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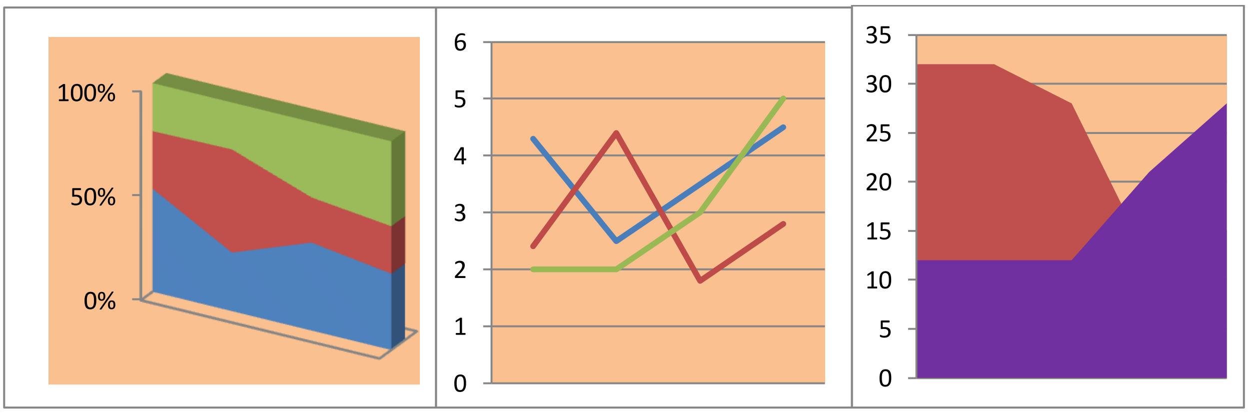
Matematikadan misol va masalalar to‘plami

o‘quv qo‘llanma



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Mazkur o‘quv qo‘llanma Oliy va o‘rta maxsus ta’lim vazirligi tomonidan tasdiqlangan “Matematika” o‘quv dasturining to‘plamlar, haqiqiy sonlar va kompleks sonlar, chiziqli algebra elementlari, vektorlar algebrasi, tekislikda analitik geometriya, matematik analizga kirish, funksiyani hosila yordamida to‘la tekshirish, ko‘p argumentli funksiya bo‘limlariga muvofiq yozilgan bo‘lib, bakalavriyat talabalari uchun mo`ljallangan. Unda qisqacha nazariy ma’lumotlar bilan birga, amaliy mashg‘ulotlar va topshiriqlar berilgan.

Ushbu o‘quv qo‘llanma Toshkent to‘qimachilik va yengil sanoat institutining ilmiy kengashi tomonidan nashrga tavsiya etilgan.

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So‘z boshi

Respublikada “Ta’lim to‘g‘risida” gi va “ Kadrlar tayyorlash milliy dasturi” haqidagi qonunlarni isloh qilish yetuk kadrlar tayyorlashda oliy o‘quv yurtlari oldiga ham bir qator vazifalar qo‘ymoqda. Jumladan, talabalar foydalanayotgan o‘quv adabiyotlarini qaytadan ko`rib chiqishni taqazo etmoqda. Shu sababli ta’lim yo‘nalishdagi talabalar uchun matematika fanidan o‘quv qo‘llanma yozishga ehtiyoj tug’ildi.

O‘quv qo‘llanmaning texnika oliy o‘quv yurtlari uchun yozilgan o‘quv qo‘llanma, darsliklardan farqi uning yozilishida, fanning o‘qitilishi uchun o‘quv rejasida o‘quv soatlartning kamligini hisobga olinishida, sodda berilishida hamda mavzularning misol va masalalar bilan o‘z-o‘zini tekshirish savollari bilan ta’minlanganligidadir. Shuning uchun mavzular qisqa, ba’zi bir teoremalar esa isbotsiz keltirilgan.

O‘quv qo‘llnma barcha bakalavriat yo‘nalishlari talabalari uchun “Matematika” fani dasturiga asosan yozilgan bo‘lib, unda to‘plamlar, haqiqiy sonlar, kompleks sonlar, ko‘phadlar, chiziqli algebra elementlari, vektorlar algebrasi, tekislikda analitik geometriya , matematik analiz, bir o‘zgaruvchili funksiyaning differensiali, ikki o‘zgaruvchili funksiyalar bayon etilgan.

Qo‘llanmaga ko‘p yillar davomida o‘qigan ma’ruza va amaliy mashg‘ulotlar materiallari asos qilib olingan. Shuningdek, shu sohaga tegishli mavjud o‘zbek, rus va ingliz tilidagi adabiyotlardan keng foydalanildi.

Qo‘llanmani o‘qib, o‘z fikr- mulohazalarini bildirgan Toshkent irrigatsiya va melioratsiya instituti dotsenti N. Yo‘ldoshev va Toshkent to‘qimachilik va yengil sanoat instituti dotsentlari X.Abduraxmanova, R.Yarkulovlarga mualliflar chuqur minnatdorchilik bildiradilar.

Mualliflar

Mundarija

1-BOB. TO‘PLAMLAR. HAQIQIY SONLAR. KOMPLEKS SONLAR.

1.1. MAVZU: TO‘PLAMLAR VA ULAR USTIDA AMALLAR

MAVZUGA OID NAZARIY MATERIALLAR

1. To‘plam tushunchasi.

To‘plam matematikadagi boshlang‘ ich tayanch tushunchalardan bo‘lib, unga ta’rif berilmaydi. To‘plam deb qandaydir ob’ektlarning birikmasi tushuniladi.

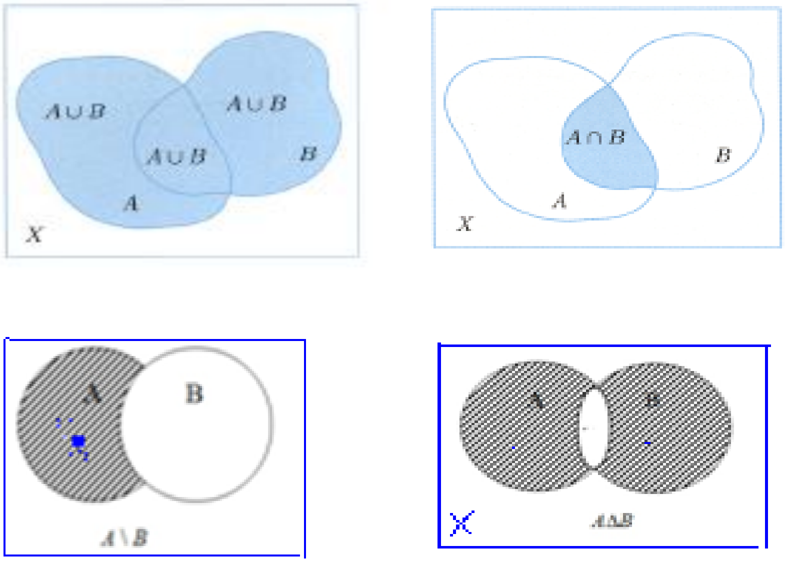
Masalan: institutdagi talabalar to‘plami, ko‘ldagi baliqlar to‘plami, Natural sonlar to‘plami va boshqalar. To‘plamni tashkil qilgan ob’ektlar, uning elementlari deyiladi. To‘plamlar katta harflar *A, B, C,…X ,Y , Z,…* bilan, uning elementlarini kichik harflar *a, b,….x, y….* bilan belgilanadi.

Agar x element X to‘plamga tegishli bo‘lsa $x \in X$, agar X to‘plamga tegishli bo‘lmasa $x \notin X$ yoziladi. Birorta ham elementga ega bo‘lmagan to‘plamni bo‘sh to‘plam deyiladi. To‘plam 3 ta 2,3,26 sonlaridan iborat degani $A = \{2, 3, 26\}$ kabi yoziladi. $A = \{x: \ 0 \le x \le 5\}$ yozuvi A to‘plam haqiqiy sonlardan tashkil topgan bo‘lib, $0 \le x \le 5$ tengsizlikni qanoatlantiradi. *A* to‘plam *B* ning to‘plamosti to‘plami deyiladi, agar *A* to‘plamning barcha elementi *B* to‘plamning ham elementi bo‘lsa va $A \subset B$ ko‘rinishda yoziladi. $A = B$ deyiladi, agar $A \subset B$ va $B \subset A$ bajarilsa [1].

A va B to‘plamlarning birlashmasi $A \cup B$ (yoki yig’indisi $A + B$) $A \cup B = \{x: \ x \in A \ \ yoki \ \ x \in B\}$ bo‘ladi.

A va B to‘plamlarning ko‘paytmasi $A \cap B$ (yoki AB) deyiladi, agar $A \cap B = \{x: \ x \in A \ \ va \ \ x \in B\}$ bo‘lsa.

Ikkita (*A* va B) to‘plam ayirmasi (farqi) deb , *A* to‘plamga tegishli, biroq *B* to‘plamga tegishli bo‘lmagan elementlar to‘plamiga aytiladi va *A**B* kabi belgilanadi (1-chizma).



1-chizma

Ikkita (A va B) to‘plamning simmetrik ayirmasi deb, faqat A to‘plamga va faqat B to‘plamga tegishli bo‘lgan elementlar to‘lamiga aytiladi va $A\Delta B$ kabi belgilanadi. Agar $A \subset B$ bo‘lsa, u holda $B\backslash A$ ayirma A to‘plamni B to‘plamgacha to‘ldiruvchi to‘plam deyiladi va $C_B A= B\backslash A$ deb yoziladi.

Agar qaralayotgan masala mohiyatidan kelib chiqib, masala yechimi sifatida hosil bo’ladigan barcha to‘plamlar biron-bir Ω to‘plamning qismi ekanligi ma`lum bo‘lsa, u holda Ω to‘plam bosh to‘plam deyiladi. Bu holda, har qanday $A \subset \Omega$ uchun A to‘plamning to‘ldiruvchisi (Ω to‘plamgacha to‘ldiruvchisi) deb $\overline{A}$ kabi belgilanib, $\Omega \setminus A = \overline{A}$ munosabat bilan aniqlanadi.

To‘plamlarning ba’zi xossalari:

1. $A \cup \varnothing = A,\ \ A \cap \varnothing = \varnothing$; 2. $A \cup \overline{A} = \Omega,\ \ A \cap \overline{A} = \varnothing$;

3. $\overline{\varnothing} = \Omega,\ \ \overline{\Omega} = \varnothing$; 4. $A \cup A = A,\ \ A \cap A = A$;.

5. $A \cup \Omega = \Omega,\ \ A \cap \Omega = A$; 6. $A \cup (A \cap B) = A,\ \ A \cap (A \cup B) = A$;

7. $A \cap B = B \cap A,\ \ A \cup B = B \cup A$; 8. $(A \cap B) \cap C = A \cap (B \cap C)$;

9. $(A \cup B) \cup C = A \cup (B \cup C)$; 10. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$;

2. Haqiqiy sonlar to‘plami.

1. N = $\{1,2,3,4,....\}$ to‘plamga natural sonlar to‘plami deyiladi.

2. Natural sonlar, ularga qarama-qarshi sonlar hamda nol soni butun sonlar to‘plamini hosil qiladi. Yani $Z = \{\dots, -n, \dots, -2, -1,0,1,2,3, \dots\}$ barcha butun sonlar to‘plamidir.

3. $\frac{p}{q}$ ko‘rinishdagi sonlar ratsional sonlar deyiladi. Bunda p biror butun, q esa natural son. Barcha ratsional sonlar to‘plamini Q orqali belgilaymiz:

$$Q = \left\{\frac{p}{q} : p \in Z, q \in N\right\}.$$

Ratsional sonlarni chekli yoki cheksiz davriy o‘nli kasr ko‘rinishida ifodalash mumkin: $34{,}6$; $352{,}0874329$; $7{,}4666 \dots = 7{,}4(6)$; $\frac{1}{3} = 0{,}333 \dots = 0,(3)$;

4. Davriy bo‘lmagan cheksiz o‘nli kasrlar bilan tasvirlanuvchi sonlar irratsional sonlar deb ataladi:

$$I=\{\sqrt{2}, \sqrt{3}, \sqrt{5},...\}.$$

5.Ratsional sonlar va irratsional sonlar birgalikda haqiqiy sonlar deyiladi. Sonlar o‘qidagi har bir nuqta haqiqiy sonlar to‘plamini tashkil etadi.

$$[Q + I = R]$$

Agar x_1 son musbat bo‘lsa, bu son nol nuqtadan o‘ng tomonda $OM_1=x_1$ masofada yotuvchi M_1 nuqta bilan tasvirlanadi; agarda x_2 son manfiy tomonda bo‘lsa, bu son nol nuqtadan chap tomonda $OM_2=x_2$ masofada yotuvchi M_2 nuqta bilan tasvirlanadi.

Sonli to‘plamlarning ba’zi turlarini ko‘rib o‘tamiz:

$a \le x \le b$ munosabatni qanoatlantiruvchi x sonlar to‘plami $[a,b]$ kesma, segment yoki yopiq oraliq deyiladi.

$a < x < b$ munosabatni qanoatlantiruvchi x sonlar to'plami *(a, b)* interval yoki ochiq oraliq deyiladi;

$a < x \leq b$ yoki $a \leq x < b$ munosabatlar orqali *(a, b]* va yarim ochiq oraliqlardagi sonlar ifodalanadi.

$-\infty<x<a$, $b<x<+\infty$, $-\infty <x< +\infty$ munosabatlarni qanoatlantiruvchi x uchun sonlar to'plami $(-\infty, a)$, (b, ∞), $(-\infty, +\infty)$ kabi belgilanadi, yarim chekli va cheksiz oraliqlar deb ataladi.

$-\infty <x< 0$ yoki $R_-=(-\infty, 0)$ – barcha manfiy haqiqiy sonlar to'plami;

$0<x<+\infty$ yoki $R_+=(0, +\infty)$ – barcha musbat haqiqiy sonlar to'plami;

$-\infty <x<+\infty$ yoki $R=(-\infty, +\infty)$ – barcha haqiqiy sonlar to'plami.

3. Haqiqiy sonning absolyut qiymati.

Ta'rif: X haqiqiy sonning absolyut qiymati $|X|$ deb , quyidagi shartlarni qanoatlantiruvchi manfiy bo'lmagan haqiqiy songa aytiladi *[1,3]*

$$|X| = \begin{cases} X; & agar \quad X > 0 \quad bo'lsa \\ 0, & agar \quad X = 0 \quad bo`lsa \\ -X; & agar \quad X < 0 \quad bo`lsa \end{cases}$$

Masalan. $|5| = 5,$ $\left|-\frac{4}{7}\right| = \frac{4}{7},$ $|0| = 0$, $|-3| = 3$, chunki $|-3| = -(-3) = 3$.

$$\sqrt{a^2} = |a|, \quad \Leftrightarrow \quad \begin{matrix} \sqrt{a^2} = +a \quad agar \quad a \geq 0 \\ \sqrt{a^2} = -a \quad agar \quad a \leq 0 \end{matrix}$$

$$|ab| = \sqrt{(ab)^2} = \sqrt{a^2b^2} = |a||b|$$

Xossalari.

