O`ZBEKISTON RESPUBLIKASI
OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

TOSHKENT TO`QIMACHILIK VA YENGIL SANOAT INSTITUTI

"OLIY MATEMATIKA"
kafedrasi

R E F E R A T

Mavzu: *Kompleks sonlar va ular ustida chiziqli amallar. Kompleks sonning tekislikdagi tasviri. Lomplers sonning moduli va argumenti. Kompleks sonning berilish usullari. Eyler formulasi. Kompleks sonning ko`rsatkichli shakli. Kompleks sondan ildiz chiqarish. Muavr formulasi.*

22-16 guruh
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Toshkent 2017

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Kompleks sonlar

Kompleks son deb

$$z = x + iy \qquad (1)$$

ifodaga aytiladi, bu erda x va y haqiqiy sonlar, i - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \qquad \text{yoki} \qquad i^2 = -1 \qquad (2)$$

x- kompleks son z ning haqiqiy qismi, iy - mavhum qismi deyiladi. Ular bunday belgilanadi: $x=Re\ z$, $y=Imz$. Agar $x=0$ bo`lsa, $0+iy=iy$ sof mavhum son deyiladi; $y=0$ agar bo`lsa, haqiqiy son hosil bo`ladi: $x+i0=x$. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: $z=x+iy$ va $z=x-iy$ bir-biriga qo`shma deyiladi.

Ushbu ikki asosiy ta`rif qabul qilinadi.

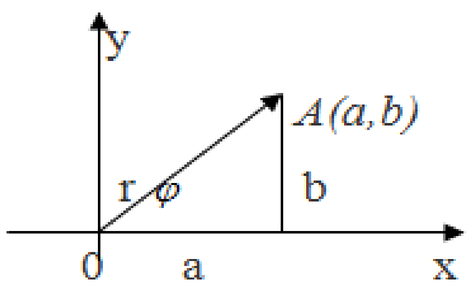
1. Agar $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ dan iborat ikki kompleks sonda $x_1=x_2$; $y_1=y_2$

Bo`lsa, ya`ni haqiqiy qismlar o`zaro va mavhum qismlar o`zaro teng bo`lsa, bunday kompleks sonlar teng deyiladi.

2. Agar $x=0$, $y=0$ bo`lsa, faqat shundagina kompleks son nolga teng bo`ladi: $z=x+iy$

Kompleks sonlarning geometrik tasviri

Har qanday kompleks son $z=x+iy$ ni OXY tekisligida koordinatalari x va y bo`lgan $A(x,y)$ nuqta shaklida tasvirlash mumkin. Aksincha, $OXY$ tekisligidagi har qanday $M(x,y)$ nuqta $z=x+iy$ kompleks songa mos keladi. O`zida kompleks son tasvirlanadigan tekislik o`zgaruvchi z ning kompleks tekisligi deyiladi



O`zgaruvchi *z* kompleks tekisligining *OX* o`qida yotuvchi nuqtalariga haqiqiy sonlar mos keladi *(y=0)*. *OY* o`qida yotuvchi nuqtalar sof mavhum sonni tasvirlaydi, chunki bu holda *x=0*. Shuning uchun kompleks sonlarni *z* ning kompleks o`zgaruvchi tekisligida tasvirlaganda *OY* o`q mavhum sonlar yoki mavhum o`q, *OX* o`q esa haqiqiy o`q deyiladi. *A(x,y)* nuqtani koordinatalar boshi bilan tutuashtirib, vektorni hosil qilamiz. Ba'zi hollarda *z=x+iy* kompleks sonning geometrik tasvirini *OA* vektor deb qabul qilish qulay bo`ladi.

