) E) (6;-4)

14. Agar $\vec{a} = \{1;2;3\}$ va $\vec{b} = \{4;-2;9\}$ bo'lsa, $\vec{c} = \vec{a} + \vec{b}$ vektorning uzunligini toping.

A) 10 B) 11 C) 12 D) 13 E) 14

15. Agar $\vec{a} = \{6;\ 2;\ 1\}$ va $\vec{b} = \{0;-1;\ 2\}$ bo'lsa, $\vec{c} = 2\vec{a} - \vec{b}$ vektorning uzunligini toping.

A) 9 B) 10 C) 11 D) 12 E) 13.

16. B(0;4;2) nuqta $\vec{a} = \{2;-3;1\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (2;-7;-1) B) (2;7;-1) C) (-2;7;1) D) (-2;-7;-1) E) (-2;-7;1)

16. N(2;0;4) nuqta $\vec{c} = \{1;-2;3\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (1;-2;-1) B) (1;2;1). C) (1;2;-1) D) (-1;-2;-1) E) (-1;2;1)

17. A(x;0;0) nuqta B(0;1;2) va C(3;1;0) nuqtalardan teng uzoqlikda bo'lsa, x ni toping.

A) $\frac{4}{5}$ B) $\frac{3}{5}$ C) $-\frac{3}{5}$ D) $\frac{5}{6}$ E) $-\frac{5}{6}$

18. $\vec{a}=\{2;-3;4\}$ va $\vec{b}=\{-2;-3;1\}$ vektorlarning skalyar ko'paytmesini toping.

A) 9 B) 10 C) 11 D) 12 E) 13

19. $\vec{m}=\{-1;5;3\}$ va $\vec{n}=\{2;-2;4\}$; vektorlarning skalyar ko'paytmesini toping.

A) -2 B) 0 C) -1 D) 2 E) 1

20. $\vec{e}=\{0;-4;2\}$ va $\vec{k}=\{-2;2;3\}$; vektorlarning skalyar ko'paytmesini toping.

A) -2 B) 0 C) -1 D) 2 E) 1

21. $\vec{a}=\{2;5\}$ va $\vec{b}=\{-7;-3\}$; vektorlar orasidagi burchakni toping.

A) 30^o B) 45^o C) 60^o D) 90^o E) 135^o

22. $\vec{c}=\{1;0\}$ va $\vec{d}=\{1;-1\}$; vektorlar orasidagi burchakni toping.

A) 30^o B) 45^o C) 60^o D) 90^o E) 135^o

23. $\vec{m}=\{5;-3\}$ va $\vec{n}=\{4;1\}$; vektorlar orasidagi burchakni toping.

A) 30^o B) 45^o C) 60^o D) 90^o E) 150^o

24. Agar $\left|\vec{a}\right|=\sqrt{137}$, $\left|\vec{a}+\vec{b}\right|=20$ va $\left|\vec{a}-\vec{b}\right|=9\sqrt{2}$ bo`lsa, $\left|\vec{b}\right|$ ni toping.

A) $8\sqrt{2}$ B) 15 C) $7\sqrt{2}$ D) 12 E) $7\sqrt{3}$

25. m ning qanday qiymatida $\vec{a}=\{1;m;-2\}$ va $\vec{b}=\{m;3;-8\}$ vektorlar perpendikulyar bo'ladi?

A) 4 B) –2 C) 2 D) –4 E) 3

26. $B(4;2;0)$ nuqta $\vec{a}=\{-2;3;-1\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) $(0;-1;1)$ B) $(-6;-1;1)$ C) $(-6;1;1)$ D) $(6;-1;-1)$ E) $(6;1;1)$

27. $\left|\vec{a}\right|=4$, $\left|\vec{b}\right|=3$, $(\vec{a}\,\hat{,}\,\vec{b})=60^0$. λ ning qanday qiymatida $(\vec{a}+\lambda\vec{b})\perp\vec{a}$ bo'ladi?

A) $2\frac{2}{3}$ B) $-2\frac{2}{3}$ C) $1\frac{2}{3}$ D) 1 E) –1

28. $\vec{a}=\{2;\ 3;\ 4\}, \vec{b}=\{-2;5;-3\}$ vektorlar berilgan . $\vec{a}+\vec{b}$ =?

A) {0;8;1} B) {0;7;1} C) {0;8;-1} D) {1;8;1} E) {0;-8;1}

29. $\vec{a}=\{2;\ 3;\ 4\}$ va $\vec{b}=\{-2;5;-3\}$vektorlarni skalyar ko'paytiring.

A) 0 B) 1 C) -1 D) 2 E) –2

30. $\vec{a}=\{-3;4;-2\}$vektorni 3 ga ko'paytiring va uzunligini toping.

A) $\sqrt{29}$ B) $\sqrt{260}$ C) $\sqrt{226}$ D) $\sqrt{261}$ E) $\sqrt{262}$

31. B(4;2;0) nuqta $\vec{a}=\{-2;\ 3;-1\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (6;-1;-1) B) (6;1;-1) C) (-6;-1;1) D) (-6;-1;-1) E) (6;-1;1)

32. $\vec{a}=\{0;1\}$ va $\vec{b}=\{2;1\}$ vektorlar berilgan. x ning qanday qiymatlarida $\vec{b}+x\vec{a}$ vektor $\vec{b}$ vektorga perpendikulyar bo'ladi?

A) 2 B) -2 C) 5 D) –5 E) 0

33. $\vec{a}=\{1;2;2\}$ vektorning birlik vektorini toping.

A) (6;-1;-1) B) (6;1;-1) C) (-6;-1;1) D) (-6;-1;-1) E) (6;-1;1)

34. $\vec{a}=\{2;-3;1\},\ \vec{b}=\{1;2;-4\},\ \vec{c}=\{5;-4;6\}$ vektorlarga qurulgan parallelepipedning hajmini toping.

A) 53 B) 54 C) 55 D) 56 E) 58

35. $|\vec{a}|=4,\ \ |\vec{b}|=3,\ \ (\vec{a}\,\hat{,}\,\vec{b})=60^0$. λ ning qanday qiymatida $(2\vec{a}-\lambda\vec{b})\perp\vec{b}$ bo'ladi ?

A) $\frac{4}{3}$ B) $\frac{3}{4}$ C) $-\frac{3}{4}$ D) 1 E) –1

36. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}|=|\vec{b}|=3$ bo'lsa, $(\vec{a}+\vec{b})$ bilan $(\vec{a}-\vec{b})$ vektorlar orasidagi burchakni toping.

A) 30^o B) 45^o C) 60^o D) 90^o E) 120^o

37. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}|=|\vec{b}|=2$ bo'lsa, $(\vec{a}-\vec{b})$ bilan $(\vec{a}+\vec{b})$ vektorlar qanday burchak tashkil etadi?

A) 30^o B) 45^o C) 60^o D) 90^o E) 120^o

38. $\vec{b}=\{0;-2\}$ va $\vec{c}=\{-3;4\}$ vektorlar berilgan. $\vec{a}=3\vec{b}-2\vec{c}$ vektorning koordinatalarini toping.

A) (6;-11) B) (6;-1) C) (-6;-14) D) (-6;14) E) (6;-14)

39. Agar $\vec{a}=\{6;\ 2;\ 1\}$ va $\vec{b}=\{0;-1;\ 2\}$ bo'lsa, $\vec{c}=2\vec{a}-\vec{b}$ vektorning uzunligini toping.

A) 9 B) 10 C) 11 D) 12 E) 13.

40. B(0;4;2) nuqta $\vec{a}=\{2;-3;1\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (2;-7;-1) B) (2;7;-1) C) (-2;7;1) D) (-2;-7;-1) E) (-2;-7;1)

41. N(2;0;4) nuqta $\vec{c}=\{1;-2;3\}$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (1;-2;-1) B) (1;2;1 C) (1;2;-1). D) (-1;-2;-1) E) (-1;2;1)

42. $\vec{a}=\{2;-3;4\}$ va $\vec{b}=\{-2;-3;1\}$ vektorlarning skalyar ko'paytmesini toping.

A) 9 B) 10 C) 11 D) 12 E) 13

43. $\vec{m}=\{-1;5;3\}$ va $\vec{n}=\{2;-2;4\}$; vektorlarning skalyar ko'paytmasini toping.

A) -2 B) 0 C) -1 D) 2 E) 1

44. $\vec{e}=\{0;-4;2\}$ va $\vec{k}=\{-2;2;3\}$; vektorlarning skalyar ko'paytmasini toping.

A) -2 B) 0 C) -1 D) 2 E) 1

45. $\vec{a}=\{2;5\}$ va $\vec{b}=\{-7;-3\}$; vektorlar orasidagi burchakni toping.

A) 30^{o} B) 45^{o} C) 60^{o} D) 90^{o} E) 135^{o}

3-BOB

TEKISLIKDA ANALITIK GEOMETRIYA

Mavzu:Tekislikdagi to`g`ri chiziq tenglamalari

Ikki nyqta orasidagi masofa.

$$d==\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad (1)$$

Masalan. $(1;-2)$ va $(5;3)$ nuqtalar orasidagi masofa topilsin.

Yechish: $d=\sqrt{(5-1)^2+(3-(-2))^2}=\sqrt{4^2+5^2}=\sqrt{41}$

Masalan. $A(3;8)$ va $B(-5;14)$ nuqtalar orasidagi masofa topilsin.

$$d=\sqrt{(-5-3)^2+(14-8)^2}=\sqrt{64+36}=10.$$

Masalan. Uchlari $A(-3;-3), B(-1;3), C(11;-1)$ nuqtalarda bo`lgan uchburchak to`g`ri burchakli uchburchak ekanligi isbotlansin.

Yechish: Tomonlari uzunliklarini topamiz:

$|AB| = \sqrt{(-1+3)^2 + (3+3)^2} = \sqrt{40}$, $|BC| = \sqrt{(11+1)^2 + (-1-3)^2} = \sqrt{160}$,

$|AC| = \sqrt{(11+3)^2 + (-13+3)^2} = \sqrt{200}$

Pifagor teoremasiga asosan: $|AB|^2 = 40, |BC|^2 = 160, |AC|^2 = 200$

$|AB|^2 + |BA|^2 = |AC|^2$, shart $40 + 160 = 200$ bajarilganligi sababli berilgan ABC uchburchak to`g`ri burchakli uchburchak , AC tomoni gipotenuzadir.

Kesmani berilgan nisbatda bo`lish.