1. Bir necha haqiqiy sonlar algebraik yigindisining absolyut qiymati qoshiluvchilar absolyut qiymatlarining yigindisidan katta emas:

$$|X+Y| \le |X| + |Y|$$

2. Ayirmaning absolyut qiymati kamayuvchi va ayriluvchining absolyut qiymatlari ayirmasidan kichik emas.

$$|X-Y| \ge |X| - |Y|$$

3. Ko‘paytmaning absolyut qiymati ko‘paytuvchilar absolyut qiymatlarining ko‘paytmasiga teng.

$$|XYZ| = |X| \cdot |Y| \cdot |Z|$$

4. Bo‘linmaning absolyut qiymati bo‘luvchi va bo‘linuvchi absolyut qiymatlarining bo‘linmasiga teng.

$$\left|\frac{X}{Y}\right| = \frac{|X|}{|Y|}$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. $A=\{2,-3,4,5\}$ va $B=\{2,-3,4,5,9,11\}$ to‘plamlar uchun $A\subset B$ munosabat o‘rinli.

2. A to‘plam $x^2-5x+6=0$ tenglama yechimlaridan tashkil topgan, ya’ni $A=\{x: x^2-5x+6=0\}$ va $B=\{2;3\}$ bo‘lsin, u holda $A=B$ munosabat o‘rinli.

3. $X=\{x,y,z\}$, $Y=\{y,b,w\}$ bo‘lsa, $X\cup Y=\{x,y,z,b,w\}$ bo‘ladi.

4. . $X=\{x,y,z\}$, $Y=\{a,b,z\}$ bo‘lsa, $X\cap Y=\{z\}$ bo‘ladi.

5. Agar $X=\{a,b,c,d,e\}$, $Y=\{b,d,e\}$ bo‘lsa, $X\backslash Y=\{a,c\}$ bo‘ladi.

6. Agar $X=\{a,b,c,d,e\}$, $Y=\{b,d,e,f\}$ bo‘lsa, $X\Delta Y=\{a,c,f\}$ bo‘ladi.

7. $A=\{2,-3,4,5\}$ va $B=\{2,-3,4,5,9,11\}$ bo‘lsa, $C_BA= B\backslash A=\{9,11\}$.

MUSTAQIL YECHISH UCHUN MASHQLAR.

1.1. $X=\{x,u,y,z\}$, $Y=\{u,b,w\}$ bo‘lsa, $X\cup Y$ *va* $X\Delta Y$ larni aniqlang.

1.2. Agar $X=\{x,y,u,z\}$, $Y=\{x,y,w,z\}$ bo‘lsa, $X\cup Y$ *va* $X\cap Y$ larni aniqlang.

1.3. $X=\{x,y,z,3\}$, $Y=\{a,b,z,2,3\}$ bo‘lsa, $X\cap Y$ *va* $X\cup Y$ larni aniqlang.

1.4. Agar $X=\{x,0,y\}$, $Y=\{x,1,y\}$ bo‘lsa, $X\cap Y$ *va* $X\cup Y$ larni aniqlang.

1.5. Agar $X=\{a,b,c,d,e\}$, $Y=\{b,d,e,t\}$ bo‘lsa, $X\backslash Y$ ni aniqlang.

1.6. $X=\{x,y,z,3\}$, $Y=\{a,b,z,2,3\}$ bo‘lsa, $X\backslash Y$ va $Y\backslash X$ larni aniqlang.

1.7. Agar $X=\{x,0,y\}$, $Y=\{x,1,y\}$ bo‘lsa, va $(X\cup Y)\backslash X\cap Y$ ni aniqlang.

1.8. Agar $X=\{a,b,c,d,e\}$, $Y=\{b,d,e,t\}$ bo‘lsa, $Y\backslash X$ va $X\cup Y$ larni aniqlang.

1.9. $X=\{1,2,3,7\}$, $Y=\{a,b,z,2,3\}$ bo‘lsa, $X\backslash Y$ va $X\cap Y$ larni aniqlang.

1.10. Agar $X=\{-2,0,3\}$, $Y=\{0,1,5\}$ bo‘lsa, va $(X\cup Y)\backslash X\cap Y$ ni aniqlang.

1.11. Quyidagi jumlalarning qaysi biri to‘g‘ri?

a) Ratsional son bilan irratsional sonning yig‘indisi irratsional son bo‘ladi;

b) Ixtiyoriy ratsional son bilan irratsional sonning ko‘paytmasi irratsional son bo‘ladi;

c) Ikkita irratsional son yig‘indisi irratsional son bo‘ladi;

d) Ikkita irratsional son ko‘paytmasi irratsional son bo‘ladi;

1.12. Quyidagi 1) $(-8;12)$ 2) $(-\infty;\infty)$ 3) $(-\infty;7)$ to‘plamlarning qaysi biri quyidan chegaralangan?

1.13. Quyidagi sonlardan qaysilari irratsional?

1) a) $\sqrt{3}$; $b)$ $\sqrt{4}$; $c)$ $7,25888\ldots = 7,25(8)$; $d)$ $56,267400137184\ldots$;

2) a) $\sqrt[4]{2,56}$; $b)$ $3,141516\ldots$; $c)$ $\sqrt{\sqrt{\sqrt{81}}+13}$;

3) a) 3,8(345); b) $\sqrt{2}$; $c)$ $\frac{1}{\sqrt{2}}$; $d)$ $\sqrt{\sqrt{\sqrt{16}}+2}$;

1.14. Ifodalarni modul belgisiz yozing:

a) $|x-1|$; b) $|2x+5|$; d) $\left||x+2|+|x-1|\right|$; e) $|3x-5|-|3x+5|$.

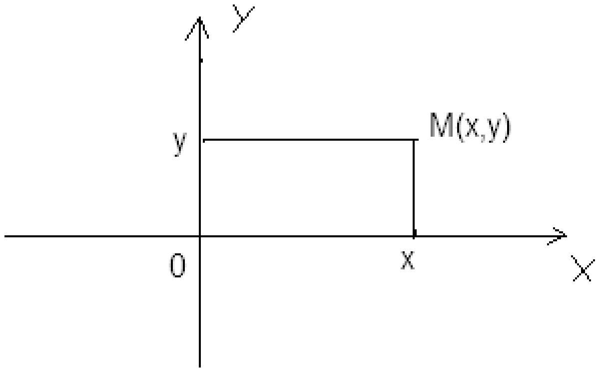
1.2. Mavzu: TEKISLIKDA DEKART, QUTB KOORDINATALAR SISTEMASI. KESMANI BERILGAN NISBATDA BO'LISH.

MAVZUGA OID NAZARIY MATERIALLAR

Hozirgi davrda har xil sohaning juda ko'p mutaxassislar tekislikda tog'ri burchakli dekart koordinatalar sistemasi haqida tasavvurga egalar, chunki bu koordinatalar yaqqol geometric ravishda bir kattalikni ikkinchi bir kattalik bilan bog'liqligini grafik ko'rinishda tasvirlab beradi. Masalan, vrach kasalni qon bosimi o'zgarishini grafik ko'rinishda tasvirlashi, ekonomist- ishlab-chiqarishning o'sishini va h.k. Tekislikda to'g'ri burchakli koordinatalar tushunchasi geometriyada eramizning boshlarida ham ishlatilgan. Dekart uni takomillashtirdi, belgilar kiritdi va ayniqsa, to'g'ri burchakli koordinatalar yordamida tekislikda analitik geometriya tuzdi. Bu bilan u geometriya va algebrani bog'lab berdi.

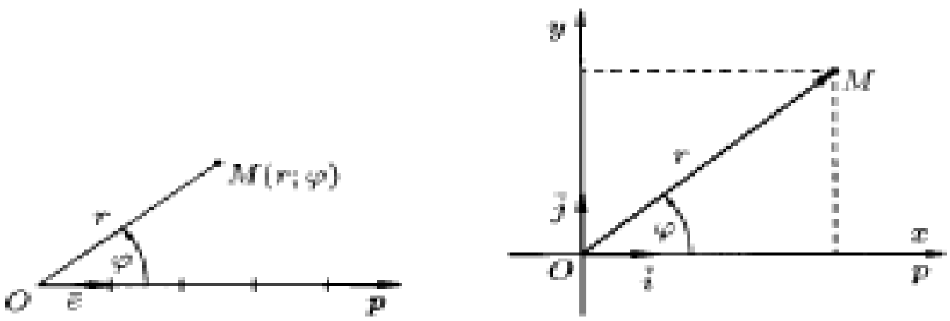
Dekart koordinatalar sistemasi ikkita o'qdan: biri gorizontal o'q-absissa o'qi, ikkinchisi vertikal o'q – ordinata o'qidan iborat bolib, ularning kesishgan nuqtasi koordinatalar boshi deyiladi hamda XOY kabi belgilanadi.

Bu sistema orqali tekislikdagi nuqta bilan bir juft haqiqiy son o'rtasida bir qiymatli moslik o'rnatiladi. Tekislikda nuqta $M(x,y)$ bilan belgilanadi (rasm 1). x, y sonlarga uning <u>*koordinata nuqtalari*</u> deyiladi. „Nuqta berilgan" degan ibora uning koordinatalarining berilganligini, ,,Nuqtani toping” degan ibora esa, shu koordinatalarni topishni tushuniladi. Koordinatalar sistemasi orqali o‘rnatilgan bunday moslikka koordinatalar usuli deyiladi 1-chizma.



1-chizma.

Amaliyotda ishlatiladigan yana bir koordinatalar sistemasi-qutb koordinatalar sistemasidir. U polyus deb atalmish O nuqtadan va qutb o‘qi deb ataladigan Op nurdan hamda Op nur tomonga yo‘nalgan birlik e vektordan iborat (2-chizma).



2-chizma

Radius r va burchak φ sonlari M nuqtaning *qutb koordinatalari* deyilib, $M(r,\varphi)$ kabi yoziladi.

Chizmadan ko‘rinib turibdiki, M nuqtaning to‘g‘ri burchakli va qutb koordinatalar sistemasi quyidagi bog‘lanishga ega:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi, \end{cases} \qquad \begin{cases} r = \sqrt{x^2 + y^2} \\ tg\varphi = \dfrac{y}{x} \end{cases}$$

Masalan. $M(-1;-\sqrt{3})$ nuqta berilgan. Shu nuqtaning qutb koordinatalarini toping.

Yechish: r, φ ni topamiz: $r = \sqrt{3+1} = 2,\quad tg\varphi = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$. Bundan $\varphi = \dfrac{\pi}{3} + \pi n,\ n \in Z.$,lekin M nuqta 3-chorakda yotgani uchun $n = -1,\ \varphi = \dfrac{\pi}{3} - \pi = -\dfrac{2\pi}{3}$. Demak $M(2,-\dfrac{2\pi}{3})$.

OXY tekisligida chiziq tenglamasi (yoki egri chiziq) deb $F(x,y) = 0$ tenglamaga aytiladi. Chiziq tenglamalari chiziqlarning geometrik xossalarini o‘rganishga qulaylik yaratadi.

1. Tekislikda berilgan $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalar orasidagi, y`ani ikki nuqta orasidagi masofa quyidagi formula orqali topiladi.

$$|AB| = d == \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

Masalan. a) $(1;-2)$ va $(5;3)$ nuqtalar orasidagi masofa topilsin.

Yechish: $d = \sqrt{(5-1)^2 + (3-(-2))^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$

b) $A(3;8)$ va $B(-5;14)$ nuqtalar orasidagi masofa topilsin.

$d = \sqrt{(-5-3)^2 + (14-8)^2} = \sqrt{64+36} = 10.$

c) Uchlari $A(-3;-3), B(-1;3), C(11;-1)$ nuqtalarda bo‘lgan uchburchak to‘g‘ri burchakli uchburchak ekanligi isbotlansin.

Yechish: Tomonlari uzunliklarini topamiz:

$$|AB| = \sqrt{(-1+3)^2 + (3+3)^2} = \sqrt{40}\ ,\ |BC| = \sqrt{(11+1)^2 + (-1-3)^2} = \sqrt{160}\ ,$$

$$|AC| = \sqrt{(11+3)^2 + (-13+3)^2} = \sqrt{200}$$

Pifagor teoremasiga asosan: $|AB|^2 = 40, |BC|^2 = 160, |AC|^2 = 200$

$|AB|^2 + |BA|^2 = |AC|^2$, shart $40 + 160 = 200$ bajarilganligi sababli berilgan

ABC uchburchak to‘g‘ri burchakli uchburchak , *AC* tomoni gipotenuzadir.

2. AB kesmani $AC : BC = \lambda$ nisbatda bo‘luvchi $C(x, y)$ nuqtani topish masalasi qo‘yilgan bo‘lsin. O‘rta maktab geometriyasidan ma’lumki

$$AC : A_1C_1 = BC : B_1C_1 = \lambda,$$

yoki

$$\frac{AC}{BC} = \frac{A_1C_1}{B_1C_1} = \lambda$$

bo‘lib,

$$A_1C_1 = x - x_1, \quad B_1C_1 = x_2 - x$$

bo‘lganligi uchun,

$$(x - x_1) : (x_2 - x) = \lambda, \; x - x_1 = \lambda\,(x_2 - x); \; x + \lambda x = x_1 + \lambda x_2;$$

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}$$

bo‘ladi. Xuddi shunday ,

$$y = \frac{y_1 + \lambda y_2}{1 + \lambda}.$$

Demak, C nuqtaning koordinatalari uchun

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}, \quad y = \frac{y_1 + \lambda y_2}{1 + \lambda} \qquad (2)$$

formulani hosil qildik. (2) formulaga AB kesmani λ nisbatda bo‘luvchi $C(x, y)$ nuqtani topish formulasi deyiladi. Xususiy holda $C(x;\ y)$ nuqta AB kesmani teng ikkiga bo‘lsa, u holda

$$\frac{AC}{CB} = \lambda = 1 \text{ bo‘lib,} \quad x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

kesmani teng ikkiga bo‘lish formulasi kelib chiqadi.

Demak, AB kesmani berilgan λ nisbatda bo‘luvchi *M(x;y)* nuqtaning koordinatalari quyidagicha aniqlanadi;

$$x = \frac{x_1 + \lambda x_2}{1+\lambda}; \quad y = \frac{y_1 + \lambda y_2}{1+\lambda}.$$

Xususan, agar $\lambda = 1$ bo‘lsa, M nuqta AB kesmaning o‘rtasida yotadi va uning koordinatalari

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2}.$$

munosabatlardan topiladi.

Masalan. $A(-2;1)$ va $B(3;6)$ nuqtalar berilgan. AB kesmani $AM : MB = 3:2$ nisbatda bo‘luvchi *M(x;y)* nuqta topilsin.

Yechish: $\lambda = AM : MB = \frac{3}{2}$ bo‘lganligi uchun formulaga asosan

$$x = \frac{-2 + 1{,}5 \cdot 3}{1 + 1{,}5} = \frac{2{,}5}{2{,}5} = 1; \ y = \frac{1 + 1{,}5 \cdot 6}{1 + 1{,}5} = \frac{10}{2{,}5} = 4$$

j: *M(1;4)*

3.To‘g‘ri burchakli koordinatalar sistemasida uchlari $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$ nuqtalarda bo‘lgan uchburchak yuzi quyidagi formula orqali topiladi:

$$S = \pm\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)] \qquad (3)$$

Masalan. Uchlari $A(2;0)$, $B(5;3)$ va $C(2;6)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

Yechish. (3) formulaga ko‘ra $x_1 = 2,\ x_2 = 5,\ x_3 = 2,\ y_1 = 0,\ y_2 = 3,\ y_3 = 6$ bo‘lganligi uchun,

$$S = \frac{1}{2}[(2\cdot 3 - 5\cdot 0) + (5\cdot 6 - 3\cdot 2) + (0\cdot 2 - 2\cdot 6)] = \frac{1}{2}(6 + 24 - 12) = \frac{1}{2}\cdot 18 = 9$$

bo‘ladi.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. $M(5;3)$ va $N(2;-1)$ nuqtalar orasidagi masofani toping.