Kompleks sonning trigonometrik shakli

Koordinatalar boshini qutb, *OX* o`qining musbat yo`nalishini qutb o`qi deb olib, *A(x,y)* nuqtaning qutb koordinatalarini φ va $r(r \geq 0)$, bilan belgilaymiz. Unda ushbu tengliklarni yozish mumkin:

$$x = r\cos\varphi, \qquad y = r\sin\varphi$$

demak, kompleks son *z* ni bunday tasvirlash mumkin:

$$x+iy=r\cos\varphi + ir\sin\varphi$$

yoki

$$z= r(\cos\varphi + i\sin\varphi) \qquad (3)$$

Bu tenglikning o`ng tomonida ifodada *z=x+iy* kompleks sonning trigonometrik shakli deb ataladi.

z kompleks sonning modulini r va argumentini φ deb belgilaymiz; ular bunday ifodalanadi:

$$r = |z|, \qquad \varphi = \arg z \qquad (4)$$

r va φ miqdorlar *x* va *y* orqali bunday ifodalanadi:

$$r = |z| \qquad \varphi = arctg\frac{y}{x}$$

Demak,

$$\left.\begin{array}{l} r = |z| = |x+iy| = \sqrt{x^2+y^2} \\ \varphi = \arg z = \arg(x+iy) = arctg\frac{y}{x} \end{array}\right\} \qquad (5)$$

Kompleks sonning argumenti φ burchak OX o`qning musbat yo`nalishidan soat strelkasi harakatiga teskari yo`nalishda hisoblansa musbat, qarama-qarshi yo`nalishda hisoblansa manfiy bo`ladi. Ravshanki, argument bir qiymatli bo`lmasdan, balki $2k\pi$ qo`shiluvchiga (k-ixtiyoriy butun son) aniqlikda belgilanadi.

Izoh. Qo`shma kompleks sonlar $z=x+iy$ va $\bar{z}=x-iy$ teng modullarga ega: $|z|=|\bar{z}|$, argumentlarning absolyut qiymatlari teng, ammo ishoralari bilan farqlanadi:

$$\arg \bar{z} = -\arg z$$

Haqiqiy son A ni ham (3) shaklda yozish mumkin, ya'ni:

$A>0$ bo`lsa, $A=|A|(\cos 0 + i\sin 0)$,

$A<0$ bo`lsa, $A=|A|(\cos\pi + i\sin\pi)$.

Nolga teng bo`lgan kompleks sonning moduli nolga teng: $|0|=0$. Nolning argumenti sifatida har qanday φ burchakni qabul qilish mumkin. Haqiqatan har qanday φ burchak uchun ushbu tenglikni yozish mumkin:

$0= 0(\cos\varphi + i\sin\varphi)$.

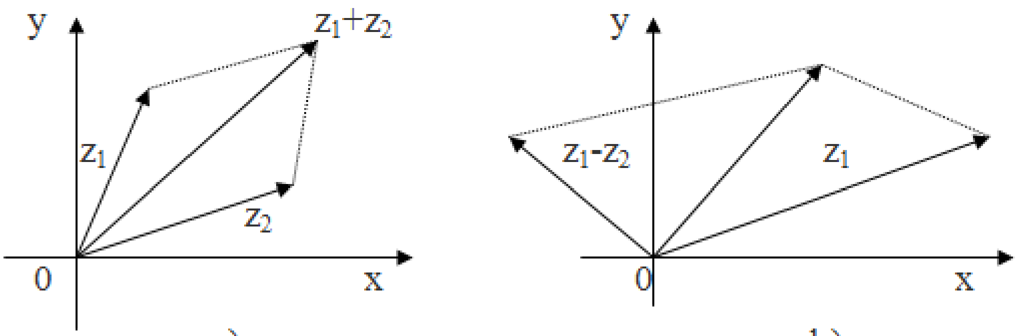
Kompleks sonlarni qo`shish

Ikki kompleks son $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ ning yig`indisi deb ushbu

$$z_1+z_2=(x_1+iy_1)+(x_2+iy_2)=(x_1+x_2)+i(y_1+y_2) \qquad (1)$$

tenglik bilan aniqlangan kompleks songa aytiladi.