$[P_1P_2]$ kesmani teng ikkiga bo`luvchi *M(x;y)* nuqtaning koordinatasi:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + r_2}{2} \quad (2)$$

$[P_1P_2]$ kesmani berilgan $\lambda = \frac{m}{n}$ nisbatda bo`luvchi *M(x;y)* nuqtaning koordinatasi.

$$x = \frac{x_1 + \lambda x_2}{1+\lambda} \qquad \text{va} \qquad y = \frac{y_1 + \lambda r_2}{1+\lambda} \quad (3)$$

Masalan. $A(1;4)$ va $B(1.1)$ kesmani $\lambda = \frac{1}{3}$ nisbatda bo`luvch $M(x;y)$nuqtaning koordinatasi topilsin.

Yechish. $$x = \frac{1 + \frac{1}{3}*1}{1+\frac{1}{3}} = \frac{\frac{3+1}{3}}{\frac{3+1}{3}} = \frac{\frac{4}{3}}{\frac{4}{3}} = 1$$

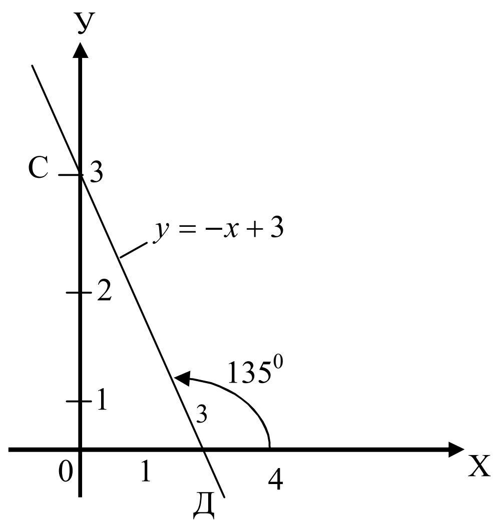
$$y = \frac{4 + \frac{1}{3}*1}{1+\frac{1}{3}} = \frac{\frac{12+1}{3}}{\frac{3+1}{3}} = \frac{\frac{13}{3}}{\frac{4}{3}} = \frac{13}{4} = 3{,}25$$

To'g'ri chiziqning burchak koeffisiyentli tenglamasi.

$$y = kx + b \qquad (5) \quad tg\varphi = k$$

Masalan: OY o'qi bilan $\varphi = 135^0$ burchak tashkil qiluvchi va OY o'qini *(0;3)* nuqtada kesib o'tuvchi to'g'ri chiziq tenglamasi tuzilsin va grafigi yasalsin.

Yechish: $k = tg(135^0) = -1$, $b = 3$. (5)-formuladan $y = -x + 3$ ni topamiz. *x=0* bo'lsa *y=3*, *y=0* bo'lsa $x = 3$



$Д(3;\ 0)$
$C(0;3)$

To'g'ri chiziqning umumiy tenglamasi

$$Ax + By + C = 0, \quad A^2 + B^2 \neq 0 \qquad (6)$$

a) *C=0*; $A \neq 0; B \neq 0$ bo`lsa, *Ax+By=0* to`g`ri chiziq koordinata boshidan o`tadi;

b) $A = 0; B \neq 0; C \neq 0$ bo`lsa , *By+C=0* to`g`ri chiziq *OX* o`qiga parallel;

v) $B = 0; A \neq 0; C \neq 0$ bo`lsa, *Ax+C=0* to`g`ri chiziq *OY* o`qiga parallel;

g) $B = C = 0; A \neq 0$ bo`lsa, Ax=0 to`g`ri chiziq *OY* o`qigidan iborat bo`ladi

d) $A = C = 0; B \neq 0$ bo`lsa, By=0 to`g`ri chiziq OX o`qidan iborat bo`ladi.

To'g'ri chiziqning kesmalarga nisbatan tenglamasi.

$Ax + By + C = 0,$ tenglamada C ni tenglamaning o'ng tomoniga o'tkazaylik, ya`ni $Ax + By = -C.$ Bu

$$-\frac{A}{C}x - \frac{B}{C}y = 1$$

ni hosil qilish mumkin. Bu yerda $-C/A = m$ va $-C/B = n$ deb belgilasak

$$\frac{x}{m} + \frac{y}{n} = 1 \,(7)$$

ni hosil qilamiz.

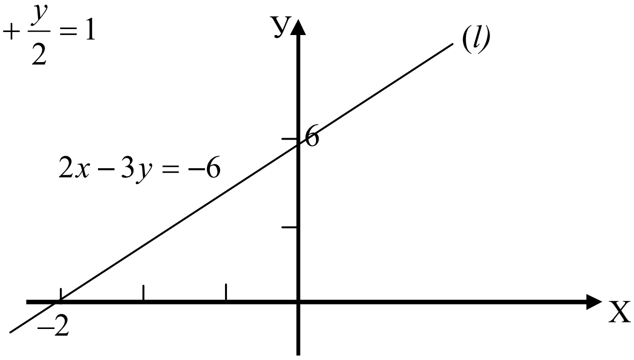
Masalan.

$2x - 3y + 6 = 0$ to'g'ri chiziq tenglamasini kesmalarga nisbatan yozing va yasang.

$2x - 3y = -6 \quad \Big| : (-6)$

Yechish: $-\frac{2}{6}x + \frac{3}{6}y = 1$

$\frac{x}{-3} + \frac{y}{2} = 1$



Ikki to'g'ri chiziq orasidagi burchak.

Tenglamalari bilan berilgan l_1 va l_2 to'g'ri chiziqlarni olaylik:

$$l_1 : y = k_1x + b_1$$

$$l_2 : y = k_2x + b_2$$

$$\text{tg}\,\varphi = \frac{k_1 - k_2}{1 + k_1 * k_2} \qquad (8)$$

To'g'ri chiziqlarning parallellik sharti:

$$k_1 = k_2 \qquad (9)$$

To'g'ri chiziqlarning o'zaro perpendikulyarlik sharti:

$$k_2 = -\frac{1}{k_1} \qquad (10)$$

Masalan. $y = 2x + 1$ va $x - y - 2 = 0$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechish:

$$l_1: y = 2x + 1\ ,\ k_1 = 2$$

$$l_2: x - y - 2 = 0.$$

$$l_2: y = x - 2,\ k_2 = 1$$

Burchakni topamiz:

$$tg\varphi = \frac{2-1}{1+2*1} = \frac{1}{3} \qquad \varphi = arctg\frac{1}{3} \approx 18{,}5^0$$

Berilgan nuqtadan o'tuvchi to'g'ri chiziqlar tenglamasi

$$y - y_o = k(x - x_0) \qquad (11)$$

Masalan. $y=3x-4$ to'g'ri chiziqqa perpendikulyar bo'lib $M(2;-3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish.

Izlanayotgan to'g'ri chiziqning burchak koeffisiyentini to'g'ri chiziqlarning perpendikulyarlik shartidan foydalanib topamiz:

Berilgan to'g'ri chiziqning burchak koeffisiyenti $k_1=3$ ga tengligidan izlanayotgan to'g'ri chiziqning burchak koeffisiyenti $k_2=-\frac{1}{3}$ bo'ladi.

Ularni dasta tenglamasiga qo'yamiz:

$$y+3=-\frac{1}{3}(x-2)$$

$$3y+9=-x+2$$

$$x+3y+7=0$$

javob: $x+3y+7=0$

Ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamasi.

$M_1(x_1;y_1)$ va $M_2(x_2;y_2)$ nuqtalar orqali o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (12)$$

Misol. $M_1(4;-2)$ va $M_2(3;-1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish. Berilgan nuqtalarni koordinatalarini (12) tenglamaga qo'yamiz:

$$\frac{y+2}{-1+2}=\frac{x-4}{3-4}, \qquad \frac{y-2}{1}=\frac{x-4}{-1},$$

bundan $y=-x+2$. Javob: $y=-x+2$.

To`g`ri chiziqlar orasidagi burchaklar bissektrisalari tenglamasi.

$A_1x+B_1y+C=0$ va $A_2x+B_2y+C=0$ to`g`ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasi formulasi quyidagicha.

$$\frac{A_1x+B_1y+C}{\sqrt{A_1^2+B_1^2}}=\pm\frac{A_2x+B_2y+C}{\sqrt{A_2^2+B_2^2}} \qquad (13)$$

Masalan: $x+y-5=0$ va $7x-y-19=0$ to`g`ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasini tuzing.

Yechish: $\frac{x+y-5}{\sqrt{1+1}}=\pm\frac{7x-y-19}{\sqrt{49+1}}$ bundan,

$$5(x+y-5) \pm (7x-y-19)=0$$

$$5(x+y-5)+(7x-y-19)=0\ ,3x+y-11=0,$$

$$5(x+y-5)-(7x-y-19)=0,\ x-3y+3=0.$$

To'g'ri chiziqning normal tenglamasi.

$$Ax+By+C=0$$

$$\mu = \pm\frac{1}{\sqrt{A^2+B^2}} \qquad (14)$$

Shunday qilib, to'g'ri chiziqning umumiy tenglamasini normallashtirish uchun bu tenglamani $\mu = \pm\frac{1}{\sqrt{A^2+B^2}}$ soniga ko'paytirish yetarli bo'lib, uning ishorasini tenglamadagi ozod had C ning ishorasiga qarama-qarshi qilib olish lozim ekan.

Masalan. $12x-5y-65=0$ to`g`ri chiziqning normal tenglamasi tuzilsin.

Yechish: $\mu = \frac{1}{\sqrt{12^2+(-5)^2}} = \frac{1}{13}$

$$\frac{12}{13}x - \frac{5}{13}y - 5 = 0$$

$$\cos\varphi = \frac{12}{13}, \sin\varphi = -\frac{5}{13}, p = 5$$

Berilgan nuqtadan to'g'ri chiziqqacha bo'lgan masofa.

$M(x_0;\ y_0)$ nuqtadan $Ax+By+C=0$ to'g'ri chiziqqacha bo'lgan masofa (d)ni ushbu formula yordamida topiladi:

$$d = \left|\frac{Ax_0+By_0+C}{\sqrt{A^2+B^2}}\right| (15)$$

Masalan. $M(3;\ -1)$ nuqtadan $3x+4y-10=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

Yechish:

$$d = \left|\frac{3\cdot 3+4\cdot(-1)-10}{\sqrt{3^2+4^2}}\right| = \left|\frac{9-4-10}{\sqrt{25}}\right| = \left|\frac{-5}{5}\right| = 1$$

Ikki to`g`ri chiziqning kesishish nuqtasi .

$$\begin{cases} A_1x+B_1y=C_1 \\ A_2x+B_2y=C_2 \end{cases} \qquad (16)$$

$$x=\frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} \qquad \text{va} \qquad y=\frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$

Misollar.

1-6. Nuqtalar orasidagi masofani toping.

1. $(1;1)$, $(4;5)$ 2. $(1;-3)$, $(5;7)$ 3. $(6;-2)$, $(-1;3)$

4. $(1;-6)$, $(-1;-3)$ 5. $(2;5)$, $(4;-7)$ 6. $(a;b)$, $(b;a)$

7. $A(-1;3)$, $B(3;11)$ va $C(5;15)$ nuqtalar berilgan. $|AB|+|BC|=|AC|$ ayniyatni isbotlang.