Yechish.Shartga ko‘ra: $x_1 = 5,\ y_1 = 3,\ x_2 = 2,\ y_2 = -1$. Bularni (1) formulaga qo‘ysak:

$$MN = \sqrt{(2-5)^2 + (-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

bo‘ladi.

2. Tekislikda $A(5;3)$, $B(2;1)$ nuqtalar berilgan. AB kesmani $\frac{AC}{CB} = \lambda = 0{,}2$ nisbatda bo‘luvchi $C(x;y)$ nuqtaning koordinatalarini toping.

Yechish. Shartga ko‘ra $x_1 = 5,\ x_2 = 2,\ y_1 = 3,\ y_2 = 1,\ \lambda = 0{,}2$.

(2) formulaga asosan:

$$x = \frac{x_1 + \lambda x_2}{1+\lambda} = \frac{5 + 0{,}2\cdot 2}{1+0{,}2} = \frac{5+0{,}4}{1{,}2} = \frac{5{,}4}{1{,}2} = \frac{54}{12} = \frac{27}{6} = \frac{9}{2} = 4{,}5;$$

$$y = \frac{y_1 + \lambda y_2}{1+\lambda} = \frac{3 + 0{,}2\cdot 1}{1+0{,}2} = \frac{3+0{,}2}{1{,}2} = \frac{3{,}2}{1{,}2} = \frac{32}{12} = \frac{16}{6} = \frac{8}{3}.$$

Shunday qilib, $C\left(4{,}5;\frac{8}{3}\right)$ bo‘ladi.

3. $A(1;3)$ va $B(7;15)$ nuqtalar orasidagi masofani teng ikkiga bo‘luvchi *M(x;y)* nuqtaning koordinatasi topilsin.. J: (4;9)

4. *A (7 ; 5)* va *B (-4 ; -2)* nuqtalar berilgan. *AB* kesmani 3 : 4 nisbatda bo‘luvchi *C(x ; y)* nuqtaning koordinatalari topilsin. J: $\left(3\frac{3}{7}:2\right)$

5. Uchlari $A(6;-7)$, $B(11;-3)$ va $C(2;-2)$ nuqtalarda bo‘lgan uchburchak to‘g‘ri burchakli uchburchak ekanligini isbotlang. J: To‘g‘ri burchakli uchburchak.

6. Quyidagi berilgan nuqtalar qaysi choraklarda joylashgan: $(-2;9)$, $(4;6)$, $(1;0)$ va $(-5;3)$. J: II, I, IV, III.

MUSTAQIL YECHISH UCHUN MASHQLAR.

2.1. Nuqtalar orasidagi masofani toping.

a) $(1;1)$, $(4;5)$ b) $(1;-3)$, $(5;7)$ v) $(6;-2)$, $(-1;3)$

g) $(1;-6)$, $(-1;-3)$ d) $(2;5)$, $(4;-7)$ e) $(a;b)$, $(b;a)$

2.2. $A(-1;3)$, $B(3;11)$ va $C(5;15)$ nuqtalar berilgan. $|AB|+|BC|=|AC|$ ayniyatni isbotlang.

2.3. $(1;3)$ va $(7;15)$ nuqtalar orasidagi masofani teng ikkiga bo‘luvchi *M(x;y)* nuqtaning koordinatasi topilsin..

2.4. *A (8 ; 4)* va *B (-5 ; -1)* nuqtalar berilgan. *AB* kesmani 3 : 4 nisbatda bo‘luvchi *C(x ; y)* nuqtaning koordinatalari topilsin.

2.5. Uchlari $A(5;-8)$, $B(11;-3)$ va $C(2;-2)$ nuqtalarda bo‘lgan uchburchak to‘g‘ri burchakli uchburchak emas ekanligini isbotlang.

2.6. A va B nuqtalarni tutashtiruvchi kesma *M(4;9)* nuqtada teng ikkiga bo'linadi. Agar $A(1;3)$ bo'lsa, B nuqta koordinatasi topilsin.

1.3. MAVZU: KOMPLEKS SONLAR

MAVZUGA OID NAZARIY MATERIALLAR

1. Kompleks son tushunchasi.

Kompleks son deb

$$z = x + iy \qquad (1)$$

ifodaga aytiladi, bu erda x va y haqiqiy sonlar, i - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \quad \text{yoki} \quad i^2 = -1 \qquad (2)$$

x- kompleks son z ning haqiqiy qismi, iy - mavhum qismi deyiladi. Ular bunday belgilanadi: $x=Re\ z$, $y=Imz$. Agar $x=0$ bo'lsa, $0+iy=iy$ sof mavhum son deyiladi; $y=0$ agar bo'lsa, haqiqiy son hosil bo'ladi: $x+i0=x$. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: $z=x+iy$ va $z=x-iy$ bir-biriga qo'shma deyiladi.

Ushbu ikki asosiy ta'rif qabul qilinadi.

1.Agar $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ dan iborat ikki kompleks sonda $x_1=x_2$; $y_1=y_2$

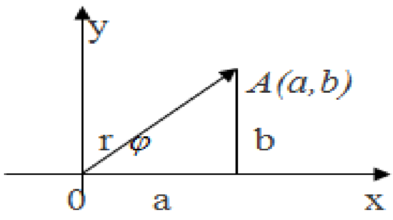
bo'lsa, ya'ni haqiqiy qismlar o'zaro va mavhum qismlar o'zaro teng bo'lsa, bunday kompleks sonlar teng deyiladi.

2.Agar $x=0$, $y=0$ bo'lsa, faqat shundagina kompleks son nolga teng bo'ladi:

$z=x+iy=0$.

2. Kompleks sonlarning geometrik tasviri.

Har qanday kompleks son $z=x+iy$ ni OXY tekisligida koordinatalari x va y bo'lgan $A(x,y)$ nuqta shaklida tasvirlash mumkin. Aksincha, OXY tekisligidagi har qanday *M(x,y)* nuqta *z=x+iy* kompleks songa mos keladi. O‘zida kompleks son tasvirlanadigan tekislik o‘zgaruvchi *z* ning kompleks tekisligi deyiladi



1-chizma

O‘zgaruvchi *z* kompleks tekisligining *OX* o‘qida yotuvchi nuqtalariga haqiqiy sonlar mos keladi *(y=0)*. *OY* o‘qida yotuvchi nuqtalar sof mavhum sonni tasvirlaydi, chunki bu holda *x=0*. Shuning uchun kompleks sonlarni *z* ning kompleks o‘zgaruvchi tekisligida tasvirlaganda *OY* o‘q mavhum sonlar yoki mavhum o‘q, *OX* o‘q esa haqiqiy o‘q deyiladi. *A(x,y)* nuqtani koordinatalar boshi bilan tutuashtirib, vektorni hosil qilamiz. Ba'zi hollarda *z=x+iy* kompleks sonning geometrik tasvirini *OA* vektor deb qabul qilish qulay bo‘ladi.

3. Kompleks sonning trigonometrik shakli.

Koordinatalar boshini qutb, *OX* o‘qining musbat yo‘nalishini qutb o‘qi deb olib, *A(x,y)* nuqtaning qutb koordinatalarini φ va $r(r \geq 0)$, bilan belgilaymiz. Unda ushbu tengliklarni yozish mumkin:

$$x = r\cos\varphi, \qquad y = r\sin\varphi$$

z kompleks sonning modulini r ($r = |z|,$) va argumentini φ ($\varphi = \arg z$) deb belgilaymiz; ular *x* va *y* orqali quyidagicha ifodalanadi:

$$\left.\begin{array}{l} r = |z| = |x+iy| = \sqrt{x^2+y^2} \\ \varphi = \arg z = \arg(x+iy) = arctg\dfrac{y}{x} \end{array}\right\}$$

Demak, kompleks son *z* ni bunday tasvirlash mumkin:

$$x+iy = r\cos\varphi + ir\sin\varphi$$

yoki

$$z= \text{r}(cos\varphi + isin\varphi) \qquad (3)$$

Bu tenglikning o‘ng tomonidagi ifodaga $z=x+iy$ kompleks sonning trigonometrik shakli deb ataladi.

Kompleks sonning argumenti φ burchak OX o‘qning musbat yo‘nalishidan soat strelkasi harakatiga teskari yo‘nalishda hisoblansa musbat, qarama-qarshi yo‘nalishda hisoblansa manfiy bo‘ladi. Ravshanki, argument bir qiymatli bo‘lmasdan, balki $2k\pi$ qo‘shiluvchi (k-ixtiyoriy butun son) aniqligida belgilanadi.

Izoh. Qo‘shma kompleks sonlar $z=x+iy$ va $\bar{z}=x-iy$ teng modullarga ega: $|z| = |\bar{z}|$ **,** argumentlarning absolyut qiymatlari teng, ammo ishoralari bilan farqlanadi:

$$\arg \bar{z} = -\arg z$$

Haqiqiy son A ni ham (3) shaklda yozish mumkin, ya'ni:

$A>0$ bo‘lsa, $A = |A|(\cos 0 + i \sin 0)$,

$A<0$ bo‘lsa, $A = |A|(\cos \pi + i \sin \pi)$.

Nolga teng bo‘lgan kompleks sonning moduli nolga teng: $|0| = 0$. Nolning

argumenti sifatida har qanday φ burchakni qabul qilish mumkin. Haqiqatan har qanday φ burchak uchun ushbu tenglikni yozish mumkin: 0= 0(cosφ+isinφ).

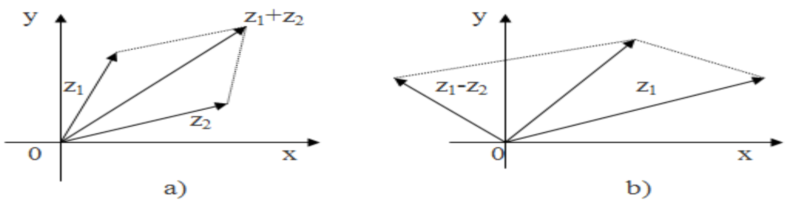
4. Kompleks sonlarni qo‘shish.

Ikki kompleks son $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ ning yig‘indisi deb ushbu

$$z_1+z_2=(x_1+iy_1)+(x_2+iy_2)=(x_1+x_2)+i(y_1+y_2) \qquad (1)$$

tenglik bilan aniqlangan kompleks songa aytiladi.

(1) formuladan vektorlar bilan tasvirlangan kompleks sonlarni qo‘shish-vektorlarni qo‘shish qoidasiga muvofiq bajarilishi kelib chiqadi.



2-chizma

5. **Kompleks sonlarni ayirish**.

Ikki $z_1=x_1+iy_1$ **va** $z_2=x_2+iy_2$ kompleks sonlarni ayirmasi deb shunday kompleks songa aytiladiki, unga z_2 kompleks sonni qo‘shganda z_1 kompleks son hosil bo‘ladi:

$$z_1 - z_2=(x_1+iy_1)- (x_2+iy_2)=(x_1 -x_2)+i(y_1 -y_2) \quad (2)$$

Ikki kompleks son ayirmasining moduli shu sonlarni kompleks o‘zgaruvchilar tekisligida tasvirlovchi nuqtalar orasidagi masofaga teng:

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

6. Kompleks sonlarni ko‘paytirish.

$z_1=x_1+iy_1$ v**a** $z_2=x_2+iy_2$ kompleks sonlar ko‘paytmasi ,

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + iy_1 x_2 + ix_1 y_2 + i^2 y_1 y_2$$

yoki $i^2 = -1$ ekanligini hisobga olib quyidagini hosil qilamiz:

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2). \quad (3)$$

Izoh: algebra qoidasiga muvofiq, $i^2 = -1,$ $i^3 = -i$**,** $i^4 = (-i)i = -i^2 = 1,$ $i^5 = i$, va hokazo, umuman k butun bo‘lganda:

$$i^{4\kappa} = -1, \quad i^{4\kappa+1} = i, \quad i^{4\kappa+2} = -1, \quad i^{4\kappa+3} = -i.$$

o’rinli.

Kompleks sonlar trigonometrik shaklda berilgan bo‘lsin:

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1, z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2).$$

Bu sonlarning ko‘paytmasini topamiz:

$$z_1z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1)r_2(\cos\varphi_2 + i\sin\varphi_2) = r_1r_2[\cos\varphi_1\cos\varphi_2 + i\sin\varphi_1\cos\varphi_2 + i\cos\varphi_1\sin\varphi_2 +$$
$$+ i^2\sin\varphi_1\sin\varphi_2] = r_1r_2[(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2) + i(\sin\varphi_1\cos\varphi_2 + \cos\varphi_1\sin\varphi_2)] =$$
$$= r_1r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)].$$

Shunday qilib,

$$z_1z_2 = r_1r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)].$$

ya’ni ikki kompleks son ko‘paytmasi shunday kompleks sonki, uning moduli ko‘paytuvchilar modullarining ko‘paytmasiga teng, argumenti esa ko‘paytuvchilar argumentlarining yig‘indisiga teng.

Izoh: $z=x+iy$ va $z=x-iy$ qo‘shma kompleks sonlar ko‘paytmasi (3) formulaga muvofiq bunday ifodalanadi:

$$z|\bar{z}| = x^2 + y^2,$$

yoki

$$z\bar{z} = |z|^2 = |\bar{z}|^2.$$

Qo‘shma kompleks sonlar ko‘paytmasi ulardan har biri modulining kvadratiga teng.

7. Kompleks sonlarni bo‘lish.

Kompleks sonlarni bo‘lish ko‘paytirishga teskari amal kabi ta’riflanadi:

$$z_1=x_1+iy_1,\ z_2=x_2+iy_2,\ |z_2| = \sqrt{x_2^2 + b_2^2} \neq 0$$

deb faraz qilamiz. U holda $\frac{z_1}{z_2} = z$ shunday kompleks sonki, unda

$z_1=z_2*z$ bo‘ladi.

Agar $\frac{x_1+iy_1}{x_2+iy_2}=x+iy$ bo‘lsa, u holda

$$x_1+iy_1=(x_2+iy_2)(x+iy)$$

yoкi

$$x_1+iy_1=(x_2x-y_2y)+i(x_2y+y_2x);$$

x va *y* ushbu

$$x_1=x_2x-y_2y, \quad y_1=y_2x+x_2y$$

tenglamalar sistemasi bilan aniqlanadi. Bundan:

$$x=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}, \qquad y=\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}.$$

Nihoyat, ushbu formulani hosil qilamiz:

$$z=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$$

Kompleks sonlarni bo‘lish amalda bunday bajariladi: $z_1=x_1+iy_1$ ni $z_2=x_2+iy_2$ ga bo‘lish uchun bo‘linuvchi va bo‘luvchini bo‘luvchiga qo‘shma songa ko‘paytiramiz.

Unda bo‘luvchi haqiqiy son bo‘ladi; unga bo‘linuvchining haqiqiy va mavhum qismlarini bo‘lamiz:

$$\frac{x_1+iy_1}{x_2+iy_2}=\frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}=\frac{(x_1x_2+y_1y_2)+i(x_2y_1-x_1y_2)}{x^2+b^2}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$$

Kompleks sonlar trigonometrik shaklda

$$z_1=r_1(\cos\varphi_1+i\sin\varphi_1),$$
$$z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$$

berilgan bo‘lsa, ushbuni hosil qilamiz:

$$\frac{z_1}{z_2}=\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2-i\sin\varphi_2)}=\frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)]$$

Bu tenglikni tekshirish uchun bo‘luvchini bo‘linmaga ko‘paytirish kifoya:

$$r_2(\cos\varphi_2 + i\sin\varphi_2)\ \frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)=$$

$$=r_2\frac{r_1}{r_2}[\cos(\varphi_2+\varphi_1-\varphi_2+i\sin(\varphi_2+\varphi_1-\varphi_2)] = r_1(\cos\varphi_1+i\sin\varphi_1).$$

Shunday qilib, ikki kompleks son bo‘linmasining moduli bo‘linuvchi va bo‘luvchi modullarining bo‘linmasiga teng; bo‘linmaning argumenti bo‘linuvchi va bo‘luvchi argumentlarining ayirmasiga teng.