(1) formuladan vektorlar bilan tasvirlangan kompleks sonlarni qo`shish-vektorlarni qo`shish qoidasiga muvofiq bajarilishi kelib chiqadi.



a) b)

Kompleks sonlarni ayirish

Ikki $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ kompleks sonlarni ayirmasi deb shunday kompleks songa aytiladiki, unga z_2 kompleks sonni qo`shganda $z_1$ kompleks son hosil bo`ladi:

$$z_1 - z_2=(x_1+iy_1)- (x_2+iy_2)=(x_1 -x_2)+i(y_1 -y_2) \quad (2)$$

Ikki kompleks son ayirmasining moduli shu sonlarni kompleks o`zgaruvchilar tekisligida tasvirlovchi nuqtalar orasidagi masofaga teng:

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Kompleks sonlarni ko`paytirish

$z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ kompleks sonlar ko`paytmasi deb, ularni ikki hadlar singari algebra qoidasiga muvofiq, lekin

$i^2 = -1,\quad i^3 = -i,\qquad i^4 = (-i)i = -i^2 = 1,\quad i^5 = i$, va hokazo, umuman k butun bo`lganda:

$$i^{4k} = -1,\qquad i^{4k+1} = i,\ i^{4k+2} = -1,\ i^{4k+3} = -i.$$

Shu qoidaga asosan quyidagi ko`paytmani hosil qilamiz:

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + iy_1 x_2 + ix_1 y_2 + i^2 y_1 y_2$$

yoki

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2). \quad (3)$$

Kompleks sonlar trigonometrik shaklda berilgan bo`lsin:

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1, z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2).$$

Bu sonlarning ko`paytmasini topamiz:

$$\begin{aligned} z_1 z_2 &= r_1(\cos\varphi_1 + i\sin\varphi_1) r_2(\cos\varphi_2 + i\sin\varphi_2) = r_1 r_2[\cos\varphi_1\cos\varphi_2 + i\sin\varphi_1\cos\varphi_2 + i\cos\varphi_1\sin\varphi_2 + \\ &+ i^2\sin\varphi_1\sin\varphi_2] = r_1 r_2[(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2) + i(\sin\varphi_1\cos\varphi_2 + \cos\varphi_1\sin\varphi_2)] = \\ &= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]. \end{aligned}$$

Shunday qilib,

$$z_1 z_2 = r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)].$$

ya`ni ikki kompleks son ko`paytmasi shunday kompleks sonki, uning moduli ko`paytuvchilar modullarining ko`paytmasiga teng, argumenti esa ko`paytuvchilar argumentlarining yig`indisiga teng.

1-izoh. $z=x+iy$ va $z=x-iy$ qo`shma kompleks sonlar ko`paytmasi (3) formulaga muvofiq bunday ifodalanadi:

$$z\left|\bar{z}\right| = x^2 + y^2,$$

yoki

$$z\bar{z} = \left|z\right|^2 = \left|\bar{z}\right|^2.$$

Qo`shma kompleks sonlar ko`paytmasi ulardan har biri modulining kvadratiga teng.

Kompleks sonlarni bo`lish

Kompleks sonlarni bo`lish ko`paytirishga teskari amal kabi ta`riflanadi:

$$z_1=x_1+iy_1,\ z_2=x_2+iy_2,\quad \left|z_2\right| = \sqrt{x_2^2 + b_2^2} \neq 0$$

deb faraz qilamiz. U holda $\frac{z_1}{z_2} = z$ shunday kompleks sonki, unda

$z_1=z_2*z$ bo`ladi.

Agar $\frac{x_1 + iy_1}{x_2 + iy_2} = x + iy$ bo`lsa, u holda

$$x_1+iy_1=(x_2+iy_2)(x+iy)$$

yoкi

$$x_1+iy_1=(x_2x-y_2y)+i(x_2y+y_2x);$$

x va y ushbu

$$x_1=x_2x-y_2y,\quad y_1=y_2x+x_2y$$

tenglamalar sistemasi bilan aniqlanadi. Bundan:

$$x = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}, \quad y = \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

Nihoyat, ushbu formulani hosil qilamiz:

$$z = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2} \qquad (4)$$

Trigonomrteik funksiyalarning qirmatlari jadvali.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Gradus	0	30°	45°	60°	0°	120°	135°	150°	180°	210°	225°	240°	270	300	315	330	360
$\sin\varphi$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\varphi$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$tg\varphi$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$ctg\varphi$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	o	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	-

Kompleks sonlarni bo`lish amalda bunday bajariladi: $z_1=x_1+iy_1$ ni $z_2=x_2+iy_2$ ga bo`lish uchun bo`linuvchi va bo`luvchini bo`luvchiga qo`shma songa ko`paytiramiz.