8. $(1;3)$ va $(7;15)$ nuqtalar orasidagi masofani teng ikkiga bo`luvch *M(x;y)* nuqtaning koordinatasi topilsin..

9. *A (7 ; 5)* va *B (-4 ; -2)* nuqtalar berilgan. *AB* kesmani 3 : 4 nisbatda bo`luvchi *C(x ; y)* nuqtaning koordinatalari topilsin.

10. Uchlari $A(6;-7)$, $B(11;-3)$ va $C(2;-2)$ nuqtalarda bo`lgan uchburchak to`g`ri burchakli uchburchak ekanligini isbotlang.

11. Quyidagi berilgan nuqtalar qaysi choraklarda joylashgan: $(-2;9)$, $(4;6)$, $(1;0)$ va $(-5;3)$.

12. Funktsiyalarning burchak koeffitsiyentli tenglamasi tuzilsin:

a) $x+3y=0$, b) $2x-5y=0$ c) $y=-2$ d) $2x-3y+6=0$ *e)* $3x-4y=12$ *f)* $4x+5y=10$

13. Burchak koeffisiyenti $k=\frac{2}{5}$; *OY* o`qini *b=4* kesmada kesib o`tuvchi to`g`ri chiziq tenglamasi tuzilsin.

14. *OY* o‘qidan $b=4$ kesma ajratib *OX* o‘qi bilan 135^0 burchak tashkil etuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

15. *OY* o‘qidan $b=-2$ kesma ajratib *OX* o‘qi bilan 60^0 burchak tashkil etuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

16. Koordinatlar boshidan o‘tib, *OY* o‘qi bilan: 1). 45^0, 2). 120^0, 3). 60^0, 4). 90^0 burchak tashkil etuvchi to‘g‘ri chiziqlarni yasang va ularning tenglamalarini yozing.

17. 1) $3x+5y+15=0$; 2) $3x+2y=0$; 3) $y=-2$; 4) $\frac{x}{4}+\frac{y}{4}=1$ to‘g‘ri chiziqlar uchun k va b parametrlarni aniqlang.

18. A (2; 3) nuqtadan o‘tib, OX o‘qi bilan 60^0 burchak hosil qiluvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

19. $y=\sqrt{3}\,x-2$ va $y=\frac{1}{\sqrt{3}}x+3$ to‘g‘ri chiziqlar berilgan. Ularning abssissa o‘qi bilan tashkil qiladigan burchaklarini toping.

20. $5x + 2y + 6 = 0$ va $x + y - 6 = 0$ to`g`ri chiziqlarning burchak koeffisiyentli tenglamasi tuzilsin.

21. Abssissa o`qidan kesgan kesmasi 3 ga, ordinata o`qidan kesgan kesmasi 1 ga teng bo`lgan to`g`ri chiziq tenglamasining burchak koeffisiyentini toping .

22. Abssissa o`qini 1, ordinate o`qini -2 nuqtada kesib o`tadigan to`g`ri chiziq tenglamasi tuzilsin.

23. Quyidagi chiziqlarni OY o`qi bilan kesishgan nuqtaning koordinatasi aniqlansin:

a) $x+3y=0$ b) $2x-5y=0$ c) $y=-2$ d) $2x-3y+6=0$ e) $3x-4y=12$

f) $4x+5y=10$ (Cal.A-15).

24. $5x - 2y + 6 = 0$ va $x + y - 6 = 0$ to`g`ri chiziq tenglamalarini burchak koeffisiyentini toping .

25. A (-1;4) nuqtadan o`tib,$OX$ o`qi bilan 45^0 li burchak tashkil qilgan to`g`ri chiziq tenglamasi tuzilsin

26. A (2;3) nuqtadan va OY o`qdan $b = 6$ kesma kesuvchi to`g`ri chiziq tenglamasi tuzilsin .

27. Quyidagi funktsiyalarni grafigini yasang.

a) $y=3$ b) $y=-2$ c) $|y|=1$

28. 1) $4x+3y-12=0$; 2) $4x+3y=0$; 3) $2x-7=0$; 4) $2y+7=0$ to‘g‘ri chiziqlarning kesmalarga nisbatan tenglamalarini yozing va ularni yasang.

29. 1) $2x-3y-6=0$; 2) $3x-2y+4=0$ to‘g‘ri chiziq tenglamalarini, kesmalar bo‘yicha tenglamasiga keltiring.

30. $Ax+5y-40=0$ to‘g‘ri chiziq A ning qanday qiymatlarida koordinata o‘qlaridan bir xil kesmalar ajratadi.

31. $y=\frac{1}{2}\cdot x+4$ to‘g‘ri chiziq berilgan. Uning koordinata o‘qlari bilan kesishish nuqtalarini toping.

32. To’g’ri chiziq *OX* o’qini *A(-6;0)* nuqtada, *OY* o’qini *B(0;7)* nuqtada kesib o’tadi. Bu to’g’ri chiziqning kesmalarga nisbatan tenglamasini tuzing.

33. *A (-2;3)* nuqtadan va *OY* o`qidan *a = 6* kesma kesuvchi to`g`ri chiziq tenglamasi tuzilsin.

34. $3x-5y+19=0$ va $10x+6y-50=0$ to`g`ri chiziqlar perpendikulyar ekanligi isbotlansin.

35. *6x-2y+5=0* va *4x+2y-7=0* to’g`ri chiziqlar orasidagi burchakni aniqlang.

36. 1) $3x-15y+16=0$, 2) $3x+15y-8=0$,

3) $6x-30y+13=0$, 4) $30x+6y+7=0$

to‘g‘ri chiziqlardan qaysilari perpendikulyar va qaysilari parallel.

37. Quyidagi to‘g‘ri chiziqlar orasidagi burchaklarni toping:

1) $\begin{matrix} y=\frac{2}{3}\cdot x-7 \\ y=5x+9 \end{matrix}$; 2) $\begin{matrix} 2x-4y+9=0 \\ 6x-2y-3=0 \end{matrix}$

3) $\begin{matrix} y=\frac{3}{7}\cdot x-2 \\ 7x+3y+5=0 \end{matrix}$; 4) $\begin{matrix} \frac{x}{4}/4-\frac{y}{5}=1 \\ \frac{x}{2}+\frac{y}{18}=1 \end{matrix}$

38. Tomonlari $4x-3y+5=0$, $3x+4y+4=0$, $x-7y+18=0$ to‘g‘ri chiziqlarda yotgan uchburchakning ichki burchaklarini toping.

39. $A(4;\,5)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing va ulardan $2x-3y+6=0$ to‘g‘ri chiziqqa perpendikulyar va parallel bo‘lganlarini ajrating.

40. Uchburchak tomonlari

$$7x-6y+9=0;\quad 5x+2y-25=0;\ 3x+10y+29=0$$

tenglamalar bilan berilgan. Uning uchlarini va balandliklarining tenglamalarini toping.

41. Uchlari $P(-4;\ 0)$, $Q(0;\ 4)$ va $R(2;\ 2)$ nuqtalarda bo‘lgan uchburchak medianalarining tenglamalarini tuzing .

42. To’g’ri chiziqlar:

$$\begin{cases} \frac{x}{4}-\frac{y}{5}=1 \\ \frac{x}{2}+\frac{y}{18}=1 \end{cases}$$ orasidagi burchakni toping.

43. $y=-\frac{2}{5}\cdot x+3$; $y=\frac{3}{7}\cdot x+\frac{2}{7}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

Uchburchakning *B* uchidan tushirilgan balandlik tenglamasi tuzilsin.

44. *A (2;9), B (4;-7)* va *C (4;9)* uchburchak uchlari bo`lsa, burchaklarini aniqlang.

45. To‘g‘ri chiziqning koordinatalar boshidan uzoqligi 3, unga koordinatlar boshidan tushirilgan perpendikulyar *OX* o‘qi bilan $\alpha=45^0$ burchak hosil qilsa, to‘g‘ri chiziq tenglamasini yozing.

46. $x-y+3=0$ to‘g‘ri chiziqqa koordinatalar boshidan tushirilgan perpendikulyarning uzunligini va uning *OX* o‘qi bilan tashkil qilgan burchagini toping.

47. *A(2;1)* nuqtadan o’tib *y=3x-4* to’g’ri chiziqqa parallel bo’lgan to’g’ri chiziq tenglamasini tuzing.

48. *A(5;-4)* nuqtadan o’tuvchi va *3x+2y-7=0* to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasini tuzing.

49. *OY* o’qiga 2 birlik kesma ajratuvchi hamda *x-2y+3=0* to’g’ri chiziq bilan 45°li burchak hosil qiluvchi to’g’ri chiziq tenglamasini tuzing

50. Uchburchak uchlarining koordinatalari berilgan.

A(-3;-1), B(2;1),C(3;5)

Uning *B* uchidan tushirilgan balandlik tenglamasini tuzing va balandligining uzunligini toping.

51. $(5;2)$ nuqtadan o`tib $4x+6y+5=0$ to`g`ri chiziqqa parallel bo`lgan to`g`ri chiziq tenglamasi tuzilsin.

52. $\left(\frac{1}{2};-\frac{2}{3}\right)$ nuqtadan o`tib $4x-8y-1=0$ to`g`ri chiziqqa perpendikulyar bo`lgan to`g`ri chiziq tenglamasi tuzilsin.

53. Uchburchakning uchlarini koordinatalari berilgan: $A(1;0), B(3;6) va C(8;2)$. A uchidan tushirilgan mediana tenglamasi tuzilsin.

54. $A(1;1)$, $B(7;4)$, $C(5;10)$ va $D(-1;7)$ nuqtalar parallelogram uchlari ekanligini ko`rsating.

55. $(4;5)$ nuqtadan o`tib $OY$ o`qiga parallel bo`lgan to`g`ri chiziq tenglamasi tuzilsin.

56. Uchburchak uchlarining koordinatalari berilgan. *A(12;-4) ,B(0;5)* va C*(-12;-11)*. Uning tomonlarining tenglamalarini tuzing.

57. *A(1;2)* va *B(4;3)* nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing hamda bu to'g'ri chiziqning koordinata o'qlari bilan kesishish nuqtalarini aniqlang.

58. *x-y-4=0* va *2x-11y+37=0* to'g'ri chiziqlarning kesishish nuqtasidan hamda koordinatalar boshidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

59. Uchburchak uchlarining koordinatalari berilgan: *A(-3;-1), B(5;3), C(6;-4)* . Uning *C* uchidan o'tkazilgan medianasining tenglamasini tuzing.