1-Teorema. Koeffisientlari haqiqiy sonlar bo‘lgan ushbu

$$A_0x^n+A_1x^{n-1}+\ldots+A^n$$

ko‘phadda x o‘rniga $x+iy$ son, so‘ngra unga qo‘shma son x-iy qo‘yilsa, o‘rniga qo‘yish natijalari ham o‘zaro qo‘shma bo‘ladi.

1-Misol. Ushbu $z_1=3-i,\ z_2=-2+3i,\ z_3=4+3i$ kompleks sonlar berilgan bo‘lsin. $z=\dfrac{z_1-z_2\cdot z_3}{z_1^3+z_3}$ ni hisoblang.

Yechish. Ketma-ket hisoblaymiz:

$$z_2\cdot z_3=(-2+3i)(4+3i)=(-8-9)+i(12-6)=-17+6i;$$
$$z_1-z_2\cdot z_3=(3-i)-(-17+6i)=(3+17)+i(-1-6)=20-7i;$$
$$z_1^3=(3-i)^3=27-27i+9i^2-i^3=(27-9)+i(-27+1)=18-26i;$$
$$z_1^3+z_3=(18-26i)+(4+3i)=(18+4)+i(-26+3)=22-23i.$$

Shunday qilib,

$$z=\frac{20-7i}{22-23i}=\frac{(20-7i)(22+23i)}{(22-23i)(22+23i)}=\frac{(440+161)+i(460-154)}{22^2+23^2}=\frac{601}{1013}+i\frac{306}{1013}.$$

2-Misol. $z=-\sqrt{3}+i$ kompleks sonning modulini , argumentini , trigonometrik va ko‘rsatkichli shakllarini toping.

Yechish. $x=-\sqrt{3}, y=1$ bo‘lganligi uchun $r=\sqrt{x^2+y^2}=2$. $tg\varphi=-\frac{1}{\sqrt{3}}$ tenglamadan φ argumentni topamiz: $\varphi=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}$

Shunday qilib, $r=2,\quad \varphi=\frac{5\pi}{6}$.

$$z=2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right),\ z=2e^{\frac{5\pi}{6}i}.$$

3-Misol. $z=\left(-\sqrt{3}+i\right)^6$ ni hisoblang.

Yechish. $x=-\sqrt{3}, y=1$ bo‘lganligi uchun $r=\sqrt{x^2+y^2}=2$. $tg\varphi=-\frac{1}{\sqrt{3}}$ tenglamadan φ argumentni topamiz:

$$\varphi=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}=$$

Demak, $r=2,\quad \varphi=\frac{5\pi}{6}$. Muavr formulasidan foydalanib quyidagi yechimga ega bo‘lamiz:

$$z=2^6\left(\cos\frac{5\pi}{6}\cdot 6+i\sin\frac{5\pi}{6}\cdot 6\right)=2^6e^{5\pi i}=64(\cos 5\pi+i\sin 5\pi)=-64.$$

4-Misol. $\sqrt[3]{-1}$ ni toping.

Yechish. z =-1 soni uchun $r=1,\ \varphi=\pi$. Shu sababli uning trigonometrik shakli quyidagicha yoziladi:

$$z=1\cdot(\cos\pi+i\sin\pi).$$

n- darajali ildiz chiqarish formulasidan foydalanib, ushbuga ega bo‘lamiz:

$$\omega_k=\sqrt[3]{\cos\pi+i\sin\pi}=\cos\frac{\pi+2\pi k}{3}+i\sin\frac{\pi+2\pi k}{3}=$$
$$=e^{\frac{i(\pi+2\pi k)}{3}},\ bunda\ k=0;1;2.$$

k ga ketma-ket 0;1;2 qiymatlar berib, ildizni uchala qiymatini topamiz:

$$\omega_0 = cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = e^{\frac{i\pi}{3}} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega_1 = \cos\pi + i\sin\pi = e^{i\pi} = -1$$
$$\omega_2 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = e^{\frac{5\pi i}{3}} = \frac{1}{2} - i\frac{\sqrt{3}}{2}.$$

8. Kompleks sonni darajaga ko‘tarish.

Bundan oldingi paragrafdagi (3) formuladan, agar *n* butun musbat son bo‘lsa, ushbu formula kelib chiqadi:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi). \qquad (1)$$

Bu Muavr formulasi deb ataladi. Bundan ko‘rinadiki, kompleks sonni butun musbat darajaga ko‘tarishda modul shu darajaga ko‘tariladi, argument esa daraja ko‘rsatkichiga ko‘paytiriladi.

Endi Muavr formulasining yana bir tadbiqini qaraymiz.

Bu formulada *r=1* deb faraz qilib,

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$$

tenglikni hosil qilamiz. Chap tomonni Nyuton binomi formulasi bo‘yicha yoyib, haqiqiy va mavhum qismlarini tenglab, $\sin n\varphi$ va $\cos n\varphi$ ni $\sin\varphi$ va $\cos\varphi$ ning darajalari orqali ifoda qila olamiz.

9. Kompleks sondan ildiz chiqarish.

Kompleks sonning *n*-darajali ildizi deb *n* -darajaga ko‘targanda ildiz ostidagi songa teng bo‘ladigan kompleks songa aytiladi, ya’ni

$$\rho^n(\cos n\psi + i\sin n\psi) = r(\cos\varphi + i\sin\varphi)$$

bo‘lsa,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \rho(\cos\psi + i\sin\psi)\,.$$

Teng kompleks sonlarning modullari teng bo‘lishi kerak, argumentlari esa 2π ga karrali songa farq qilishi mumkin bo‘lgani uchun

$$\rho^n = r,\ n\psi = \varphi + 2\kappa\pi$$

Bundan

$$\rho = \sqrt[n]{r},\ \psi = \frac{\varphi + 2\kappa\pi}{n},$$

bu yerda *k* - ixtiyoriy butun son, - musbat *r* son ildizining arifmetik qiymati. Demak,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}(\cos\frac{\varphi + 2\kappa\pi}{n} + i\sin\frac{\varphi + 2\kappa\pi}{n}) \qquad (2)$$

k ga *0,1,2,…,n-1* qiymatlarni berib, ildizning *n* ta har xil qiymatlarini topamiz.

Shunday qilib, kompleks sonning *n*- darajali ildizi *n* ta har xil qiymatga ega bo‘ladi.

Ikki hadli tenglamani yechish.

$x^n=A$ shakldagi tenglama ikki hadli tenglama deyiladi. Bu tenglamaning ildizlarini topamiz.

Agar *A* haqiqiy musbat son bo‘lsa,

$$x = \sqrt[n]{A}(\cos\frac{2\kappa\pi}{n} + i\sin\frac{2\kappa\pi}{n})\ (k=0,1,2,\ldots,\ n-1).$$

Qavs ichidagi ifoda *1* ning *n*-darajali ildizining hamma qiymatlarini beradi.

Agar *A* haqiqiy manfiy son bo‘lsa,

$$x = \sqrt[n]{|A|}(\cos\frac{2\kappa\pi}{n} + i\sin\frac{2\kappa\pi}{n})$$

Qavs ichidagi ifoda - *1* ning *n*-darajali ildizining hamma qiymatlarini beradi.

Agar *A* kompleks son bo‘lsa, *x* ning qiymatlari (2) formula bo‘yicha topiladi.

$z=x+iy$ bo‘lsin. Agar x va y haqiqiy o‘zgaruvchilar bo‘lsa, z kompleks o‘zgaruvchi deb ataladi. Kompleks o‘zgaruvchi z ning har bir qiymatiga XOY tekisligida ma’lum nuqta mos keladi.

1-Ta’rif. Kompleks o‘zgaruvchi z ning biror kompleks qiymatlar sohasida har bir qiymatiga boshqa ω kompleks miqdorning aniq qiymati mos kelsa, ω kompleks o‘zgaruvchi z ning funksiyasi bo‘ladi. Kompleks argumentning funksiyasi $\omega = f(z)$ yoki $\omega = \omega(z)$ bilan belgilanadi.

ω funksiyaning kompleks qiymatlari bunday belgilanadi:

$$e^{x+iy}=e^x(\cos y+i\sin y), \tag{1}$$

ya'ni

$$\omega(z)=e^x(\cos y+i\sin y), \tag{2}$$

10. Ko‘rsatkichli funksiyaning ba’zi xossalari.

Agar z_1 va z_2 - ikkita kompleks son bo‘lsa, unda

$$e^{z+z} = e^z e^z \tag{3}$$

Isbot. $z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$ bo‘lsin. U holda

$$e^{z_1+z_2} = e^{(x_1+iy_1)+(x_2+iy_2)} = e^{(x_1+x_2)+i(y_1+y_2)} = e^{x_1}e^{x_2}[\cos(y_1+y_2)+i\sin(y_1+y_2)]. \tag{4}$$

Ikkinchi tomondan, trigonometrik shakldagi ikki kompleks sonning ko‘paytmasi haqidagi teoremaga asosan:

$$\begin{aligned} e^{z_1}e^{z_2} &= e^{x_1+iy_1}e^{x_2+iy_2} = e^{x_1}(\cos y_1 + i\sin y_1)e^{x_2}(\cos y_2 + i\sin y_2) = \\ &= e^{x_1}e^{x_2}[\cos(y_1+y_2)+i\sin(y_1+y_2)]. \end{aligned} \tag{5}$$

(4) va (5) tengliklarda o‘ng tomonlar teng, demak, chap tomonlar ham teng bo‘ladi:

$e^{z+z} = e^z e^z$. Shunga o‘xshash

$$e^{z - z_2} = \frac{e^{z_1}}{e^{z_2}} \qquad (6)$$

formula ham isbotlanadi.

Agar *m* butun son bo‘lsa, $e^{(z)m} = e^{mz}$ tenglik orinli bo‘ladi. Agar *m>0* bo‘lsa, bu formula (3) formulaga asosan osongina hosil bo‘ladi; agar *m<0* bo‘lsa, bu formula (3) va (6) formulalarga asosan hosil qilinadi.

11.Eyler formulasi.

Kompleks ildizlarga ega bo‘lgan ko‘phadni ko‘paytuvchilarga ajratish**.**

Agar o‘tgan paragrafning (1) formulasida *x=0* desaк, unda

$$e^{iy} = \cos y + i\sin y \qquad (1)$$

formulani hosil qilamiz. Bu – ko‘rsatkichi mavhum son bo‘lgan ko‘rsatkichli funksiyani trigonometrik funksiyalar orqali ifodalovchi Eyler formulasidir.

(1) formulada *y* ni *–y* bilan almashtirib,

$$e^{-iy} = \cos y - i\sin y \qquad (2)$$

formulani hosil qilamiz. (1) va (2) tengliklardan *cosy* va *siny* ni topamiz:

$$\left.\begin{aligned} \cos y &= \frac{e^{iy} + e^{-iy}}{2} \\ \sin y &= \frac{e^{iy} - e^{iy}}{2i} \end{aligned}\right\} \qquad (3)$$

So‘nggi formulalardan foydalanib, jumladan, $\cos\varphi$ va $\sin\varphi$ ni ularning har qanday butun musbat darajalarini va shu darajalar ko‘paytmasini karrali yoylarning sinuslari va cosinuslari orqali tasvirlashimiz mumkin.

12. Kompleks sonning ko‘rsatkichli shakli.

Kompleks sonni trigonometrik shaklda tasvirlaymiz:

$$z = r(\cos\varphi + i\sin\varphi),$$

bu erda r kompleks sonning moduli, φ kompleks sonning argumenti. Eyler formulasiga ko‘ra

$$(\cos\varphi + i\sin\varphi) = e^{i\varphi}.$$

Demak, har qanday kompleks sonni ushbu ko‘rsatkichli shaklda tasvirlash mumkin:

$$z = re^{i\varphi}.$$

Ushbu kompleks sonlar berilgan bo‘lsin:

$z_1 = r_1 e^{i\varphi_1}$ $z_2 = r_2 e^{i\varphi_2}$

quyidagi tengliklar o‘rinli

$$z_1 z_2 = r_1 e^{i\varphi_1} r_2 e^{i\varphi_2} = r_1 r_2 e^{i(\varphi_1+\varphi_2)}; \quad (5)$$

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}} = \frac{r_1}{r_2} e^{i(\varphi_1-\varphi_2)} \quad (6)$$

$$z^n = (re^{i\varphi})^n = r^n e^{in\varphi}; \quad (7)$$

$$\sqrt[n]{re^{i\varphi}} = \sqrt[n]{r}e^{i\frac{\varphi+2\kappa\pi}{n}} \quad (k = 0,1,2,\ldots,n-1); \quad (8)$$

13. Ko‘phadni ko‘paytuvchilarga ajratish. Bezu teoremasi

Ma’lumki,

$$f(x) = A_0 x^n + A_1 x^{n-1} + \ldots + A_{n-1}x + A_n$$

funksiya, n butun musbat son bo‘lganda, ko‘phad yoki x ning butun ratsional funksiyasi deb ataladi; n soni ko’phadning darajasi deyiladi. Bu erda $A_0, A_1, \ldots, A_n$ koeffisientlar haqiqiy yoki kompleks sonlar; erkli o‘zgaruvchi x ham haqiqiy, ham kompleks qiymatlar olishi mumkin.

O‘zgaruvchi x ning ko‘phadni nolga aylantiradigan qiymati ko’phadning ildizi deb ataladi;

1-**Teorema**.(Bezu teoremasi). Ko‘phad $f(x)$ ni x-a ayirmaga bo‘lganda $f(a)$ ga teng qoldiq hosil bo‘ladi.

Isbot. $f(x)$ ni x-a ga bo‘lganda bo‘linmada darajasi $f(x)$ darajasidan bitta kam bo‘lgan $f_1(x)$ ko‘phad hosil bo‘lib, qoldiq o‘zgarmas R bo‘ladi. Shunga ko‘ra bunday tenglikni yozish mumkin:

$$f(x) = (x-a)f_1(x) + R. \quad (1)$$

Bu tenglik x ning a ga teng bo‘lmagan hamma qiymatlari uchun to‘g‘ri

Endi x ni a ga intilishga majbur qilamiz. Unda (1) tenglik chap tomonining limiti $f(a)$ ga, o‘ng tomonining limiti R ga teng bo‘ladi. $f(x)$ va $(x-a)f_1(x)+R.$ funksiyalar o‘zaro teng bo‘lgani uchun ularning $x \to a$ dagi limitlari ham teng bo‘ladi, ya’ni $f(a)$ =R.

Natija. Agar a ko‘phadning ildizi, ya’ni $f(a)$ =0 bo‘lsa, $f(x)$ ko‘phad x-a ga

qoldiqsiz bo‘linadi, demak, ko‘paytma shaklida tasvirlanadi:

$$f(x) = (x-a)f_1(x),$$

bu yerda $f_1(x)$ - ko‘phad.

Endi quyidagi teoremalarni isbotsiz keltiramiz, ular oliy algebra kursida isbot qilingandi.

2-**Teorema.** Har qanday butun ratsional $f(x)$ funksiya eng kamida bitta haqiqiy yoki kompleks ildizga egadir.