Unda bo`luvchi haqiqiy son bo`ladi; unga bo`linuvchining haqiqiy va mavhum qismlarini bo`lamiz:

$$\frac{x_1+iy_1}{x_2+iy_2}=\frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}=\frac{(x_1x_2+y_1y_2)+i(x_2y_1-x_1y_2)}{x^2+b^2}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$$

Kompleks sonlar trigonometrik shaklda

$$z_1=r_1(\cos\varphi_1+i\sin\varphi_1),$$
$$z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$$

berilgan bo`lsa, ushbuni hosil qilamiz:

$$\frac{z_1}{z_2}=\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2-i\sin\varphi_2)}=\frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2) \quad (5)$$

Bu tenglikni tekshirish uchun bo`luvchini bo`linmaga ko`paytirish kifoya:

$$r_2(\cos\varphi_2 + i\sin\varphi_2)\ \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2) =$$

$$= r_2\frac{r_1}{r_2}[\cos(\varphi_2 + \varphi_1 - \varphi_2 + i\sin(\varphi_2 + \varphi_1 - \varphi_2)] = r_1(\cos\varphi_1 + i\sin\varphi_1).$$

Shunday qilib, ikki kompleks son bo`linmasining moduli bo`linuvchi va bo`luvchi modullarining bo`linmasiga teng; bo`linmaning argumenti bo`linuvchi va bo`luvchi argumentlarining ayirmasiga teng.

Teorema. Koeffisientlari haqiqiy sonlar bo`lgan ushbu

$$A_0x^n + A_1x^{n-1} + \ldots + A^n$$

Ko`phadda $x$ o`rniga $x+iy$ son, so`ngra unga qo`shma son $x-iy$ qo`yilsa, o`rniga qo`yish natijalari ham o`zaro qo`shma bo`ladi.

Misol. Ushbu $z_1 = 3 - i,\ z_2 = -2 + 3i,\ z_3 = 4 + 3i$ kompleks sonlar berilgan bo`lsin. $z = \dfrac{z_1 - z_2 \cdot z_3}{z_1^3 + z_3}$ ni hisoblang.

Yechish. Ketma-ket hisoblaymiz:

$$z_2 \cdot z_3 = (-2 + 3i)(4 + 3i) = (-8 - 9) + i(12 - 6) = -17 + 6i;$$
$$z_1 - z_2 \cdot z_3 = (3 - i) - (-17 + 6i) = (3 + 17) + i(-1 - 6) = 20 - 7i;$$
$$z_1^3 = (3 - i)^3 = 27 - 27i + 9i^2 - i^3 = (27 - 9) + i(-27 + 1) = 18 - 26i;$$
$$z_1^3 + z_3 = (18 - 26i) + (4 + 3i) = (18 + 4) + i(-26 + 3) = 22 - 23i.$$

Shunday qilib,

$$z = \frac{20 - 7i}{22 - 23i} = \frac{(20 - 7i)(22 + 23i)}{(22 - 23i)(22 + 23i)} = \frac{(440 + 161) + i(460 - 154)}{22^2 + 23^2} =$$
$$= \frac{601}{1013} + i\frac{306}{1013}.$$

Misol. $z = -\sqrt{3} + i$ kompleks sonning moduli , argumentini , trigonometrik va ko`rsatkichli shakllarini toping.

Yechish. $x = -\sqrt{3}, y = 1$ bo`lganligi uchun $r = \sqrt{x^2 + y^2} = 2.$ $tg\varphi = -\dfrac{1}{\sqrt{3}}$ tenglamadan φ argumentni topamiz:

$$\varphi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$$

Shunday qilib, $r=2, \quad \varphi=\frac{5\pi}{6}$.

$$z=2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right), \quad z=2e^{\frac{5\pi}{6}i}.$$

Misol. $z=\left(-\sqrt{3}+i\right)^6$ ni hisoblang.