60. *ABCD* to'g'ri to'rtburchak *AB* tomonining uchlari *A(3;2)* va *B(-3;0)* nuqtalarda yotadi. *AD* tomonning uzunligi 8 sm.ga teng. Bu to'g'ri to'rtburchak tomonlarining tenglamalarini tuzing.

61. *2x-3y-12=0* va *3x+y-12=0* to'g'ri chiziqlar orasidagi burchaklar bissyektrisalarining tenglamalarini tuzing.

62. *3x-4y-20=0* va *8x-6y-5=0* to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini tuzing.

63. *A(2;5)* nuqtadan *6x+8y-6=0* to'g'ri chiziqqacha bo'lgan masofani toping.

64. Ushbu 1) $5x+12y-26=0$, 2) $3x-4y+10=0$,

3) $y=3x+5$, 4) $2x+2y+7=0$

to'g'ri chiziq tenglamalarini normal ko'rinishga keltiring.

65. Ushbu 1) $\frac{2}{5}x+\frac{3}{4}y-6=0$, 2) $\frac{12}{13}x-\frac{5}{13}-7=0$

3) $\frac{3}{5}x+\frac{3}{4}y-2=0$, 4) $\frac{1}{3}x+\frac{2}{3}y-4=0$

to‘g‘ri chiziq tenglamalaridan qaysilari normal ko‘rinishda?

66. *R(3;-4)* nuqta koordinatalar boshidan to’g’ri chiziqqa tushirilgan perpendikulyarning asosi. To’g’ri chiziqning normal tenglamasini tuzing.

67. Uchlari $P(0;\ 5)$, $Q(-3;\ 1)$ va $R(-1;\ -2)$ nuqtalarda bo‘lgan uchburchakning R nuqtasidan o‘tkazilgan balandligining uzunligini toping.

68. $5x-12y-26=0$, $5x-12y-65=0$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

69. *M(1;2)* nuqtadan *20x-21y-58=0* to’g’ri chiziqqacha bo’lgan masofani toping.

70. Uchburchakning tomonlari tenglamalari berilgan: *x+3y-7=0(AB)*, *4x-y-2=0(BC)*, *6x+8y-35=0*. *B* uchidan tushirilgan balandlik uzunligi topilsin.

3. $3x+y-3\sqrt{10}=0$ va $6x+2y+5\sqrt{10}=0$ to`g`ri chiziqlar orasidagi masofa topilsin.

71. *M(4;-1)* nuqtadan hamda *x-3y+2=0* va *y-4=0* to’g’ri chiziqlarning kesishish nuqtasidan o’tuvchi to’g’ri chiziq tenglamasini tuzing.

72. *3x-y+5=0* va *2x+3y+1=0* to’g’ri chiziqlarning kesishish nuqtasidan o’tuvchi hamda $7x-3y+5=0$ to’g’ri chiziqqa parallel bo’lgan to’g’ri chiziq tenglamasini tuzing.

73. *3x-y=0* va *x+4y-2=0* to’g’ri chiziqlarning kesishish nuqtasidan o’tib, *2x+7y=0* to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq Tenglamasini tuzing.

74. Trapetsiya asoslarining tenglamalari $3x-4y-15=0$, $3x-4y-35=0$ berilgan. Trapetsiyaning balandligini toping.

75. Uchlari $A(-2;\ 0)$, $B\ (2;\ 4)$ va $C(4;\ 0)$ nuqtalarda bo‘lgan uchburchak tomonlarining, AE medianasining, AD balandligining tenglamalarini hamda AE mediananing uzunligini toping.

76. ΔABC: $A(1;-2)$, $B(7;1)$, $C(3;7)$ uchlari berilgan bo`lsa: a) $BC$- tomon tenglamasi? b) $A$- uchidan tushirilgan balandlik tenglamasi tuzilsin? c) $A$- uchidan o`tuvchi va BC ga parallel to`g`ri chiziq tenglamasi tuzilsin?

77. $A(-2;-8)$, $B(-18;-8)$, $C(0;5)$ nuqtalar berilgan bo`lsin. $A$ va $C$ nuqtalardan o`tuvchi to`g`ri chiziqqa 1) parallel bo`lgan,2) perpendikulyar bo`lgan, B nutadan o`tuvch to`g`ri chiziq tenglamasi tuzilsin.

Ikkinchi darajali chiziqlar.

Tekislikdagi ikkinchi tartibli chiziqning umumiy tenglamasi quyidagi ko'rinishda bo'ladi:

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0 \qquad (17)$$

Bu yerda A,B,C,D,E,F lar o'zgarmas koeffisiyentlar bo'lib, A,B,C lardan kamida bittasi noldan farqli bo'lishi zarur, aks holda to'g'ri chiziqqa ega bo'lamiz.

Aylana tenglamasi.

$$(x-a)^2+(y-b)^2=R^2 \qquad (18)$$

Bunda qavslarni ochib

$x^2+y^2-2ax-2by+(a^2+b^2-R^2)=0$ ni hosil qilamiz. Agarda $-2a=2D;$ $-2b=2E;$ $a^2+b^2-R2=F$ deb belgilash kiritsak, aylana tenglamasi:

$$x^2+y^2+2Dx+2Ey+F=0 \qquad (19)$$

Markazi koordinata boshida radiusi R ga teng aylana tenglamasi quyidagicha bo'ladi:

$$x^2+y^2=R^2 \qquad (20)$$

Masalan.

Markazi $C(2:-3)$ nuqtada, radiusi $R=4$ ga teng aylana tenglamasi yozilsin.

Yechish:

$$(x-2)^2+(y+3)^2=4^2$$

$$x^2-4x+4+y^2+6y+9=16$$

$$x^2+y^2-4x+6y-3=0$$

Masalan.

$x^2+y^2-6x+8y=0$ aylananing markazi va radiusi topilsin.

Yechish:

$$x^2+y^2-6x+8y=0 \Leftrightarrow (x^2-6x)+(y^2+8y)=0$$

$$(x^2-6x+9)+(y^2+8y+16)-9-16=0$$

$$(x-3)^2+(y+4)^2=5^2$$

Demak, aylana markazi $M(3;-4)$ va radiusi esa $R=5$

Aylanaga o'tkazilgan urinma tenglamasi

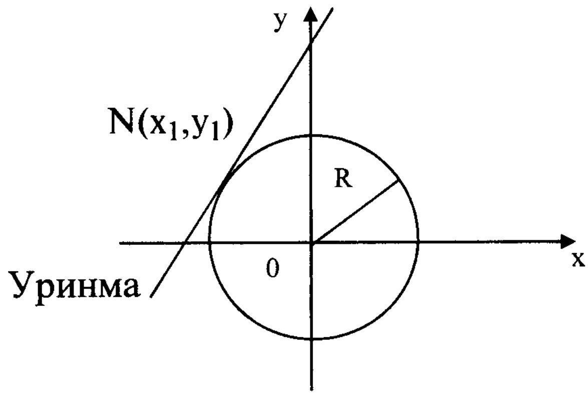
Agar $N\ (x_1;y_1)$ nuqta aylananing biror nuqtasi bo'lsa, u holda bu nuqtadan aylanaga o'tkazilgan urinma tenglamasi

$$(x-a)(x_1-a)+(y-b)(y_1-b)=R^2 \quad (21)$$

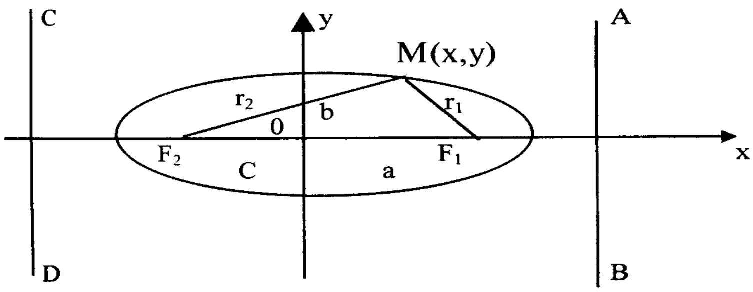
yoki

$$x \cdot x_1 + y \cdot y_1 = R^2 \quad (22)$$

dan iborat bo'ladi.



Ellips



$F_1(c,o)$:
$F_2(-C:O)$
$d(F_1,F_2)=2c$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (23)$$

$$a^2=b^2+c^2 \qquad (24)$$

Bu ellipsning *kanonik* tenglamasidir. Bu yerda

a – ellipsning katta yarim o'qi va $a=\sqrt{b^2+c^2}$

b – ellipsning kichik yarim o'qi deyiladi va $b=\sqrt{a^2-c^2}$

Ellipsning fokusi $c=\sqrt{a^2-b^2}$ dan topiladi.

Ellipsning ekssentrisiteti

$$\varepsilon=\frac{2c}{2a}\Rightarrow \qquad \varepsilon=\frac{c}{a} \qquad (25)$$

bunda $0\leq\varepsilon<1$

Ekssentrisitet ellipsning cho'ziqligi darajasini xarakterlaydi.

Ellipsning ixtiyoriy nuqtasidan (F_1 va F_2) fokuslarigacha bo'lgan masofalar uning fokal radius-vektorlari (r_1 va r_2) deyiladi.

Ixtiyoriy $M(x,y)$ nuqta uchun

$$r_1=a-\varepsilon x, \qquad r_2=a+\varepsilon x, \qquad r_1+r_2=2a. \qquad (26)$$

Ellipsning kichik o'qiga parallel bo'lgan va undan $\frac{a}{\varepsilon}$ masofadan o'tgan ikki to'g'ri chiziq ellipsning direktrisalari deyiladi:

$$x=-\frac{a}{\varepsilon} \quad \text{va} \quad x=\frac{a}{\varepsilon} \qquad (27)$$

Ellipsning ixtiyoriy $M(x,y)$ nuqtasiga o'tkazilgan urinma tenglamasi

$$\frac{x\cdot x_1}{a^2}+\frac{y\cdot y_1}{b^2}=1 \qquad (28)$$

ko'rinishda bo'ladi.

Masalan.

Katta o'qi 10 ga teng va ekssentrisiteti ε=0,8 ga teng bo'lgan ellipsning sodda tenglamasini tuzing.

Yechish:

$$2a=10\Rightarrow a=5$$

(25)-formuladan $c=\varepsilon\, a=4$

(24)- formulalardan $b^2=a^2-c^2=5^2-4^2=25-16=9 \Leftrightarrow b=3.$

$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1\Rightarrow\frac{x^2}{25}+\frac{y^2}{9}=1$ ni hosil qilamiz.

Masalan.

$4x^{2+}9y^{2=}16$ ellipsning katta va kichik yarim o'qlarini, fokuslarini hamda ekssentrisitetini toping.