3-**Teorema.** n- darajali har qanday ko‘phad x-a shakldagi n ta chiziqli ko‘paytuvchiga va x^n oldidagi koeffisientga teng ko‘paytuvchiga ajraladi.

4-Teorema. Agar x ning $n+1$ ta, $a_0, a_1, \ldots, a_n$ har xil qiymatlarida n-darajali ikki $\varphi_1(x)$ va $\varphi_2(x)$ ko‘phadning qiymatlari bir xil bo‘lsa, bu ko‘phadlar bir-biriga aynan tengdir.

5-Teorema. Agar $P(x) = A_0x^n + A_1x^{n-1} + \ldots + A_{n-1}x + A_n$ ko‘phad aynan nolga teng bo‘lsa, uning hamma koeffisientlari nolga teng.

6-Teorema. Agar ikki ko‘phad bir-biriga aynan teng bo‘lsa, ko‘phadlardan birining koeffisientlari ikkinchisining mos koeffisientlariga tengdir.

Bu berilgan ko‘phadlarning ayirmasi aynan nolga teng ko‘phad bo‘lishidan kelib chiqadi. Demak, bundan oldingi teoremaga asosan, uning hamma koeffisientlari nolga tengdir.

Agar n- darajali ko‘phadning chiziqli ko‘paytuvchilarga ajralmasida

$f(x) = A_0(x - a_1)(x - a_2)\ldots(x - a_n)$

ba’zi chiziqli ko‘paytuvchilar bir xil bo‘lsa, ularni birlashtirish mumkin va unda ko‘phadning ko‘paytuvchilarga yoyilmasi ushbu ko‘rinishda bo‘ladi.

$$f(x) = A_0(x - a_1)^{\kappa_1}(x - a_2)^{\kappa_2}\ldots(x - a_m)^{\kappa_m}$$

Bunda

$$\kappa_1 + \kappa_2 + \ldots + \kappa_m = n.$$

Bu holda a_1 ildiz k_1 karrali ildiz, a_2 ildiz k_2 karrali ildiz va hokazo deb ataladi.

Har qanday n- darajali ko‘phad n ta ildizga ega bo‘ladi.

Izoh. $$f(x) = A_0x^n + A_1x^{n-1} + \ldots + A_{n-1}x + A_n$$

ko‘phadning ildizlari haqida aytilganlarning hammasini ushbu

$$A_0x^n + A_1x^{n-1} + \ldots + A_{n-1}x + A_{n-1}x + A_n = 0$$

algebraik tenglamaning ildizlari terminlarida ifodalash mumkin.

7-**Teorema**. Agar a_1 son $f(x)$ ko‘phadning $k_1>1$ karrali ildizi bo‘lsa, shu son $f'(x)$ hosila uchun k_1-1 karrali ildiz bo‘ladi.

8-**Teorema**. Agar koeffisientlari haqiqiy bo‘lgan $f(x)$ ko‘phad x+iy kompleks ildizga ega bo‘lsa, shu ko‘phad $x-iy$ qo‘shma kompleks ildizga ega bo‘ladi.

Shunday qilib, haqiqiy koeffisientli ko‘phad tegishli darajadagi chiziqli va kvadrat uchhad shaklidagi haqiqiy ko‘paytuvchilarga ajraladi, ya’ni

$$f(x)= A_0(x-a_1)^{l_1}(x-a_2)^{l_2}\ldots(x-a_r)^{l_{\hat{e}}}(x^2+p_1x+q_1)^{k_1}\ldots(x^2+p_sx+q_s)^{k_l}$$

bunda $\kappa_1+\kappa_2+\ldots+\kappa_r+2l_1+\ldots+2l_s=n$.

14.Ratsional kasrlarni eng soda kasrlarga yoyish .

$$P_n(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$$

funksiya darajali ko‘phad deyiladi. Bunda $a_0,a_1,a_2,\ldots,a_n$ -ko‘phadning koeffisiyentlari, n- daraja ko‘rsatkichi.

1-**Ta’rif**. Ikki ko‘pghadning nisbati kasr- ratsional funksiya yoki ratsional kasr deyiladi:

$$P(x)=\frac{Q_m(x)}{P_n(x)}=\frac{b_0x^m+b_1x^{m-1}+\ldots+b_{m-1}x+b_m}{a_0x^n+a_1x^{n-1}+a_{n-1}x+a_n}$$

Agar $m<n$ bo‘lsa, u holda ratsional kasrm to‘g‘ri, agar $m\geq n$ bo‘lsa, u holda ratsional kasr noto‘ri kasr bo‘ladi.

R(x) ratsional kasr noto‘g‘ri b o‘lgan hollarda kasrning $Q_m(x)$ suratini $P_n(x)$ maxrajiga odatdagidek bo‘lish yo‘li bilan uning butun qismini ajratish kerak:

$$\begin{array}{l|l} Q_m(x) & \dfrac{P(x)}{q(x)} \end{array}$$

....

____

$r(x)$

q(x) bo‘linma va *r(x)* qoldiq ko‘phad bo‘ladi, bunda *r(x)* qoldiqning darajasi $P_n(x)$ bo‘luvchining darajasidan kichikdir. $Q_m(x)$ bo‘linuvchi $P_n(x)$ bo‘luvchi hamda bo‘linmaning ko‘paytmasi bilan qoldiqning *r(x)* yig‘indisiga teng bo‘lgani uchun

$$Q_m(x) = P_n(x)\cdot q(x) + r(x) \text{ yoki } \frac{Q_m(x)}{P_n(x)} = q(x) + \frac{r(x)}{P_n(x)}$$

ayniyatni hosil qilamiz.

Bunda *q(x)*- butun qismi; $\frac{r(x)}{P_n(x)}$ esa to‘g‘ri kasr bo‘ladi.

Shunday qilib, noto‘g‘ri ratsional kasr bo‘lgan holda, undan *q(x)* butun qismini va $\frac{r(x)}{P_n(x)}$ to‘g‘ri kasrni ajratish mumkin.

Masalan: $P_m(x) = \frac{2x^4 - 3x^3 + 1}{x^2 + x - 2}$ noto‘g‘ri ratsional kasrni butun qismini ajrating.

Yechish: $R(x)$- ratsional kasr noto‘g‘ri kasr, chunki suratning darajasi maxrajning darajasidan kata (4>2).

Ko‘phadlarni bo‘lish qoidasi bo‘yicha suratni maxrajga bo‘lamiz.

$$\begin{array}{l|l}
2x^4 - 3x^3 + 1 & \underline{x^2 + x - 2} \\
\underline{2x^4 + 2x^3 - 4x} & 2x^2 - 5x + 9 \\
-\begin{cases} -5x^3 + 4x^2 + 1 \\ \underline{-5x^3 - 5x^2 + 10x} \end{cases} & \\
\qquad -\begin{cases} 9x^2 - 10x + 1 \\ \underline{9x^2 + 9x - 18} \end{cases} & \\
\qquad\qquad -19x + 19 &
\end{array}$$

Shunday qilib, $R(x) = 2x^2 - 5x + 9 + \frac{-19x + 19}{x^2 + x - 2}$ ni hosil qilamiz.

2-Ta’rif: Quyidagi ko‘rinishdagi kasrlar eng sodda ratsional kasrlar deyiladi.

$I.\ \frac{A}{x-a};$

$II.\ \frac{A}{(x-a)^k};(k \geq 2\ va\ butunson)$

$III.\ \frac{Ax+B}{x^2+px+q}(D<o)$

$IV.\ \frac{Ax+B}{(x^2 px+q)^s}(s \geq 2\ va\ butunsonlar, hamda D<0)$

Bunda *A, B* – haqiqiy koeffisiyentlar, *a,p,q* lar ham haqiqiy sonlar.

Ushbu $P(x)=\frac{Q_m(x)}{P_n(x)}$ to‘g‘ri ratsional kasrni qarab chiqamiz, bu kasrning $P_n(x)$ maxraji $(x-a)^k, (x^2+px+q)^s$ ko‘rinishdagi chiziqli va kvadrat ko‘paytuvchilarga yuoyiladi, bunda $(x-a)^k$ ko‘rinishdagi ko‘paytuvchi *k* karralikdagi haqiqiy ildizga mos keladi. $(x^2+px+q)^s$ ko‘rinishdagi ko‘paytuvchi *s* karralikdagi kompleks qo‘shma ildizlarga mos keladi.

$$P_n(x)=a_1(x-a)...(x-a)^k(x^2+px+q)...(x^2 px+q)^s$$

1-**Teorema**. Har qanday $P(x)=\frac{Q_m(x)}{P_n(x)}$ ratsional kasr *I,II, III, IV* turdagi oddiy kasrlarning yig‘indisi ko‘rinishida ifodalash mumkin. Bunda

a) (1) yoyilmaning *(x-a)* ko‘rinishdagi ko‘paytuvchiga I turdagi bitta $\frac{A}{x-a}$ kasr mos keladi.

b) (1) yoyilmaning *(x-a)*^{k} ko‘rinishdagi ko‘paytuvchiga *I* va *II* turdagi *k* ta kasr mos keladi.

$$\frac{A_1}{(x-a)^k}+\frac{A_2}{(x-a)^{k-1}}+...+\frac{A_q}{(x-a)}$$

c) (1) yoyilmasining (x^2+px+q) ko‘rinishdagi ko‘paytuvchiga *III* turdagi kasr mos keladi.

d) (1) yoyilmaning $\left(x^2+px+q\right)^s$ ko‘rinishdagi kopaytuvchiga *III* va *IV* turdagi *s* ta kasr mos keladi.

$$\frac{A_1x+B_1}{x^2+px+q}+\frac{A_1x+B_1}{\left(x^2+px+q\right)^2}+\ldots+\frac{A_ix+B_i}{\left(x^2+px+q\right)^s}$$

1-Misol. $\frac{x^3+3x^2+5x+7}{x^2+2}$ noto‘g‘ri kasrning to‘g‘ri kasrga yoying.

Yechish.

$$\begin{array}{r|l} 2x^3+3x^2+5x+7 & \underline{x^2+x-2} \\ \underline{x^3\qquad\quad +2x} & x+3 \\ -\begin{cases} 3x^2+3x+7 \\ \underline{3x^2\qquad\ +6} \end{cases} & \\ 3x+1 & \end{array}$$

Shunday qilib,

$$\frac{x^3+3x^2+5x+7}{x^2+2}=x+3+\frac{3x+1}{x^2+2}.$$

2-Misol. $\frac{x+4}{x^3+6x^2+11x+6}$ soda ratsional kasrlarga yoying.

Kasrning maxraji *x=-1* da nolga tengligi sababli bu ko‘phad *x+1* ga qoldiqsiz bo‘linadi.

$$\begin{array}{r|l} x^3+6x^2+11x+6 & \underline{x+1} \\ \underline{x^3+x^2\qquad\qquad} & x^2+5x+6 \\ -\begin{cases} 5x^2+11x \\ \underline{5x^2+5x} \end{cases} & \\ 6x+6 & \\ \underline{6x+6} & \end{array}$$

$$x^3+6x^2+11x+6=(x+1)\left(x^2+5x+6\right)=(x+1)(x+2)(x+3)$$

$$\frac{x+4}{x^3+6x^2+11x+6}=\frac{x+4}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$$

x=-1 ligidan, *3=2A* ni topamiz, $A=\frac{3}{2}$. Agar *x=-2* ligidan *2=-B, B=-2.*x=-3 ligidan 1=2C, $C=\frac{1}{2}$.

$$\frac{x+4}{x^3+6x^2+11x+6}=\frac{x+4}{(x+1)(x+2)(x+3)}=\frac{3}{2(x+1)}-\frac{2}{x+2}+\frac{1}{2(x+3)}.$$

3-Misol. $\frac{7x-x^2-4}{(x+1)(x^2-5x+6)}$ soda ratsional kasrlarga yoying.

Yechish: Integral ostidagi funksiya ratsional kasrla kasrlardan iborat. Ularning maxrajini ko‘paytuvchilarga ajratamiz: $(x+1)(x-2)(x-3)$. To‘g‘ir kasrni eng sodda ratsional kasrlar yig‘indisi ko‘rinishida yozishdan foydalanamiz, ya’ni

$$\frac{7x-x^2-4}{(x+1)(x^2-5x+6)}=\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x-3}\ .$$

Bu tyenglikning o‘ng tomonini umumiy maxrajga keltirib, suratlarini o‘zaro tenglab quyidagi ayniyatga ega bo‘lamiz:

$$7x-x^2-4=A(x-2)(x-3)+B(x+1)(x-3)+C(x+1)(x-2)\ .$$

A,B,C koeffisiyentlarni xususiy qiymatlar berish usuli bilan aniqlaymiz:

$$\left.\begin{array}{r|l} x=-1 & -12=12A \\ x=2 & 6=-3B \\ x=3 & 8=4C \end{array}\right\}$$

Bundan: *A=-1, B=-2, C=2.* Bu qiymatlarni o‘rniga qo‘ysak, berilgan kasr eng sodda ratsional kasrlarni yig‘indisi ko‘rinishiga keladi.

$$\frac{7x-x^2-4}{(x+1)(x^2-5x+6)}=-\frac{1}{x+1}-\frac{2}{x-2}+\frac{2}{x-3}.$$

4-Misol. $\frac{15x-x^2-11}{(x-1)(x^2+x-2)}$ soda ratsional kasrlarga yoying.

Yechish.

$$\frac{15x-x^2-11}{(x-1)(x^2+x-2)}=\frac{15x-x^2-11}{(x-1)^2(x+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$$

$$\left|\begin{array}{ll} 15x-x^2-11\equiv A(x-1)(x+2)+B(x+2)+C(x-1)^2 & \\ x=1 & |3=3B,\ B=1 \\ x=-2 & |-45=9C,\quad C=-5 \\ x^2 & |-1=A+C,\ A=4 \end{array}\right|$$

$$\frac{15x-x^2-11}{(x-1)(x^2+x-2)}=\frac{15x-x^2-11}{(x-1)^2(x+2)}=\frac{4}{x-1}+\frac{1}{(x-1)^2}-\frac{5}{x+2}\quad .$$

5-Misol. $\frac{x^4-8x^3+23x^2-43x+27}{(x-1)(x^2-2x+5)}$ ko‘phadni to‘g‘ri kasr ko‘rinishida yozing.

Yechish: Noto‘g‘ri kasr bo‘lganligi uchun uning suratini maxrajiga bo‘lib, ko‘phad va to‘g‘ri ratsional kasr yig‘indisi ko‘r`nishida yozib olish mumkin:

$$\frac{x^4-8x^3+23x^2-43x+27}{(x-1)(x^2-2x+5)}=x-4+\frac{-2x^2+3x+13}{(x-2)(x^2-2x+5)}=$$

$$=\frac{x^2}{2}-4x+\frac{A}{x-2}+\frac{Bx+C}{x^2-2x+5}$$

$$\left|\begin{array}{ll} -2x+3x-13\equiv A(x^2-2x+5)+(Bx+C)(x-2) & \\ x=2 & |-15=5A,\ A=-3 \\ x^2 & |\ A+B=-2,\ B=1, \\ x^0 & |\ 5A-2C=-13, C=-1 \end{array}\right|$$

$$\frac{x^4-8x^3+23x^2-43x+27}{(x-1)(x^2-2x+5)}=x-4+\frac{-2x^2+3x+13}{(x-2)(x^2-2x+5)}=$$

$$=\frac{x^2}{2}-4x-\frac{3}{x-2}+\frac{x-1}{x^2-2x+5}$$

6-Misol. $\frac{2x^3-5x^3+8x-32}{x^4+9x^2+20}$ sodda kasrlarga yoying.