Yechish. $x=-\sqrt{3}, y=1$ bo`lganligi uchun $r=\sqrt{x^2+y^2}=2$. $tg\varphi=-\frac{1}{\sqrt{3}}$

tenglamadan φ argumentni topamiz:

$$\varphi=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}=$$

Shunday qilib, $r=2, \quad \varphi=\frac{5\pi}{6}$.

Muavr formulasidan foydalanib quyidagi yechimga ega bo`lamiz:

$$z=2^6\left(\cos\frac{5\pi}{6}\cdot 6+i\sin\frac{5\pi}{6}\cdot 6\right)=2^6e^{5\pi i}=$$
$$=64\left(\cos 5\pi+i\sin 5\pi\right)=-64.$$

Misol. $\sqrt[3]{-1}$ ni toping.

Yechish. z =-1 soni uchun $r=1, \ \varphi=\pi$. Shu sababli uning trigonametrik shakli quyidagicha yoziladi:

$$z=1\cdot\left(\cos\pi+i\sin\pi\right).$$

n- darajali ildiz chiqarish formulasidan foydalanib, ushbuga ega bo`lamiz:

$$\omega_k=\sqrt[3]{\cos\pi+i\sin\pi}=\cos\frac{\pi+2\pi k}{3}+i\sin\frac{\pi+2\pi k}{3}=$$
$$=e^{\frac{i(\pi+2\pi k)}{3}}, \ bunda \ \ k=0;1;2.$$

k ga ketma-ket 0;1;2 qiymatlarn berib, ildizni uchala qiymatini topamiz:

$$\omega_0=cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=e^{\frac{i\pi}{3}}=\frac{1}{2}+i\frac{\sqrt{3}}{2}$$
$$\omega_1=\cos\pi+i\sin\pi=e^{i\pi}=-1$$
$$\omega_2=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}=e^{\frac{5\pi i}{3}}=\frac{1}{2}-i\frac{\sqrt{3}}{2}.$$

Kompleks sonni darajaga ko`tarish

Bundan oldingi paragrafdagi (3) formuladan, agar n butun musbat son bo`lsa, ushbu formula kelib chiqadi:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi). \quad (1)$$

Bu Muavr formulasi deb ataladi. Bundan ko`rinadiki, kompleks sonni butun musbat darajaga ko`tarishda modul shu darajaga ko`tariladi, argument esa daraja ko`rsatkichiga ko`paytiriladi.

Endi Muavr formulasining yana bir tadbiqini qaraymiz

Bu formulada *r=1* deb faraz qilib,

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$$

tenglikni hosil qilamiz. Chap tomonni Nyuton binomi formulasi bo`yicha yoyib, haqiqiy va mavhum qismlarini tenglab, $\sin n\varphi$ va $\cos n\varphi$ ni $\sin\varphi$ va $\cos\varphi$ ning darajalari orqali ifoda qila olamiz.

Kompleks sondan ildiz chiqarish.

Kompleks sonning n-darajali ildizi deb n -darajaga ko`targanda ildiz ostidagi songa teng bo`ladigan kompleks songa aytiladi, ya`ni

$$\rho^n(\cos n\psi + i\sin n\psi) = r(\cos\varphi + i\sin\varphi)$$

bo`lsa,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \rho(\cos\psi + i\sin\psi).$$

Teng kompleks sonlarning modullari teng bo`lishi kerak, argumentlari esa $2\pi$ ga karrali songa farq qilishi mumkin bo`lgani uchun

$$\rho^n = r, \qquad n\psi = \varphi + 2\kappa\pi$$

Bundan

$$\rho = \sqrt[n]{r}, \qquad \psi = \frac{\varphi + 2\kappa\pi}{n},$$

bu yerda k - ixtiyoriy butun son, - musbat r son ildizining arifmetik qiymati. Demak,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2\kappa\pi}{n} + i\sin\frac{\varphi + 2\kappa\pi}{n}\right) \quad (2)$$

k ga *0,1,2,…,n-1* qiymatlarni berib, ildizning *n* ta har xil qiymatlarini topamiz.

Shunday qilib, kompleks sonning *n*- darajali ildizi *n* ta har xil qiymatga ega bo`ladi.

Foydalaniladigan adabiyotlar ro`yxati

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