Yechish:

$$\frac{4x^2}{16}+\frac{9y^2}{16}=1 \Rightarrow \frac{x^2}{4}+\frac{y^2}{\frac{16}{9}}=1.$$

Bundan $a^2=4 \Rightarrow a=2,\qquad b^2=\frac{16}{9} \Rightarrow b=\frac{4}{3}$

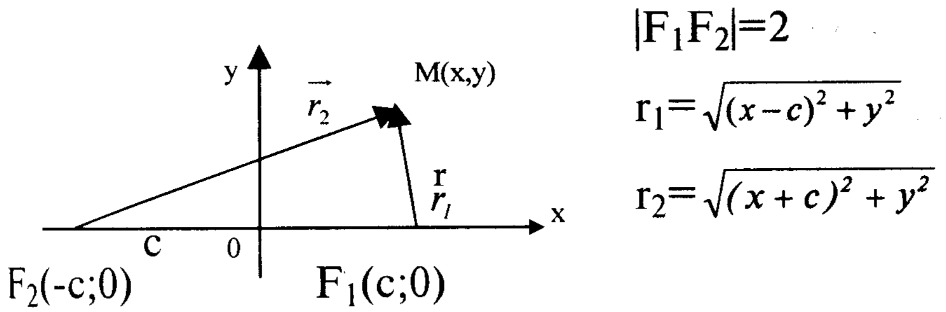
$$a^2=b^2+c^2 \text{ dan}$$

$$c^2=a^2-b^2=4-\frac{16}{9}=\frac{36-16}{9}=\frac{20}{9}$$

$$c=\sqrt{\frac{20}{9}}=\frac{2\sqrt{5}}{3}$$

$$\varepsilon=\frac{2\sqrt{5}}{3}*\frac{1}{4}=\frac{\sqrt{5}}{6}.$$

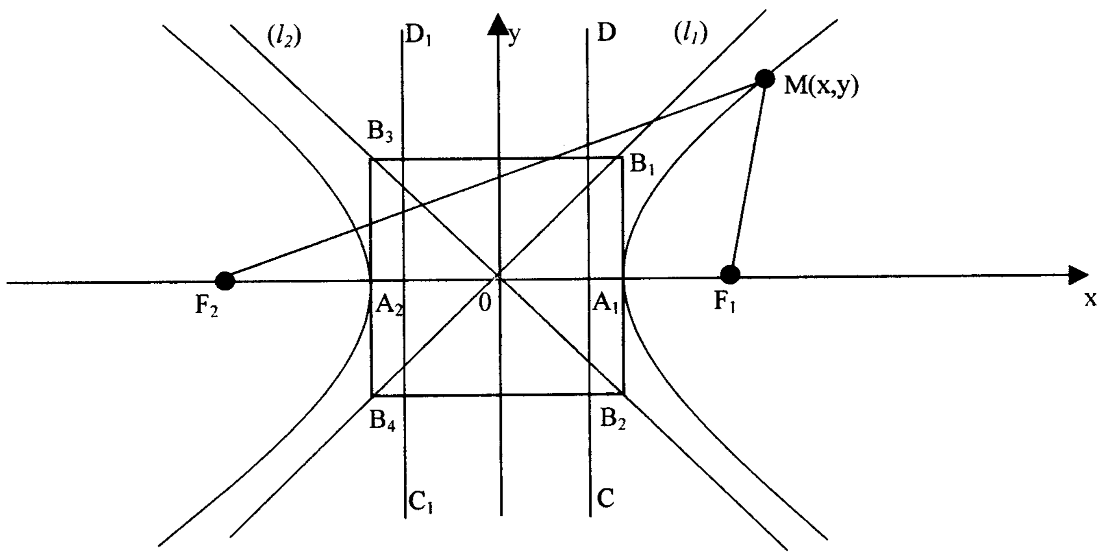
Giperbola.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad (29)$$

giperbolaning kanonik tenglamasi.

$$b^2=c^2-a^2$$



A_1 *(a:0)* va A_2 *(-a:0)* nuqtalar giperbolaning uchlari deyiladi.

$[A_1A_2]$ kesmaga giperbolaning haqiqiy o'qi deyiladi.

$[B_1B_2]$ kesmaga giperbolaning mavhum o'qi deyiladi.

a - haqiqiy yarim o'q, *в* – mavhum yarim o'q deyiladi.

Mos ravishda

$$y=\pm\frac{в}{a}x \quad (30)$$

formula bilan aniqlanuvchi ikki (l_1) va (l_2) to'g'ri chiziqlarga asimptotalar deyiladi. Formula

$$\varepsilon=\frac{c}{a} \quad (31)$$

bilan aniqlanuvchi kattalikka giperbolaning ekssentrisiteti deyiladi. c>*a* bo'lganligidan ε>1. Agar ε birga yaqin bo'lsa, giperbola tarmoqlari shuncha siqiq va ε birdan qancha katta bo'lsa, giperbola tarmoqlari shuncha yoyiq joylashgan bo'ladi.

Giperbolaning fokal radiuslari :

$$x<0 \text{ da } \left.\begin{matrix} r_1 = a - \varepsilon x \\ r_2 = -a - \varepsilon x \end{matrix}\right\} \text{(chap tarmoq uchun). (32)}$$

$$x>0 \text{ da } \left.\begin{matrix} r_1 = -a + \varepsilon x \\ r_2 = a + \varepsilon x \end{matrix}\right\} \text{(o'ng tarmoq uchun). } (32^1)$$

Giperbolaning direktrisalari tenglamasi:

$$x=\frac{a}{\varepsilon} \text{ va } x=-\frac{a}{\varepsilon} \quad (33)$$

Yarim o'qlari teng (*a=в*) bo'lgan giperbolaga teng tomonli giperbola deyiladi va

$$x^2-y^2=a^2 \quad (34)$$

formula bilan ifodalanadi.

Giperbolaning $(x_1;y_1)$ nuqtasiga o'tkazilgan urinmaning tenglamasi:

$$\frac{x\cdot x_1}{a^2}+\frac{y\cdot y_1}{b^2}=1 \quad (35)$$

Masalan.

Fokuslari orasidagi masofa *2c=8* bo'lgan, uchlari orasidagi masofa *2a=6* bo'lgan giperbolaning kanonik tenglamasi tuzilsin.

Yechish:

$$2c = 8 \Rightarrow c = 4$$

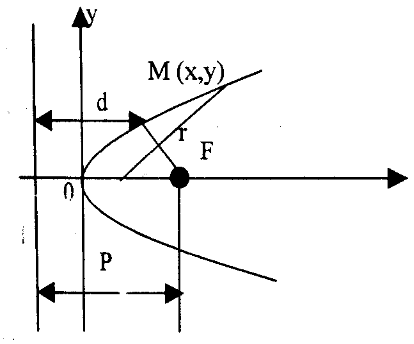
$$2a = 6 \Rightarrow a = 3$$

$$b = \sqrt{c^2 - a^2} = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7} \Rightarrow b^2 = 7$$

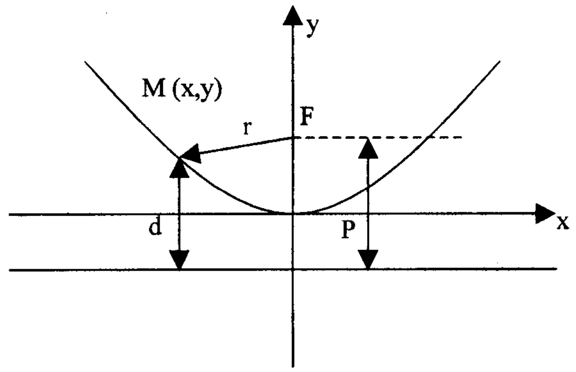
(3) formuladan $\frac{x^2}{9} - \frac{y^2}{7} = 1$ hosil qilamiz.

Parabola

$$y^{2}=2px \quad (36)$$



1-cnizma



2-chizma

Direktrisa tenglamasi:

$$x = -\frac{P}{2} \quad (37)$$

Parabolaning fokusi koordinatasi: $F\left(\frac{P}{2};0\right)$

Parabolaning *M(x,y)* nuqtasining fokal radiusi

$$r = x + \frac{P}{2} \quad (38)$$

Agar parabola koordinata boshidan o’tib *OY* o’qiga simmetrik bo’lsa uning tenglamasi:

$$x^2=2py \quad (39)$$

Uning direktrisasi tenglamasi:

$$y = -\frac{P}{2}, \quad (40)$$

Fokusi koordinatasi $F\left(0;\frac{P}{2}\right)$ nuqtada, *M(x,y)* nuqtasining fokal radiusi

$$r = y + \frac{P}{2} \quad (41)$$

$A(x_1,y_1)$ nuqtasiga o'tkazilgan urinma tenglamalari mos ravishda

$$yy_1 = p(x+x_1) \text{ va } xx_1 = p(y+y_1) \quad (42)$$

Masalan.

$y = \frac{1}{4}x^2$ parabola fokusining koordinatalarini toping va direktrisasining tenglamasini tuzing.

Yechish:

$y = \frac{1}{4}x^2$ ni kanonik ko'rinishda yozamiz:

$y = \frac{1}{4}x^2 \Rightarrow x^2 = 4y$ dan $2p=4 \Rightarrow p=2$ ekanligi kelib chiqadi. Direktrisa tenglamasini $y = -\frac{P}{2}$ dan topamiz. $y = -\frac{P}{2} = -\frac{2}{2} = -1 \Rightarrow y = -1$ bo'ladi. Parabola fokusining koordinatasi: $F\left(0;\frac{P}{2}\right)$ yoki $F(0;1)$.

Misollar.

1. Aylana tenglamasi tuzilsin:

a) Markazi $(3;-1)$, radiusi 5; b) Markazi $(-2;-8)$, radiusi 10.

c) Markazi ordinatada va radiusi $(4;7)$ nuqtadan o`tuvchi aylana tenglamasi tuzilsin.

d) Markazi $(-1;5)$ va $(-4;-6)$ nuqtadan o`tuvchi aylana tenglamasi tuzilsin.

2. Aylana markazi va radiusi topilsin:

a) $x^2 + y^2 - 4x + 10y + 13 = 0$

b) $x^2 + y^2 + 6y + 2 = 0$

c) $x^2 + y^2 + x = 0$

d) $16x^2 + 16y^2 + 8x + 32y + 1 = 0$

e) $2x^2 + 2y^2 - x + y = 1$

3. Radiusi 3 ga, markazi $(2;-5)$ nuqtada bo`lgan chiziq tenglamasi tuzilsin.

4. $x^2 + y^2 + 2x - 6y + 7 = 0$ aylananing radiusi va markazi aniqlansin.

5. $N\,(7;\ -2)$ nuqtadan o'tib, markazi $C\,(3;\ -5)$ nuqtada bo'lgan aylana tenglamasini yozing.