Yechish. $\frac{2x^3-5x^3+8x-32}{(x^2+4)(x^2+5)}=\frac{Ax+B}{x^2+4}+\frac{Cx+D}{x^2+5}$

$$2x^3-5x+8x-32\equiv(Ax+B)(x^2+5)+(Cx+D)(x^2+4)$$

$$\begin{array}{l|l} x^3 & 2=A+C,\ \ A=-3 \\ x^2 & -5=B+D, \\ x & 8=5A+4C \\ x^0 & -22=5B+4D \end{array}$$

$$A=0,\ B=-2,\ C=2,\ D=-3$$

$$\frac{2x^3-5x^3+8x-32}{(x^2+4)(x^2+5)}=-\frac{2}{x^2+4}+\frac{2x-3}{x^2+5}\ .$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR

Quyidagi kompleks sonlar ustida chiziqli amallar bajaring.

1. $(5+6i)+(7-6i)$; J. 12
2. $(3+2i)+(4-3i)+(-6+4i)$; J. 1+3i
3. $\left(\frac{3}{8}-\frac{1}{3}i\right)+\left(\frac{1}{8}-\frac{2}{3}i\right)-\left(1\frac{1}{2}+2i\right)$; J. -1+i
4. $(4+i)(4-i)$; J. 17
5. $2i\cdot 5i$; J. -10
6. $(1-i)(-3)$; J -3+3i
7. i^{98}; J. -1
8. $(1+i)^3-(1-i)^3$; J. 4i

9. $2 \div (1+i)$; J. 1-i

10. $(\sqrt{5}-i) \div (\sqrt{5}-2i)$; J. (7+i√5) / 9

11. Quyidagi tenglamalarni yechung.

a) $x^2 - 4x + 5 = 0$; J.. 2+i; 2-i.

b) $x^3 + 1 = 0$;

c) $x^6 - 9x^3 + 8 = 0$.

12. Ildizlari berilgan bo‘lsa , kvadrdt tenglamani tuzing:

a) $x_1 = 2 - i$; $x_2 = 3 - 2i$; J. x^2-(5-3i)x+4-7i=0;

b) $x_1 = -2i$; $x_2 = 3 + i$; J. x^2-(3-i)x+2-6i=0;

13. Ko‘paytuvchilarga alrating.

a) $x^4 - x^3$; J. x^2(x-1);

b) $x^3 - 8$; J. (x-2)(x^2+2x+4);

c) $x^4 - x^2$; J. x^2(x-1)(x+1);

) $x^5 - x^3 - x^2 + 1$; J. $(x-1)^2$(x+1)(x^2+x+1);

4. Ratsionak kasrlarni to‘g‘ri kasrlarga aylantiring.

a) $\frac{x^5 + 5x^2 + 2x + 1}{x^2 - x}$; J. x^3 +x^2 +x+6+(8x+1) /(x^2-x);

b) $\frac{4x^4 + 8x^3 - 1}{x^3 + x^2 + 4}$; J. 4x +4 – (4$x^2$ + 16x +17) / (x^3 +x^2+4);

c) $\frac{x^3 + 5x^2 - 2x + 1}{x^2 - x}$; J. x + 6 +(4x+1) / ($x^2$ - x);

15. Ratsional kasrlarni soda kasrlarga yoying.

a) $\frac{x+4}{(x+8)(x-11)}$; J. 4 / 19(x+8) + 15 / 19(x-11);

b) $\frac{2x+1}{(x-1)(x+1)(x-2)}$; J. – 1 / 2(x-1) – 1 / 2(x+1) + 1 / (x-2);

MUSTAQIL YECHISH UCHUN MASHQLAR.

Kompleks sonlar ustida chiziqli amallarni bajaring.

3.1. $(4+9i)+(-4+i)$;

3.2. $2+(3+4i)+2i+(-6-7i)$;

3.3. $(0.5-3.2i)+(1.5-0.8i)+(-4-i)$;

3.4. $\left(1\frac{3}{4}+\frac{3}{2}i\right)+\left(1\frac{1}{2}-\frac{5}{6}i\right)+\left(-\frac{3}{4}-2i\right)$;

3.5. $(0.12-1.4i)+(1.08+0.7i)+(2.5-0.2i)$;

3.6. $(-2a+3bi)+(a+bi)+(-4a-2bi)+(-2a+5bi)$.

3.7. $(5+3i)-(5-3i)$;

3.8. $\left(\frac{2}{5}+\frac{3}{4}i\right)+\left(\frac{3}{5}-\frac{3}{4}i\right)-(1-i)-(-2-7i)$.;

3.9. $(1.2-0.4i)+(0.3-1.4i)-(1.5+0.7i)-(2.3+0.6i)$;

3.10. $(m+ni)+(-3m+2ni)-[(m-ni)-(4m-10ni)]$;

3.11. $(2+3i)(6-5i)$;

3.12. $(2-5i)(8-3i)$;

3.13. $(5+4i)(-2-i)(5-4i)(-2+i)$;

3.14. $(\sqrt{3}-i)(\sqrt{2}+i\sqrt{3}$;

3.15. $(0.5+0.2i)(2+3i)$;

3.16. $5i\cdot(-3i)$;

3.17. $[(a^2+b^2)+2abi][(a^2+b^2)-2abi]$.

3.18. $(2+3i)^2-(2-3i)^2$;

3.19. $\left(\frac{1}{2}-\frac{i\sqrt{3}}{\sqrt{2}}\right)^2$;

3.20. i^6;

3.21. 3.23. i^{459};

3.22. $(1+i)^3+(1-i)^3$;

Quyidagi kompleks sonlarni trigonometrik va ko‘rsatkichli shaklida yozing.

3.23. $1+\sqrt{3}i;$ 3.24. $-5-5i;$ 3.25. $-\frac{1}{2}+i\frac{\sqrt{3}}{2};$

3.26. $2-2i;$ 3.27. $\cos\frac{\rho}{6}-i\cos\frac{\pi}{6};$ 3.28. $-\cos\frac{\pi}{9}+i\sin\frac{\pi}{9};$

3.29. $1+i;$ 3.30. $-i;$ 3.31. $i^{195}-1;$

3.32. $-5\sqrt{3+5i};$

3.33. $z^2=3+4i.$ kompleks sonning haqiqiy va mavhum qismini toping.

3.34. x va y ning qanday qiymatida tenglik o‘rinli bo‘ladi:

$$\frac{x+1+(y-3)i}{5+3i}=1+i;$$

3.35. Quyidagi kvadrat tenglamalarni yechung:

a) $9x^2-18x+13=0;$ b) $4x^2+3x+1=0;$

3.36. Tenglamalarni yeching.

a) $x^4-3x^3+4x^2+x-15=0;$

b) $x^4-16=0;$

c) $x^3+4x+6=0;$

3.37. Ildizlari x_1 va x_2 berilgan , kvadrat tenglamani tuzing:

a) $x_1=1+i, x_2=1-i;$ b) $x_1=\frac{-1+4i\sqrt{3}}{3}, x_2=\frac{-1-4i\sqrt{3}}{3};$

3.38. Agar $x_1; x_2; x_3$ lar $x^3-1=0$ tenglamani yechimi bo‘lsa, quyidagi tengliklar bajarilishini ko‘rsating:

a) $x_1+x_2+x_3=0;$ b) $x_1\cdot x_2\cdot x_3=1.$

3.39. Ko‘paytuvchilarga ajrating.

a) $x^4-1;$ l) $x^3-2x^2-x+2;$

b) $x^2-9x;$ m) $x^3-8x^2+4x-32;$

c) x^4-81;

n) $x^4+5x^2+x^3+5x$;

d) $1-\frac{1}{8}x^3$;

o) $25x^2-9$;

e) $7x^2+10x+20$;

p) $x^5+3x^3+x^2+3$;

f) $\frac{1}{4}x^2-x+1$;

r) x^3-7x^2-x+7;

j) x^3-x^2+5x-5;

s) $x^6+2x^5+x^4+2x^3$;

h) $2x^3-6x+x^2+3$;

t) $4x^4+8x^3-x-2$;

Ratsional kasrlarni to‘g‘ri kasrlarga aylantiring.

3.40. $\frac{x^3}{x-2}$;

3.41. $\frac{x^4}{x^2+2}$;

3.42. $\frac{x^3+1}{x^3-x^2}$;

3.43. $\frac{2x^5-2x+1}{1-x^4}$;

3.44. $\frac{x^4+x^3+2x^2+x+2}{x^4+5x^2+4}$;

3.45. $\frac{x^4+x^3+2x^2+x+2}{x^4+5x^2+4}$;

3.46. $\frac{x^3-2x^2-2x+1}{x^3-x^2}$;

3.47. $\frac{4x^4+8x^3-3x-3}{x^3+2x^2+x}$;

3.52. $\frac{2x^5+5x+1}{1-x^2}$;

3.53. $\frac{4x^4+8x^3-3x-3}{2x^2-x}$;

3.54. $\frac{x^4+3x-1}{x^4+x^2+4}$;

3.55. $\frac{4x^5+8x^3-1}{x^2-4}$;

3.56. $\frac{2x^4-5x+1}{1+x^2}$;

3.57. $\frac{x^4+3x^3+x^2+x+2}{x^2+4}$;

3.58. $\frac{x^3+2x^2+4x-2}{x^2+4}$;

3.59. $\frac{2x^5+2x^3+x^2+x}{1-x^2}$;

3.48. $\frac{2x^5 - 2x^3 + x^2}{1 - x^2}$;

3.49. $\frac{2x^4 - 4x^3 + 2x^2 - 4x + 1}{x^3 - 2x^2 + x}$;

3.50. $\frac{x^4 + x^3 + x^2 + x + 1}{x^2 - 4}$;

3.51. $\frac{x^4 + x^3 + 2x^2 - 4x + 1}{x^2 + x}$;

3.60. $\frac{x^4 + 2x^2 + 4x - 2}{3x^2 - 4}$;

3.61. $\frac{4x^4 + 8x^3 - 1}{x + 4}$;

3.62. $\frac{x^4 + 3x^3 + x^2 + x + 2}{x^3 + 4}$;

3.63. $\frac{x^4 + x^3 - 3x + 1}{x^2 - x}$;

Ratsional kasrlarni soda kasrlarga yoying.

3.64. $\frac{4x + 4}{x^3 + x^2 + 4x + 4}$;

3.65. $\frac{x - 4}{(x - 2)(x - 3)}$;

3.66. $\frac{11x + 16}{(x - 1)(x + 2)}$;

3.67. $\frac{2x - 1}{(x - 1)(x - 2)}$;

3.68. $\frac{2x^2 - 5x + 1}{x^3 - 2x^2 + x}$;

3.69. $\frac{2x^2 + x + 4}{x^3 + x^2 + 4x + 4}$;

3.70. $\frac{7x - 15}{x^3 - 2x^2 + 5x}$;

3.71. $\frac{5x - 14}{x^3 - x^2 - 4x + 4}$;

2-BOB. **CHIZIQLI ALGEBRA ELEMENTLARI**

Algebra-matematikaning bo‘limi bo‘lib, faqat sonlar ustida qo‘shish, ayirish, ko‘paytirish, bo‘lish amallari bo‘lib qolmay, balki boshqa matematik ob’ektlar, masalan ko‘phadlar, vektorlar, matritsalar, operatorlarda ham qo‘llaniladi.

Chiziqli algebrada asosan uch turdagi ob’ektlar: matritsalar, fazo va algebraik shakllar o‘rganiladi. Geometriya va mexanikada chiziqli algebraning ko‘pgina masalalari algebraik shakllar ko‘rinishida bo‘ladi. Matritsa ko‘rinishidagi ifodasi esa hisoblashga qulaydir.

Matritsa –jadvaldir. Chiziqli differensial tenglamalar sistemasi va ularning yechimlari matritsalar orqali, ayniqsa hozirgi vaqtda elektrotexnika, radiotexnika, avtomatikaga doir ko‘pgina kitoblarda tenglamalar sistemasi va differensial tenglamalar sistemalarining yechimlari matritsalar nazariyasi apparatidan foydalanib tekshiriladi.

Matritsa sonlardan tuzilgan to‘g‘ri burchakli jadval. Ko‘pincha ma’lumotlarni to‘g‘ri burchakli jadval ko‘rinishda joylashtirishga to‘g‘ri keladi. Masalan, agar uchta zavod beshta har xil turdagi mahsulot chiqarayotgan bo‘lsa, u holda yillik ishlab chiqarish haqidagi hisobot ushbu

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} \end{pmatrix}$$

jadval ko‘rinishida berilishi mumkin, bu yerda x_{ij} bilan *i*-nchi zavod tomonidan yil davomida ishlab chiqarilgan *j*-turdagi mahsulot miqdori belgilanadi, qisqacha $X = (x_{ij})$. Agar kelgusi yil davomida mahsulot assortimenti o‘zgargan bo‘lsa, u holda ikkinchi yil uchun ishlab chiqarish hisoboti ham $Y = (y_{ij})$ matritsa ko‘rinishida bo‘ladi. Unda ikki yillik mahsulot chiqarish $X + Y = (x_{ij} + y_{ij})$ matritsa bilan ifodalanadi.

Mahsulot chiqarishni donalarda, metrlarda, tonnalarda, so‘mlarda va h.k. ifodalab matritsalar tuzish mumkin.

2.1.Mavzu: **MATRITSALAR. MATRITSALAR USTIDA AMALLAR**

MAVZUGA OID NAZARIY MATERIALLAR

To‘rtta sondan iborat

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

kvadrat jadval *ikkinchi tartibli kvadrat matritsa* deyiladi.

Sonlarning *m* ta satr va *n* ta ustundan iborat to‘g‘ri to‘rtburchakli jadvalga

$m \times n$ o‘lchamli matritsa deyiladi. Bu matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko‘rinishda yoziladi.

Agar *m=1* bo‘lsa *satr matritsa, n=1* bo‘lsa- *ustun matritsa, m=n* bo‘lsa- *kvadrat matritsa* hosil bo‘ladi. Kvadrat *A* matritsa uchun shu matritsaning elementlaridan tuzilgan *n* tartibli determinantni hisoblash mumkin. Bu determinant $\det A$ yoki $|A|$ orqali belgilaniladi:

$$\det A = |A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{mn} \end{vmatrix}$$

Agar $\det A = 0$ bo‘lsa, u holda *A* matritsa *maxsus*, $\det A \neq 0$ bo‘lsa, *maxsusmas* deyiladi.

Bosh diagonalida turgan elementlari birga, qolgan elementlari nolga teng bo‘lgan kvadrat matritsa *birlik matritsa* deb ataladi va *E* bilan belgilanadi:

$$E = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots. & \dots. & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Ravshanki, $\det E = 1$. Barcha elementlari nollardan iborat matritsani nol matritsa deyiladi va Q bilan belgilanadi:

$$Q = \begin{pmatrix} 0\ 0\dots0 \\ 0\ 0\dots0 \\ \vdots\ \vdots\ \vdots \\ 0\ 0\dots0 \end{pmatrix}$$

$A = \lfloor a_{jk} \rfloor$ mxn matritsa satr elementlarini mos ustun elementlari o‘rinlarini almashtirib yozsak, transponirlangan $A^T = \lfloor a_{kj} \rfloor$ nxm matritsa hosil bo‘ladi. Misol uchun

$$A = \begin{pmatrix} 1 & 5 & 9 \\ 4 & 6 & 3 \\ 0 & 8 & -2 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 4 & 0 \\ 5 & 6 & 8 \\ 9 & 3 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 5 & 6 \\ 2 & -5 & 8 \end{pmatrix} \qquad B^T = \begin{pmatrix} 2 & 2 \\ 5 & -5 \\ 6 & 8 \end{pmatrix}$$

$$C = \begin{pmatrix} 7 & -4 & 3 \end{pmatrix} \qquad C^T = \begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix}$$

Bu tushuncha uchun quyidagi xossal o‘rinli:

1) $(A^T)^T = A$

2) $(A+B)^T = A^T + B^T$

3) $(kA)^{\mathrm{T}} = kA^T$

4) $(AB)^T = B^T A^T$

Bir xil $m \times n$ o‘lchamli A va B matritsaning *yig‘indisi* deb, o‘sha o‘lchamli shunday C $= A + B$ matritsaga aytiladiki, uning har bir elementi A va B matritsalarning mos elementlari yig‘indisidan iborat bo‘ladi.

Masalan: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ va $B = \begin{pmatrix} m & n \\ l & k \end{pmatrix}$ matritsalarning yig‘indisi va ayirmasi quyidagicha topiladi:

$$\text{a) C} = A + B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} m & n \\ l & k \end{pmatrix} = \begin{pmatrix} a+m & b+n \\ c+l & d+k \end{pmatrix}$$

$$\text{b) } A - B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} m & n \\ l & k \end{pmatrix} = \begin{pmatrix} a-m & b-n \\ c-l & d-k \end{pmatrix}$$

$m \times n$ o‘lchamli A matritsaning λ songa ko‘paytmasi deb, o‘sha o‘lchamdagi $B = \lambda \cdot A$ matritsaga aytiladiki, bu matritsa elementlari A matritsa elementlarini λ ga ko‘paytirishdan hosil bo‘ladi.

Masalan: $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ matritsani λ soniga ko‘paytirish quyidagicha bo‘ladi:

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{pmatrix}$$

$m \times k$ o‘lchamli A matritsaning $k \times n$ o‘lchamli B *matritsaga ko‘paytmasi* deb, $m \times n$ o‘lchamli shunday $C = A \cdot B$ matritsaga aytiladiki, uning c_{ij} elementi A matritsaning *i*- satr elementilarini B matritsaning *j*- ustunidagi mos elementlariga ko‘paytmalari yig‘indisiga teng, yani

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{ik}b_{kj}$$

Agar $AB=BA$ bo‘lsa, u holda A va B matritsalar o‘rin almashinadigan yoki kommutativ matritsalar deyiladi.

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ va $B = \begin{pmatrix} m & n \\ l & k \end{pmatrix}$ ikkinchi tartibli matritsalarning ko‘paytmasi quyidagicha topiladi:

1. $A \cdot B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} m & n \\ l & k \end{pmatrix} = \begin{pmatrix} am+bl & an+bk \\ cm+dl & cn+dk \end{pmatrix}$

2. $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ va $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ uchinchi tartibli matritsalarnuing ko‘paytmasi quyidagicha topiladi:

$$A \cdot B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} =$$

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{22}b_{23} + a_{33}b_{33} \end{pmatrix}$$

3. $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$ va $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{33} \end{pmatrix}$ matritsalarnuing ko‘paytmasi quyidagicha topiladi:

$$A \cdot B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix}$$

4. $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ va $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$ matritsalarnuing ko‘paytmasi quyidagicha topiladi:

$$A\cdot B=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} \\ & \end{pmatrix}$$

2-Misol. $A=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$; $B=\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}$ matritsa berilgan:

1) *A+2B*, *2) 3A-B*, *3) A·B* amallar bajarilsin.

1) $A+2B=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}+2\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}+\begin{pmatrix} 10 & 6 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} 12 & 9 \\ 6 & 9 \end{pmatrix}$

2). $3A-B=3\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}-\begin{pmatrix} 10 & 15 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} 6 & 9 \\ 12 & 3 \end{pmatrix}-\begin{pmatrix} 10 & 15 \\ 2 & 8 \end{pmatrix}=\begin{pmatrix} -4 & -6 \\ 10 & -5 \end{pmatrix}$

3)$A-B=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}-\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 3 & -3 \end{pmatrix}$

3-Misol. Quyidagi matritsalar berilgan: $A\cdot B$ matritsani toping.

$A=\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}; B=\begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}$

Yechish.

$A\cdot \mathbf{B}=\begin{pmatrix} 2 & 3 \\ 6 & 8 \end{pmatrix}\cdot\begin{pmatrix} 5 & 7 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} 2\cdot 5+3\cdot 1 & 2\cdot 7+3\cdot 4 \\ 6\cdot 5+8\cdot 1 & 6\cdot 7+8\cdot 4 \end{pmatrix}=\begin{pmatrix} 13 & 26 \\ 38 & 74 \end{pmatrix}$

2). $A=\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & 2 & 3 \end{pmatrix}$ va $B=\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ matritsalar berilgan: $A\cdot B$ va $B\cdot A$ ko‘paytmani toping.

Yechish.

$$A\cdot B=\begin{pmatrix} 1\cdot 2+3\cdot 1+1\cdot 3 & 1\cdot 1+3\cdot(-1)+1\cdot 2 & 1\cdot 0+3\cdot 2+1\cdot 1 \\ 2\cdot 2+0\cdot 1+4\cdot 3 & 2\cdot 1+0\cdot(-1)+4\cdot 2 & 2\cdot 0+0\cdot 2+4\cdot 1 \\ 1\cdot 2+2\cdot 1+3\cdot 3 & 1\cdot 1+2\cdot(-1)+3\cdot 2 & 1\cdot 0+2\cdot 2+3\cdot 1 \end{pmatrix}=\begin{pmatrix} 8 & 0 & 7 \\ 16 & 10 & 4 \\ 13 & 5 & 7 \end{pmatrix}$$

$$B\cdot A=\begin{pmatrix} 2\cdot 1+1\cdot 2+0\cdot 1 & 2\cdot 3+1\cdot 0+0\cdot 2 & 2\cdot 1+1\cdot 4+0\cdot 3 \\ 1\cdot 1+1\cdot 2+0\cdot 1 & 1\cdot 3+1\cdot 0+2\cdot 2 & 1\cdot 1-1\cdot 4+2\cdot 3 \\ 3\cdot 1+2\cdot 2+1\mp\cdot 1 & 3\cdot 3+2\cdot 0+1\cdot 2 & 3\cdot 1+2\cdot 4+1\cdot 3 \end{pmatrix}=\begin{pmatrix} 4 & 6 & 6 \\ 1 & 7 & 3 \\ 8 & 11 & 14 \end{pmatrix}$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

Matritsalar ustida amallarni bajaring.

1. A= $\begin{pmatrix} 3 & -2 & 4 \\ -5 & 3 & 8 \\ 2 & 1 & -4 \end{pmatrix}$ B= $\begin{pmatrix} 5 & 3 & -7 \\ 2 & -4 & 5 \\ 3 & 7 & -8 \end{pmatrix}$ $2A-5E+4B^2=?$ J:

$\begin{pmatrix} 41 & -188 & 136 \\ 138 & 229 & -364 \\ -88 & -298 & 339 \end{pmatrix}$

2. A= $\begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 4 & -5 & 0 \end{pmatrix}$ B= $\begin{pmatrix} 5 & 2 & -3 \\ 4 & -5 & 2 \\ 3 & 1 & -4 \end{pmatrix}$ $2B^2+3A-2E=?$ J: $\begin{pmatrix} 52 & -15 & 14 \\ 27 & 71 & 54 \\ 26 & -24 & 16 \end{pmatrix}$

3. A= $\begin{pmatrix} 5 & -3 & 2 \\ 4 & 1 & -3 \\ 7 & -2 & 4 \end{pmatrix}$ B= $\begin{pmatrix} 2 & 5 & -4 \\ 3 & -2 & 1 \\ 5 & 3 & -7 \end{pmatrix}$ $(A+B)^2-3B=?$ J: $\begin{pmatrix} 33 & -5 & -12 \\ 2 & -4 & 5 \\ 3 & 7 & -8 \end{pmatrix}$

4. $A=\begin{pmatrix} 1 & 3 \\ 4 & 2 \\ 0 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 1 & -1 \\ 2 & 0 & 3 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping. J: $\begin{pmatrix} 8 & 1 & 8 \\ 12 & 4 & 2 \\ -2 & 0 & -3 \end{pmatrix}$; $\begin{pmatrix} 6 & 9 \\ 2 & 3 \end{pmatrix}$.

5. $A=\begin{pmatrix} 2 & 3 \\ 4 & 2 \\ 1 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 3 & 1 & -1 \\ 2 & 1 & 0 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping.J: $\begin{pmatrix} 12 & 5 & -2 \\ 16 & 6 & -4 \\ 1 & 0 & -1 \end{pmatrix}$; $\begin{pmatrix} 9 & 12 \\ 8 & 8 \end{pmatrix}$

6. $A=\begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 4 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \end{pmatrix}$ $A\cdot B$ va $B\cdot A$ ni toping.

J: $\begin{pmatrix} -2 & 0 & -4 \\ 4 & 1 & 6 \\ -2 & 3 & -7 \end{pmatrix}$; $\begin{pmatrix} -4 & 3 \\ 14 & -5 \end{pmatrix}$

Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

7. $A=\begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{pmatrix}$ $A^{-1}=?$ J: $-\frac{1}{6}\begin{pmatrix} 14 & 15 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{pmatrix}$

8. $A=\begin{pmatrix} 5. & 7 & -3 \\ 2 & -8 & 4 \\ 1 & 9 & -7 \end{pmatrix}$ $A^{-1}=?$ J: $\frac{1}{5}\begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -12 & 1 & 7 \end{pmatrix}$

9. $A=\begin{pmatrix} 5 & -3 & 7 \\ 9 & 1 & -2 \\ 4 & -7 & 8 \end{pmatrix}$ $A^{-1}=?$ J: $-\frac{1}{259}\begin{pmatrix} -6 & -25 & -1 \\ -78 & 12 & 73 \\ -67 & 23 & 32 \end{pmatrix}$

MUSTAQIL YECHISH UCHUN MASHQLAR.

Matritsalar ustida amallarni bajaring

1.1. $A=\begin{pmatrix} 3 & -5 & 2 \\ -4 & 3 & 7 \\ 8 & -2 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 7 & -3 \\ 5 & 1 & -4 \\ 3 & -8 & 9 \end{pmatrix}$ $3A-5E+2B^2=?$ J: $\begin{pmatrix} 60 & 75 & 116 \\ -6 & 145 & 131 \\ 10 & -124 & 211 \end{pmatrix}$

1.2. $A=\begin{pmatrix} 3 & 5 & -7 \\ 2 & -4 & 8 \\ 7 & 3 & -5 \end{pmatrix}$ $B=\begin{pmatrix} 1 & -3 & 0 \\ 1 & 5 & -2 \\ 3 & -1 & 4 \end{pmatrix}$ $3B^2+4E+2A=?$ J: $\begin{pmatrix} 4 & -44 & 4 \\ 4 & 64 & -38 \\ 70 & -48 & 44 \end{pmatrix}$

1.3. $A=\begin{pmatrix} 4 & 5 & -3 \\ 1 & -2 & 4 \\ 3 & 4 & -7 \end{pmatrix}$ $B=\begin{pmatrix} 5 & -3 & 4 \\ -3 & 5 & 7 \\ 4 & -5 & 8 \end{pmatrix}$ $2A^2-4B+3E=?$ J: $\begin{pmatrix} 7 & 8 & 16 \\ 40 & 33 & -18 \\ -26 & -22 & 77 \end{pmatrix}$

1.4. $A=\begin{pmatrix} 5 & -3 & 4 \\ 1 & 2 & -7 \\ 3 & -4 & 6 \end{pmatrix}$ $B=\begin{pmatrix} 4 & 5 & -7 \\ 2 & -3 & 5 \\ 1 & 0 & -2 \end{pmatrix}$ $2A+B-3E^2=?$ J: $\begin{pmatrix} 52 & -15 & 14 \\ 27 & 71 & 54 \\ 26 & -24 & 16 \end{pmatrix}$

1.5. $A=\begin{pmatrix} 3 & -5 & 4 \\ 2 & 7 & -5 \\ 4 & -8 & 9 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 4 & -7 \\ 3 & -5 & 7 \\ 3 & 2 & -6 \end{pmatrix}$ $3A-2B^2+4E=?$ J: $\begin{pmatrix} 3 & 37 & -100 \\ -18 & -77 & 181 \\ 24 & -4 & -27 \end{pmatrix}$

1.6. $A=\begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 7 & -3 & 4 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 2 & -3 \\ 4 & -3 & 5 \\ 2 & 4 & 7 \end{pmatrix}$ $2E+A-2B^2=?$ J: $\begin{pmatrix} -1 & -16 & 9 \\ 6 & 33 & -17 \\ 10 & -8 & 18 \end{pmatrix}$

1.7. $A=\begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -6 \\ 7 & -2 & 5 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 2 & -1 \\ 4 & -3 & 2 \\ 5 & 7 & -3 \end{pmatrix}$ $(A+B)^2-2B=?$ J: $\begin{pmatrix} 117 & 3 & 26 \\ 16 & 7 & 37 \\ 131 & 46 & 26 \end{pmatrix}$

1.8. $A=\begin{pmatrix} 3 & 2 & -5 \\ 4 & -3 & 7 \\ -8 & 2 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -5 & 8 \\ 3 & 4 & -6 \end{pmatrix}$ $A^2-4E+3B=?$ J: $\begin{pmatrix} 72 & 2 & 13 \\ -50 & 12 & -58 \\ 1 & -12 & 33 \end{pmatrix}$

2.2. Mavzu: IKKINCHI VA UCHINCHI TARTIBLI DETERMINANTLAR. DETERMINANTLARNING ASOSIY XOSSALARI. YUQORI TARTIBLI DETERMINANTLAR.

MAVZUGA OID NAZARIY MATERIALLAR

Ikkinchi tartibli kvadrat matritsaga mos keluvchi ikkinchi tartibli determinant deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

Uchinchi tartibli kvadrat matritsaga mos keluvchi uchinchi tartibli

determinant deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{33} - a_{31}a_{22}a_{13} - a_{21}a_{12}a_{33} - a_{32}a_{23}a_{11}$$

Determinantlarning asosiy xossalari:

Determinantda mos satr va ustun elementlari o‘rnini almashtirishga uni transponirlash deyiladi.

1-xossa. Transponirlash natijasida determinantning qiymati o‘zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

<u>2-xossa</u>. Determinantda istalgan ikkita satr yoki ikkita ustunining o‘rnini almashtirsak, uning qiymati o‘z ishorasini o‘zgartiradi, ammo absolyut qiymat o‘zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{13} & a_{21} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{23} & a_{31} \end{vmatrix}$$

<u>1-natija.</u> Ikkita satri yoki ustuni bir xil bo‘lgan (yoki proporsional) determinantning qiymati nolga teng.

<u>3-xossa</u>. Determinantning satri yoki ustunidagi elementlari umumiy λ ko‘paytuvchiga ega bo‘lsa, λ ni determinant belgisidan tashqariga chiqarish mumkin.

<u>2-natija.</u> Determinantning biror satri yoki ustuni boshqa satri yoki ustuniga parallel bo‘lsa bunday determinantning qiymati nolga teng.

<u>4-xossa</u>. Agar determinantning biror qatorining har bir elementi ikki qo‘shiluvchining yig’indisidan iborat bo‘lsa, u holda bu determinant ikki determinant yig‘indisidan

$$\begin{vmatrix} a_{11}+b_{11} & a_{12} & a_{13} \\ a_{21}+b_{21} & a_{22} & a_{23} \\ a_{31}+b_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & a_{12} & a_{13} \\ b_{21} & a_{22} & a_{23} \\ b_{31} & a_{32} & a_{33} \end{vmatrix}$$

iborat bo‘ladi.