6. M (4; 2) va N (12; 8) nuqtalar berilgan. Diametri MN kesmadan iborat bo‘lgan aylana tenglamasini yozing.

7.Aylananing markazi va radiusini toping: 1) $x^2+y^2-4x+8y-16=0$;

2) $3x^2+3y^2-6x+8y-29/3=0$;

3) $x^2+y^2+7x=0$;

4) $5x^2+5y^2+9y=0$

8. $x^2+y^2-4x+8y-16=0$, va $x^2+y^2+8x+12y-14=0$ aylanalar markazlaridan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

9. $4x-3y-10=0$, $3x-4y-5=0$, $3x-4y-15=0$ to’g’ri chiziqlarga urinuvchi aylananing tenglamasini tuzing.

10. Markazi $2x+y=0$ tog’ri chiziqda yotib, $4x-3y+10=0$ va $4x-3y-30=0$ to’g’ri chiziqlarga urinuvchi aylananing tenglamasini yozing.

11. *A(-1;5)* nuqtadan o’tib $3x+4y-35=0$ va $4x+3y+14=0$ to’g’ri chiziqlarga urinuvchi aylananing tenglamasini tuzing.

12. *A(1;1) B(1;-1) C(2;0)* nuqtalardan o’tuvchi aylananing tenglamasini tuzing.

13. $(x-3)^2-(y-7)^2=169$ aylananing *M(8,5;3,5)* nuqtada teng ikkiga bo’linuvch vatarini tenglamasini tuzing.

14. $(x-2)^2+(y+1)^2=16$ aylananing *A(1;2)* nuqtada teng ikkiga bo’linuvch vatarini tenglamasini tuzing.

15. $x^2+y^2-10x-10y=0$, $x^2+y^2+6x+2y-40=0$ aylanalarning umumiy vatarini uzunligini toping.

16. *A(1;6)* nuqtadan $x^2+y^2+2x-19=0$ aylanaga o’tkazilgan urinmaning tenglamalarini yozing.

17. *A(4;2)* nuqtadan $x^2+y^2=10$ aylanaga o’tkazilgan urinmalar orasidagi burchakni toping.

18. $x^2+y^2+10x-2y+6=0$ aylananing $2x+y-7=0$ to’g’ri chiziqqa parallel bo’lgan urinmasining tenglamasini toping.

19. $x^2 + y^2 - 6x + 5 = 0$ aylananing kanonik tenglamasini yozing.

20. $4x - 3y - 10 = 0$, $3x - 4y - 5 = 0$, $3x - 4y - 15 = 0$ to'g'ri chiziqlarga urinuvchi aylananing tenglamasini tuzing.

21. Markazi $2x + y = 0$ to'g'ri chiziqda yotib, $4x - 3y + 10 = 0$ va $4x - 3y - 30 = 0$ to'g'ri chiziqlarga urinuvchi aylananing tenglamasini yozing.

22. *A(-1;5)* nuqtadan o'tib $3x + 4y - 35 = 0$ va $4x + 3y + 14 = 0$ to'g'ri chiziqlarga urinuvchi aylananing tenglamasini tuzing.

23. *A(1;1,) B(1;-1,)C(2;0)* nuqtalardan o`tuvchi aylananing tenglamasini tuzing.

24. Ellipsning parametrlari aniqlansin, grafigi yasalsin:

a) $x^2 + 4y^2 = 16$, b) $4x^2 + y^2 = 1$, c) $25x^2 + 4y^2 = 100$

d) $16x^2 + 25y^2 = 400$, e)$9y^2 + x^2 = 9$ f) $y^2 + x^2 = 1$ j) $9x^2 + 25y^2 = 225$.(CAL.a-23)

25.$9x^2 + 25y^2 = 225$, 2) $9x^2 + y^2 = 36$ ellipslar uchun o'qlarining uzunliklarini, fokuslarini va ekssentrisitetlarini toping va yasang.

26. Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips M $(2;\ \sqrt{3})$ va N (0; 2) nuqtalardan o'tadi. Ellips tenglamasini yozing. M nuqtadan fokuslargacha masofalarni toping.

27. Ellipsning ekssentrisiteti ε berilgan. Ellips yarim o'qlarining $\frac{b}{a}$ nisbatini toping.

28. Ikkita uchi $x^2 + 5y^2 = 20$ ellipsning fokuslarida, qolgan ikkitasi kichik yarim o'qlarining oxirlarida bo'lgan to'rtburchkning yuzini toping.

29. $M_1\left(2;-\frac{5}{3}\right)$ nuqta $\frac{x^2}{9} + \frac{y^2}{5} = 1$ ellipsda yotadi. M_1 nuqtaning fokal radiuslari yotadigan to'g'ri chiziqlarning tenglamalarini yozing.

30. *M(-4;2,4)* nuqtani $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ellipsda yotishini tekshirib, shu nuqtaning fokal radiuslarini toping.

31. O'ng fokusdan 14 masofa narida joylashgan $\frac{x^2}{100} + \frac{y^2}{36} = 1$ ellipsning nuqtasini toping.

32. quyidagi tenglamalarning har biri ellipsni ifodalashini ko'rsating.

1) $5x^2+9y^2-30x+18y+9=0$

2) $16x^2+25y^2+32x-100y-284=0$

Ularning yarim o’qlari va ekssentrisitetini toping.

33. $x+2y-7=0$ to’g’ri chiziqni $x^2+4y^2=25$ ellips bilan kesishish nuqtalarini toping.

34. m ning qanday qiymatlarida $y=-x+m$ to’g’ri chiziq $\frac{x^2}{20}+\frac{y^2}{5}=1$ a) ellips bilan kesishadi. b) ellipsga urinadi, c) ellipsdan tashqarida yotadi.

35. $\frac{x^2}{10}+\frac{2y^2}{5}=1$ ellipsning $3x+2y+7=0$ to’g’ri chziqqa parallel bo’lgan urinmalarini toping.

36. $\frac{x^2}{30}+\frac{y^2}{24}=1$ ellipsning $4x-2y+23=0$ to’g’ri chziqqa parallel bo’lgan urinmalarining tenglamalarini yozing.

37. *A(4;-1)* nuqtadan o’tuvchi $x+4y-10=0$ to`g`ri chiziqqa urinuvchi ellipsning tenglamasini yozing.Ellipsning o’qlarri koordinata o’qlari bilan utma-ut tushadi.

38. Fokuslari orasidagi masofa 24, katta o‘qi 26 ga teng bo‘lgan ellipsning kanonik tenglamasini yozing va uni yasang.

39. Quyidagilar berilganda ellipsning kanonik tenglamasini toping:

1) katta yarim o‘q 10, ekssentrisitet $0{,}8$;

2) kichik yarim o‘q 12, ekssentrisitet $\frac{5}{13}$

3) ekssentrisitet $0{,}6$, fokuslar orasidagi masofa 6.

40. Fokuslari abssissa o`qida yotuvchi va quyidagi shartlarni qanoatlantiruvchi ellipsning kanonik tenglamasini tuzing:

a) uning kichik o`qi 24 ga , fokuslar orasidagi masofa 10 ga teng;

b) Direktrisalari orasidagi masofa 32 ga, ekssentrisiteti 0,5 ga teng.

41. Ellipsning fokuslari ordinatalar o`qida yotib:

a) uning kichik o`qi 16 ga, ekssentrisiteti esa 0,6 ga teng;

b) uning fokuslari 6 ga va direktrisalari orasidagi masofa $16\frac{2}{3}$ ga teng bo`lsa, uning kanonik tenglamasini tuzing.

42. Giperbolaning parametrlarini aniqlang va grafigini yasang.

a) $16x^2 - 25y^2 = 400$, b) $9y^2 - x^2 = 9$ c) $y^2 - x^2 = 1$ d) $9x^2 - 25y^2 = 225$.

43. $9x^2 - 4y^2 = 36$ giperbolaning parametrlarini aniqlang va grafigini yasang.

44. Quyidagilar berilganda giperbolaning kanonik tenglamasini yozing:

1) fokuslari orasidagi masofa 10, ekssentrisitet $5/3$;

2) haqiqiy yarim o‘qi $\sqrt{20}$ va giperbola $N\ (-10;\ \ 4)$ nuqtadan o‘tadi;

3) fokuslar orasidagi masofa 10, uchlari orasidagi masofa 4.

45.1) $144x^2 - 25y^2 = 3600$; 2) $9x^2 - y^2 = 144$ giperbolalar uchun o‘qlarning uzunliklarini, fokuslarini va ekssentrisitetini toping.

46. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ giperbolada abssissasi 3 ga teng nuqta olingan. Bu nuqtaning fokal radiuslarini toping.

47. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ ellips berilgan. Uchlari ellipsning fokuslarida, fokuslari esa uning uchlarida bo‘lgan giperbola tenglamasini yozing va uni yasang.

48. Giperbola biror uchidan fokuslarigacha bo‘lgan masofalar 9 va 1 bo‘lsa, uning tenglamasini yozing.

49. $x^2 - 4y^2 = 16$ giperbolani va uning asimptotalarini yasang. Fokuslarini, ekssentrisitetini va asimptotalari orasidagi burchakni toping.

50. $M_1\left(10;-\sqrt{5}\right)$ nuqta $\frac{x^2}{80} - \frac{y^2}{20} = 1$ giperbolada yotadi. M_1 nuqtaning fokal radiuslari yotgan to’g’ri chiziqlarning tenglamasini yozing.

51. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ giperbolaning o’ng fokusidan 4,5 birlik masofada yotuvchi nuqtalarni toping.

52. Quyidagi tenglamalar qanday chiziqlarni ifodalashini aniqlang.

1) $y = -1 + \frac{2}{3}\sqrt{x^2 - 4x - 5}$

2) $y = 7 - \frac{3}{2}\sqrt{x^2 - 6x + 13}$

3) $x = 9 - 2\sqrt{y^2 + 4y + 8}$

4) $x = 5 - \frac{3}{4}\sqrt{y^2 + 4y - 12}$

53. Agar giperbolaning ekssentrisiteti $E = \sqrt{5}$ fokusi $F(2;-3)$ va unga mos direktrisasi $3x - y + 3 = 0$ bo'lsa, uning tenglamasini yozing.

54. $2x - y - 10 = 0$ to'g'ri chiziq va $\frac{x^2}{20} - \frac{y^2}{5} = 1$ giperbolaning kesishish nuqtalarini toping.

66.Fokuslari orasidagi masofa 10 ga teng va mavhum yarimm o`qi 3 ga teng bo`lgan giperbolaning kanonik tenglamasinin tuzing.

55. $16x^2 - 9y^2 = 144$ parabola berilgan. Uning yarim o`qini, fokuslari koordinatalarini, ekssentrisitetini, direktrisasi va asimptotalari tenglamasini to;ping.

56. Giperbolaning fokuslari abssissala o`qida byotib:

a) uning fokuslari orasidagi masofa 6 ga va ekssentrisiteti 1,5 ga teng;

b) Uning haqiqiy yarim o`qi 5 ga teng, uchlari esa markazi bilan fokuslar orasidagi masofani teng ikkiga bo`lsa, uning kanonik tenglamasini tuzing.

57. Giperbolaning fokuslari *Oy* o`qida yotib:

a) asimptotalari $y = \pm\frac{12}{5}x$ va uchlari orasidagi masofa 48 ga teng;

b) Fokuslari orasidagi masofa 10 ga, ekssentrisiteti $\frac{5}{3}$ ga teng bo`lsa, uning kanonik tenglamasini tuing.

58. Mavhum o`qi 4 ga teng va fokusi $F(-\sqrt{13};0)$ nuqtada bo`lgan giperbola tenglamasi tuzilsin.

59. Parabola chizilsin:

a) $y = -x^2$, b) $x = -2y^2$, c) $x = y^2 - 1$, d) $y = x^2 - 6x + 13$, e) $x = 4 - y^2$, d) $y = x^2 + 2$,

e) $y = x^2 + 2x$.

60. $y = x^2$ parabolaning grafigi chizilsin.

61. $x = y^2$ parabolaning grafigi chizilsin.

62. $y = 2x^2 - 4x + 1$ parabolaning parametrlarini aniqlagn va grafigini chizing.

63. $x = y^2 - 1$ parabolaning parametrlarini aniqlang va grafigini chizing.

64. $y^2 = 6x$ parabola berilgan. Uning p parametrini, direktrisasi tenglamasini toping va shaklini chizing.

65. Parabolaning kanonik tenglamasini tuzing: a) parabolaning fokusi $F(0;4)$;b) parabola OY o`qqa nisbatan simmetrik va *A(9;6)* nuqtalarni aniqlang

66. Direktrisasi $y = -3$ bo`lgan parabolaning kanonik tenglamasi tuzilsin.

67. Koordinatalar boshidan va $N(-3;\ 6)$ nuqtadan o‘tib, OY o‘qiga simmetrik bo‘lgan parabola tenglamasini yozing va uni yasang.

68. Koordinatalar boshidan va $N(6;\ 3)$ nuqtadan o‘tib, OY o‘qiga simmetrik bo‘lgan parabola tenglamasini yozing va uni yasang.

69. 1) $y^2 = 6x$; 2) $y^2 = -6x$; 3) $x^2 = -4y$; 4) $x^2 = 4y$ parabolalar uchun fokuslarini va direktrisalarining tenglamalarini toping.

70. $y^2 = 16x$ parabolada fokal radiusi 5 ga teng bo‘lgan nuqtani toping.

71. OX o‘qiga nisbatan simmetrik bo‘lgan parabola, $x + y = 0$ to‘g‘ri chiziq va $x^2 + y^2 + 4y = 0$ aylana kesishgan nuqtalardan o‘tadi. Parabola tenglamasini yozing. Aylanani, to‘g‘ri chiziqni va parabolani yasang.

72. *E(0;-3)* fokusga ega bo’lib, koordinatalar boshidan o’tuvchi parabolaning tenglamasini, OY o’qi parabolaning simmetriya o’qi ekanligini hisobga olib, tuzilsin.

73. Quyidagi tenglamalar qaysi chiziqlarni ifodalaydi.

1) $y = +2\sqrt{x}$

2) $y = -2\sqrt{x}$

3) $x = +\sqrt{5y}$

4) $x = -\sqrt{3y}$

74. *M* nuqtaning ordinatasi 7 ga teng bo’lib $y^2 = 20x$ parabolada yotadi. *M* ning fokal radiuslarini toping.

75. *F(-7;0)* fokusga va *x-7=0* direktrisaga ega bo’lgan parabolaning tenglamasi yozilsin.

76. $y^2 = 16x$ parabolada fokal radiusi 13 ga teng bo’lgan nuqtani toping.

1-Topshiriq

1. *ABC* uchburchak uchlarining koordinatalari berilgan.

a) uchidan o`tkazilgan mediana tenglamasini tuzing va uning uzunligini toping;

b) B uchidan o`tkazilgan balandlik tenglamasini tuzing va shu balandlik uzunligini toping;

v) B burchak bissektrisasi tenglamasini tuzing va uning uzunligini toping.

1. $A(4;1), B(0;-2), C(-5;10)$
2. $A(-7;3), B(5;-2), C(8;2)$
3. $A(5;-1), B(1;-4), C(-4;8)$
4. $A(-14;6), B(-2;1), C(1;5)$
5. $A(6;0), B(2;-3), C(-3;9)$
6. $A(-9;2), B(3;-3), C(6;1)$
7. $A(7;-4), B(3;-7), C(-2;5)$
8. $A(-8;4), B(4;-1), C(7;3)$
9. $A(3;-3), B(-1;-6), C(-6;6)$
10. $A(-6;5), B(6;0), C(9;4)$
11. $A(4;11), B(-1;-1), C(7;5)$
12. $A(3;13), B(-2;1), C(6;7)$
13. $A(7;11), B(2;-1), C(10;5)$
14. $A(6;13), B(1;1), C(9;7)$
15. $A(4;14), B(-1;2), C(7;8)$
16. $A(6;10), B(1;-25), C(9;4)$
17. $A(4;13), B(-1;1), C(7;7)$
18. $A(6;11), B(1;-1), C(9;5)$
19. $A(4;10), B(-1;-2), C(7;4)$
20. $A(-4;10), B(1;7), C(0;4)$
21. $A(-10;-1), B(-6;-4), C(6;1)$
22. $A(18;8), B(12;0), C(0;5)$
23. $A(-6;-3), B(-6;-2), C(10;-1)$
24. $A(14;10), B(8;2), C(-4;7)$
25. $A(-2;-1), B(2;-4), C(14;1)$
26. $A(8;7), B(2;-4), C(14;1)$
27. $A(1;0), B(5;-3), C(17;2)$
28. $A(20;2), B(14;-6), C(26;-1)$
29. $A(-1;7), B(3;4), C(15;9)$
30. $A(7;6), B(1;2), C(-11;3)$

2-Topshiriq

1. ABC uchburchakning uchlari berilgan. Quyidagilarni toping:

a) AB, BC, AC tomon tenglamasini tuzing va $|AB|, |BC|, |AC|$ tomon uzunliklarini toping.

b) C uchidan AB tomonga tushirilgan balandlik tenglamasini;

v) A uchidan BC tomonga tushiriluan mediana tehglamasini va balandlik uzunligini, medianasini uzunligini;

g) “b” va “v” bandlarda topilgan balandlik va medianning kesishish nuqtasi topilsin;

d) *C* nuqtadan o`tuvchi *AB* tomonga parallel to`g`ri chiziq tenglamasini;

e) *C* uchidan *AB* to`g`ri chiziqqacha bo`lgan masofani toping.

1. $A(4;-5), B(6;9), C(-4;-1)$
2. $A(1;-3), B(-5;4), C(-2;10)$
3. $A(1;8), B(-5;-4), C(-1;-3)$
5. $A(6;-4), B(-8;3), C(-2;-7)$.
6. $A(2;3), B(-4;-7), C(2;0)$
7. $A(-4;-8), B(4;1), C(0;7)$
8. $A(4;-2), B(7;0), C(-3;1)$
9. $A(4;1), B(-2;8), C(1;-5)$
10. $A(4;0), B(1;-3), C(5;2)$
11. $A(7;10), B(1;3), C(4;-2)$
13. $A(11;-3), B(-1;-3), C(7;1)$
14. $A(5;9), B(4;-1), C(0;1)$
15. $A(7;3), B(1;7), C(-2;1)$
16. $A(6;-4), B(-8;3), C(-2;-7)$
17. $A(2;6), B(6;-6), C(2;-4)$
18. $A(10;1), B(3;7), C(-3;4)$
19. $A(8;3), B(2;8), C(-4;-4)$
20. $A(7;7), B(-7;5), C(-3;-3)$
21. $A(3;-3), B(4;3), C(-6;1)$
22. $A(6;2), B(-6;8), C(2;-4)$
23. $A(7;5), B(-4;0), C(2;-5)$
24. $A(8;-1), B(2;6), C(-4;4)$
25. $A(-5;0), B(2;-6), C(8;-3)$
26. $A(1;-4), B(-1;10), C(-0;6)$
27. $A(-3;7), B(-1;3), C(2;-4)$
28. $A(10;4), B(-4;6), C(-1;3)$
29. $A(2;-6), B(3;11), C(-1;3)$
30. $A(-5;5), B(4;-7), C(-2;-7)$

Ikkinchi darajali chiziqlar.

3-Topshiriq

Chiziq tenglamasini kanonik ko`rinishga keltiring va uning shaklini chizing.

1. $x^2 + y^2 - 16x + 4y - 13 = 0$
2. $x^2 + y^2 - 6x + 4y - 3 = 0$
3. $x^2 + y^2 - 14x + 6y - 6 = 0$
4. $x^2 + y^2 + 16x - 17 = 0$
5. $x^2 + y^2 - 10x + 4y - 7 = 0$
6. $x^2 + y^2 - 18x + 6y - 10 = 0$
7. $x^2 + y^2 - 14x + 6y - 6 = 0$
8. $x^2 + y^2 - 22x + 8y + 16 = 0$
9. $x^2 + y^2 - 18x - 6y - 10 = 0$
10. $x^2 + y^2 + 18x + 6y - 10 = 0$
11. $x^2 + y^2 - 2x + 2y - 8 = 0$
12. $x^2 + y^2 - 28x + 6y - 20 = 0$

13. $x^2+y^2+28x+6y-20=0$

14. $x^2+y^2-28x+6y-20=0$

15. $x^2+y^2-18x+2y+18=0$

16. $x^2+y^2-18x+6y-31=0$

17. $x^2+y^2-4x+2y-11=0$

18. $x^2+y^2-10x+4y-20=0$

19. $x^2+y^2-10x+2y-1=0$

20. $2x^2+2y^2-16x+8y-5=0$

22. $x^2+y^2-4x-2y-11=0$

23. $x^2+y^2-6x+6y+7=0$

24. $x^2+y^2+6x+6y-7=0$

25. $x^2+y^2-12x+6y-4=0$

25. $x^2+y^2+12x+6y-4=0$

26. $x^2+y^2-2x+6y-6=0$

27. $x^2+y^2-4x+6y-3=0$

28. $x^2+y^2+4x-6y-10=0$

29. $x^2+y^2-28x+2y-1=0$

30. $x^2+y^2+18x+2y+17=0$

4-Topshiriq

Chiziq tenglamasini kanonik ko`rinishga keltiring va uning shaklini chizing.

1. $9x^2+4y^2=36$

2. $2x^2+3y^2=6$

3. $4x^2+3y^2=12$

4. $3x^2+2y^2=6$

5. $5x^2+4y^2=20$

6. $8x^2+5y^2=40$

7. $3x^2+y^2=30$

8. $7x^2+5y^2=35$

9. $x^2+4y^2=4$

10. $3x^2+5y^2=15$

11. $2x^2+6y^2=12$

12. $7x^2+4y^2=28$

13. $4x^2+7y^2=28$

14. $2x^2+17y^2=34$

15. $2x^2+4y^2=24$

16. $2x^2+4y^2=16$

17. $x^2+2y^2=8$

18. $4x^2+9y^2=36$

19. $x^2+3y^2=6$

20. $4x^2+y^2=12$

21. $3x^2+y^2=6$

22. $5x^2+y^2=20$

23. $8x^2+y^2=40$

24. $5x^2+15y^2=30$

25. $5x^2+7y^2=35$

26. $4x^2+2y^2=8$

27. $4x^2+7y^2=28$

28. $2x^2+9y^2=18$

29. $3x^2+4y^2=96$

30. $4x^2+22y^2=44$

5-Topshiriq

Chiziq tenglamasini kanonik ko`rinishga keltiring va uning shaklini chizing.

1. $9x^2 - 4y^2 = 36$
2. $2x^2 - 3y^2 = 6$
3. $4x^2 - 3y^2 = 12$
4. $3x^2 - 2y^2 = 6$
5. $5x^2 - 4y^2 = 20$
6. $8x^2 - 5y^2 = 40$
7. $3x^2 - y^2 = 30$
8. $7x^2 - 5y^2 = 35$
9. $x^2 - 4y^2 = 4$
10. $3x^2 - 5y^2 = 15$
11. $2x^2 - 6y^2 = 12$
12. $7x^2 - 4y^2 = 28$
13. $4x^2 - 7y^2 = 28$
14. $2x^2 - 17y^2 = 34$
15. $2x^2 - 4y^2 = 24$
16. $2x^2 - 4y^2 = 16$
17. $x^2 - 2y^2 = 8$
18. $4x^2 - 9y^2 = 36$
19. $x^2 - 3y^2 = 6$
20. $4x^2 - y^2 = 12$
21. $3x^2 - y^2 = 6$
22. $5x^2 - y^2 = 20$
23. $8x^2 - y^2 = 40$
24. $5x^2 - 15y^2 = 30$
25. $5x^2 - 7y^2 = 35$
26. $4x^2 - 2y^2 = 8$
27. $4x^2 - 7y^2 = 28$
28. $2x^2 - 9y^2 = 18$
29. $3x^2 - 4y^2 = 96$
30. $4x^2 - 22y^2 = 44$

6-Topshiriq

Chiziq tenglamasini kanonik ko`rinishga keltiring va uning shaklini chizing.

1. $y^2 = 4x - 3$
2. $y^2 = 4x$
3. $4x^2 + 5y = 0$
4. $6y^2 - x = 0$
5. $x^2 + 4y = 0$
6. $5y^2 - x = 0$
7. $2y^2 + 5y = 0$
8. $6y - x^2 = 0$
9. $y^2 = 5x - 7$
10. $6y^2 - x = 8$

7-Topshiriq

Chiziq tenglamasini kanonik ko`rinishga keltiring va uning shaklini chizing.

1. Quyidagilar ma`lum:

A,B – egri chiziqda yotuvchi nuqtalar;

a- katta yarim o`q(yoki haqiqiy yarim o`q);

b- kichik(yoki mavhum) yarim o`q;

ε - ekssentrisitet;

$y = \pm kx$ giperbola asimptotalari tenglamasi;

D- egri chiziq direktrisasi;

2c- fokus masofasi.

a) ellipsning; b) giperbolaning; v)parabolaning kanonik tenglamasini tuzing

1. a) $a = 9, \varepsilon = \frac{\sqrt{17}}{9}$; b) $b = 7; F(-\sqrt{130};0)$; v) simmetriya o`qi $OY, A(-4;32)$

2. a) $b = 3, F(-\sqrt{55};0)$; b) $a = 8, \varepsilon = \frac{5}{4}$; v) D:*x=3*

3. a) $A\left(5;\frac{5}{6}\sqrt{11}\right), B\left(-4;\frac{5\sqrt{5}}{3}\right)$; b) $k = \frac{2}{7}; \varepsilon = \frac{\sqrt{53}}{7}$; v) D: $y = -4$

4. a) $\varepsilon = \frac{4}{5}, A\left(-4;\frac{9}{5}\right)$; b) $A\left(-5;\frac{9}{4}\right) va B\left(\frac{20}{3};-4\right)$; v) simmetriya o`qi $OX, A(-6;10)$.

5. a) $2a = 18, \varepsilon = \frac{\sqrt{77}}{9}$; b) $k = \frac{6}{7}; c = \sqrt{85}$; v) D: *x=-3*

6. a) $b = 5; \varepsilon = \frac{2\sqrt{6}}{7}$; b) $k = \frac{4}{7}; 2a = 14$; v) D:*x=-3*

7. a) $a = 6, \varepsilon = \frac{7\sqrt{3}}{2}$; b) $b = 1, F(-\sqrt{17};0)$; v) simmetriya o`qi OY, $A(-4;-10)$

8. a) $b = 4, F(-3;0)$; b) $a = 3, \varepsilon = \frac{\sqrt{13}}{3}$; v) D: *x=8.*

9. a) $A(-3\sqrt{5};4) va \quad B(6;-2\sqrt{5})$; b) $k = \frac{5}{9}, \varepsilon = \frac{\sqrt{106}}{9}$; v) D: *y=-16*

10. a) $\varepsilon = \frac{\sqrt{39}}{8}; A\left(-4;\frac{5\sqrt{3}}{2}\right)$; b) $A\left(-6;\frac{7\sqrt{7}}{4}\right) \; va \; B\left(\frac{16\sqrt{6}}{7};5\right)$; v) simmetriya o`qi *OX,* *A(-3;6).*

11. a) $2a=12, \varepsilon=\frac{\sqrt{5}}{3}$; 2 b) $k=\frac{1}{3}; 2c=4\sqrt{10};$ v) D: *x=8.*

12. a) $b=2, \varepsilon=\frac{\sqrt{3}}{2}$; b) $k=\frac{1}{3}, 2a=18;$ v) D: *x=-5.*

13. a) $a=9, \varepsilon=\frac{\sqrt{65}}{9};$ b) $b=4, F(-4\sqrt{5};0);$ v) simmetriya o`qi *OY, A (-3;4).*

14. a) $b=2, F\left(-2\sqrt{15};0\right);$ b) $a=5, \varepsilon=\frac{\sqrt{29}}{5};$ v) D: $x=\frac{5}{8}.$

15. a) $A\left(-3;\frac{6}{7}\sqrt{10}\right) va B\left(\frac{7}{3}\sqrt{5};-2\right);$ b) $k=\frac{1}{3}; \varepsilon=\frac{\sqrt{10}}{3};$ v) D: $y=-\frac{3}{8}$

16. a) $\varepsilon=\frac{4\sqrt{2}}{9}; A\left(6;-\frac{7\sqrt{5}}{3}\right);$ b) $A\left(-\frac{9\sqrt{5}}{2};4\right) va B\left(3;-\frac{8\sqrt{10}}{3}\right);$ v) simmetriya o`qi *OX, A(-3;8).*

17. a) $2a=16, \varepsilon=\frac{\sqrt{7}}{4}$; b) $k=\frac{3}{8}, 2c=2\sqrt{73};$ v) D: $y=6$

18. a) $b=2; \varepsilon=\frac{3\sqrt{5}}{7};$ b) $k=\frac{5}{6}, 2a=12;$ v) D: $x=-\frac{5}{9}$

19. a) $a=4, \varepsilon=\frac{\sqrt{7}}{4}$; b) $b=3, F(-\sqrt{34};0);$ v) simmetriya o`qi *OY, A(-3;-4).*

20. a) $b=6, F\left(\sqrt{13};0\right);$ b) $a=9, \varepsilon=\frac{\sqrt{85}}{9};$ v) D: $x=6$.

22. a) $\varepsilon=\frac{\sqrt{15}}{4}, A\left(-3;\frac{\sqrt{7}}{4}\right);$ b) $A\left(8;-\sqrt{17}\right) va B(10;4);$ v) D: $y=-8.$

23. a) $2a=6, \varepsilon=\frac{\sqrt{5}}{3};$ b) $k=\frac{4}{5}; 2c=2\sqrt{41};$ v) simmetriya o`qi *OX, A(-2;6).*

24. a) b=5, $\varepsilon=\frac{2\sqrt{14}}{9}$; b) $k=\frac{2}{3}, 2a=18;$ v) D; $x=-5.$

25. a) $a=8, \varepsilon=\frac{\sqrt{15}}{8};$ b) $b=5, F\left(-\sqrt{89};0\right);$ V) simmetriya o`qi *OY, A(-2;6).*

26. a) $b=2, F\left(-4\sqrt{2};0\right);$ b) $a=6, \varepsilon=\frac{\sqrt{13}}{3};$ v) D: $x=9$

27. a) $A\left(6;-\sqrt{5}\right) va B\left(-3\sqrt{5};2\right);$ b) $k=\frac{1}{2}; \varepsilon=\frac{\sqrt{5}}{2};$ v) D: $y=-3$

28. a) $\varepsilon=\frac{\sqrt{3}}{2}, A\left(-6;-\sqrt{7}\right)$; b) $A\left(10;\frac{4\sqrt{19}}{9}\right), B\left(\frac{9\sqrt{5}}{2};-2\right)$; v) D: $y=9$

29. a) $2a=10, \varepsilon=\frac{\sqrt{21}}{5}$; b) $k=\frac{1}{4}, 2c=4\sqrt{17}$; v) simmetriya o`qi *OX, A(3;-5).*

30. a) $b=1, \varepsilon=\frac{2\sqrt{2}}{3}$; b) $k=\frac{3}{7}; 2a=14$; v) D: $x=-\frac{3}{4}$;

<u>**Adabiyotlar**</u>

1. T.Sh. Shodiyev. «Analitik geometriya va chiziqli algebra». T. «O'qituvchi» nash. 1984 yil. 56-61 betlar.
2. Yo.U. Soatov. «Oliy matematika» 1-j. T. «O'qituvchi». 1992 yil. 48-55 betlar.
3. F. Rajabov, A. Nurmatov. «Analitik geometriya va chiziqli algebra». T. «O'qituvchi». 1990 yil. 59-62 betlar.
4. http;//docs.ttesi/uz/sim/htme/Oliy matematika,2005y.

MUNDARIJA