<u>5-xossa</u>. Agar biror satr elementlariga boshqa parallel satrning elementlari istalgan ko‘paytuvchiga ko‘paytirib qo‘shilsa, determinant o‘zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} + \lambda a_{12} & a_{12} & a_{13} \\ a_{21} + \lambda a_{22} & a_{22} & a_{23} \\ a_{31} + \lambda a_{32} & a_{32} & a_{33} \end{vmatrix}$$

Algebraik to'ldiruvchilar va minorlar

1-Ta'rif. Determinant berilgan elementining minori deb, shu element turgan satr va ustunni bir vaqtda o'chirishdan hosil bo'lgan determinantga aytiladi.

Masalan, ushbu $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ determinant a_{12} turgan satr va ustunni o'chirish natijasida hosil bo'lgan $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ 2-tartibli determinant a_{12} elementining minoridan iborat bo'ladi va M_{12} deb beriladi:

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

Shunday qilib, yuqorida tuzilgan uchinchi tartibli Δ determinantning har bir $a_{i,j} (i, j = 1, 2, 3)$ elementiga mos minori $M_{i,j}$ bo'ladi.

2-Ta'rif. Determinant biror elementining algebraik to'ldiruvchisi deb, uning bu determinantda juft yoki toq joy egallaganiga bog'liq ravishda musbat yoki manfiy ishora bilan olingan minoriga aytiladi:

$$A_{i,j} = (-1)^{i+j} M_{i,j}$$

Masalan, a_{22} elementning algebraik to'ldiruvchisi

$A_{22} = (-1)^{2+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = (-1)^{2+2} M_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$ son bo'ladi, chunki a_{22} juft joyda turibdi, a_{32} element algebraik to'ldiruvchisi

$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = -(a_{11}a_{23} - a_{13}a_{21})$ son bo'ladi, chunki a_{32} toq o'rinda turibdi.

Ixtiyoriy tartibli determinantni hisoblashning uchta usulini keltiramiz:

1.Determinant *tartibini pasaytirish usuli*- determinant biror satr (ustun) elementlarining bittasidan boshqalarini oldindan nolga aylantirib olib, shu satr (ustun) bo'yicha yoyish usuli.

2. Determinantni *uchburchak ko'rinishiga* keltirish usuli - determinantning bosh diagonalidan bir tomonida yotuvchi hamma elementlari nolga aylantiriladi va uchburchaksimon shaklga keltiriladi, masalan

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{vmatrix}$$

Ravshanki, uchburchak shaklidagi determinantning qiymati bosh diagonallari elementlari ko'paytmasiga teng: $\Delta = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$

Masalan.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 8 \end{vmatrix} = 1 \cdot 2 \cdot 3 \cdot 8 = 48$$

Determinantni *satr yoki ustun bo'yicha yoyib hisoblash* quyidagicha bo'ladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Masalan.

$$\Delta = \begin{vmatrix} 1 & 7 & 3 & 0 \\ 0 & 10 & 2 & 3 \\ 0 & -14 & -8 & 2 \\ 0 & -8 & -6 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 10 & 2 & 3 \\ -14 & -8 & 2 \\ -8 & -6 & 1 \end{vmatrix} = 2 \cdot 2 \cdot 2 \cdot \begin{vmatrix} 5 & 1 & 3 \\ -7 & -4 & 2 \\ -4 & -3 & 1 \end{vmatrix} = 8 \cdot \begin{vmatrix} 17 & 10 & 0 \\ 1 & 2 & 0 \\ -4 & -3 & 1 \end{vmatrix} = 8 \begin{vmatrix} 17 & 10 \\ 1 & 2 \end{vmatrix} = 192$$

3. Sarrius usuli.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \begin{matrix} a_{11}a_{12} \\ a_{21}a_{22} \\ a_{31}a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{12}a_{21}a_{33}.$$

$$\begin{matrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

Masalan.

1. $$\begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} \begin{matrix} 3 & 2 \\ 1 & 3 \\ 5 & 3 \end{matrix} = 3 \cdot 3 \cdot 4 + 2 \cdot 1 \cdot 5 + 2 \cdot 1 \cdot 3 - 2 \cdot 3 \cdot 5 - 3 \cdot 1 \cdot 3 - 2 \cdot 1 \cdot 4 =$$
$$= 36 + 10 + 6 - 30 - 9 - 8 = 52 - 47 = 5$$

2. $$\begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 3 \cdot 3 \cdot 4 + 2 \cdot 1 \cdot 5 + 2 \cdot 1 \cdot 3 - 2 \cdot 3 \cdot 5 - 3 \cdot 1 \cdot 3 - 2 \cdot 1 \cdot 4 =$$
$$\begin{matrix} 3 & 2 & 2 \\ 1 & 3 & 1 \end{matrix}$$
$$= 36 + 10 + 6 - 30 - 9 - 8 = 52 - 47 = 5$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

Determinantlarni hisoblang.

1. $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$ *J:* 1 2. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ *J: -2* 3. $\begin{vmatrix} 3 & 2 \\ 8 & 5 \end{vmatrix}$ *J: -1* 4. $\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$ *J: 1*

5. $\begin{vmatrix} \frac{1-t^2}{1+t^2} & \frac{2t}{1+t^2} \\ \frac{-2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{vmatrix}$ *J:* 1

Quyidagi determinantlarni ixtiyoriy ustun yoki satr elementlari bo'yicha yoyib hisoblang.

6. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ *J: -10;* 7. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$ *J: 4a;* 8. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & -4 & 8 \end{vmatrix}$ *J: 68 ;*

Determinantni tartibini pasaytirish usulidan foydalanib hisoblang:

9. $\begin{vmatrix} 1 & -4 & 0 & 3 \\ -4 & 3 & 2 & -3 \\ -2 & 3 & -1 & 4 \\ 3 & 2 & 5 & 0 \end{vmatrix}$ J: 175; 10. $\begin{vmatrix} 2 & -1 & 0 & 5 \\ -1 & -3 & 2 & -4 \\ 4 & 2 & -1 & 3 \\ 3 & 0 & -4 & -2 \end{vmatrix}$ J:-203;

11 $\begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix}$ J: 0; 12. $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$ J: 0; 13. $\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$ J: -1; 14. $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$ J: 4ab

15. $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$ J: 1; 16. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$ J: -2x; 17. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$ J: 72;

18. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$ J: (x-y)(y-z)(x-z); 19. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 2 & -7 & -5 \end{vmatrix}$ J: 605; 20. $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$ J: -5;

21. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$ J: 1 22. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ J: 1; 23. $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$ J: -8;

Quyidagi determinantlarni ixtiyoriy ustun yoki satr elementlari bo‘yicha yoyib hisoblang.

24. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & b \end{vmatrix}$ J: $-2b^2$; 25. $\begin{vmatrix} 1 & 2 & 5 \\ 0 & 5 & 7 \\ 0 & -4 & 8 \end{vmatrix}$ J: 68; 26. $\begin{vmatrix} 0 & 0 & 1 \\ 2 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ J: -19;

27. $\begin{vmatrix} 1 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & -1 & 8 \end{vmatrix}$ J:-23; 28. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$ J: 0; 29. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$ J:-2x;

30. $\begin{vmatrix} 3 & -1 & -2 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{vmatrix}$ J: -9; 31. $\begin{vmatrix} -1 & 2 & 5 \\ 2 & 0 & 6 \\ 4 & 0 & 7 \end{vmatrix}$ J: 20; 32. $\begin{vmatrix} 1 & 7 & -1 \\ 2 & 6 & 2 \\ 1 & 1 & 4 \end{vmatrix}$ J: -16

Determinantni tartibini pasaytirish usulidan foydalanib hisoblang:

33. $\begin{vmatrix} 3 & -1 & 0 & 3 \\ 5 & 1 & 4 & -7 \\ 5 & -1 & 0 & 2 \\ 1 & -8 & 5 & 3 \end{vmatrix}$ J: 260; 34. $\begin{vmatrix} 6 & -3 & 4 & 2 \\ -1 & 0 & 4 & 5 \\ 2 & 7 & 3 & 4 \\ 0 & -5 & -1 & 3 \end{vmatrix}$ J: -74;

2.3. MAVZU: CHIZIQLI TENGLAMALAR SISTEMASINI KRAMER, MATRITSALAR VA GAUSS USULLARIDA YECHISH.

MAVZUGA OID NAZARIY MATERIALLAR

1. Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi berilgan.

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$ shart bajarilganda

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

yechimga ega.

Masalan. Ushbu $\begin{cases} 3x + 2y = 7 \\ 4x - 5y = 40 \end{cases}$ chiziqli tenglamalar sistemasini yeching.

$$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & -5 \end{vmatrix} = 3 \cdot (-5) - 2 \cdot 4 = -15 - 8 = -23$$

$$x = \frac{\begin{vmatrix} 7 & 2 \\ 40 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -5 \end{vmatrix}} = \frac{-35 - 80}{-15 - 8} = \frac{-115}{-23} = 5$$

$$y = \frac{\begin{vmatrix} 3 & 7 \\ 4 & 40 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -5 \end{vmatrix}} = \frac{120 - 28}{-15 - 8} = \frac{92}{-23} = -4$$

J: (5;-4)

1-Teorema. Agar $\Delta \neq 0$ bo'lsa, u holda bir jinsli sistema yagona $x=y=z=0$ yechimga ega.

2-Teorema. Bir jinsli sistema noldan farqli yechimlarga ega bo'lishi uchun uning koeffisiyentidan tuzilgan determinantning nolga teng bo‘lishi zarur va yetarlidir.

2. Bir jinsli uch noma’lumli ikkita tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \end{cases}$$

ushbu

$$x = k\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, y = -k\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, z = k\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

formula bilan aniqlanuvchi yechimlarga ega, bunda k- ixtiyoriy son.

Masalan: Ushbu $\begin{cases} 2x - 5y + 2z = 0 \\ x + 4y - 3z = 0 \end{cases}$ tenglamalar sistemesini yeching.

$$x = k\begin{vmatrix} -5 & 2 \\ 4 & -3 \end{vmatrix} = k(15 - 8) = 7k,\ y = k\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix} = k(-6 - 2) = -8k,\ z = k\begin{vmatrix} 2 & -5 \\ 1 & 4 \end{vmatrix} = k(8 + 5) = 13k.$$

J: *x=7k;y=-8k;z=13k.*

3. Bir jinsli uch noma’lumli uchta tenglamalar sistemasi berilgan.

$$\begin{cases} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3x + b_3y + c_3z = 0 \end{cases}$$

Uning determinanti $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ bo‘lsa, tenglamalar sistemasi cheksiz ko‘p yechimga ega.

Masalan. Ushbu $\begin{cases} x + 2y + 3z = 4 \\ 2x + y - z = 3 \\ 3x + 3y + 2z = 10 \end{cases}$ tenglamalar sistemasini yeching.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & 3 & 2 \end{vmatrix} = 2 - 6 + 18 - 9 + 3 - 8 = 23 - 23 = 0$$

J: Sistema birgalikda emas.

4. Ikki noma’lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \\ a_3x + b_3y = c_3 \end{cases}$$

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ bo‘lganda va uning hech qaysi ikkita tenglamasi o‘zaro zid bo‘lmasa, birgalikda bo‘ladi.

Masalan. Ushbu $\begin{cases} 2x - 3y = 6 \\ 3x + y = 9 \\ x + 4y = 3 \end{cases}$ tenglamalar sistemasini yeching.

Yechish: $\Delta = \begin{vmatrix} 2 & -3 & 6 \\ 3 & 1 & 9 \\ 1 & 4 & 3 \end{vmatrix} = 6 - 27 + 72 - 6 - 72 + 27 = 0$

J:Tenglamalar sistemasi birgalikda.

5. Uch noma’lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1, \\ a_{21}x + a_{22}y + a_{23}z = b_2, \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

ning bosh determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

bo‘lganda yagona yechimga ega bo‘lib, bu yechim Kramer formulalari bilan hisoblanadi:

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta},$$

bunda

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Masalan: Ushbu

$$\begin{cases} x-2y+z=-4, \\ 3x+2y-z=8, \\ 2x-3y+2z=-6 \end{cases}$$

chiziqli tenglamalar sistemasini yeching.

Yechilishi: asosiy va yordamchi determinantlarni topamiz:

$$\Delta=\begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix}=1\cdot2\cdot2+3\cdot(-3)\cdot1+2\cdot(-2)\cdot(-1)-[2\cdot2\cdot1+1\cdot(-3)\cdot(-1)+2\cdot3\cdot(-2)]=-1-(-5)=4.$$

Determinant $\Delta=4\neq0$ bo‘lgani uchun sistema yagona yechimga ega va Kramer formulasini qo‘llab, uni topamiz:

$$\Delta_x=\begin{vmatrix} -4 & -2 & 1 \\ 8 & 2 & -1 \\ -6 & -3 & 2 \end{vmatrix}=-4\cdot2\cdot2+8\cdot(-3)\cdot1+(-6)\cdot(-2)\cdot(-1)-[(-6)\cdot2\cdot1+(-4)\cdot(-3)\cdot(-1)+2\cdot8\cdot(-2)]=$$

$$=-52-(-56)=4;$$

$$\Delta_y=\begin{vmatrix} 1 & -4 & 1 \\ 3 & 8 & -1 \\ 2 & -6 & 2 \end{vmatrix}=1\cdot8\cdot2+3\cdot(-6)\cdot1+2\cdot(-4)\cdot(-1)-[2\cdot8\cdot1+1\cdot(-6)\cdot(-1)+2\cdot3\cdot(-4)]=$$

$$=6-(-2)=8;$$

$$\Delta_z=\begin{vmatrix} 1 & -2 & -4 \\ 3 & 2 & 8 \\ 2 & -3 & -6 \end{vmatrix}=1\cdot2\cdot(-6)+3\cdot(-3)\cdot(-4)+2\cdot(-2)\cdot8-[2\cdot2\cdot(-4)+1\cdot(-3)\cdot8+(-6)\cdot3\cdot(-2)]=$$

$$=-8-(-4)=-4.$$

$$x=\frac{\Delta_x}{\Delta}=\frac{4}{4}=1,\quad y=\frac{\Delta_y}{\Delta}=\frac{8}{4}=2,\quad z=\frac{\Delta_z}{\Delta}=\frac{-4}{4}=-1$$

J: $x=1,\quad y=2,\quad z=-1.$

6. Agar kvadrat matritsa maxsusmas bo‘lsa, u holda $AA^{-1}=A^{-1}A=E$ tenglikni qanoatlantiruvchi yagona A^{-1} matritsa mavjud bo‘ladi va u A matritsaga teskari matritsa deyiladi. A matritsaga A^{-1} teskari matritsa quyidagicha aniqlanadi:

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}$$

$$A_{ik} = (-1)^{ij} M_{ij}$$

Bu erda A_{ik} a_{ik} elementining *algebraik to'ldiruvchisi*, M_{ij} esa a_{ik} elementining minori deyiladi.

Masalan. Berilgan $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$ matritsaga teskari matritsani toping.

Yechish. Matritsaning determinantini hisoblaymiz:

$$\det A = \begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 27 + 2 - 24 = 5$$

Demak, A matritsa maxsusmas matritsa ekan. Endi A_{ik} algebraik to'ldiruvchilarni hisoblaymiz:

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 3 & 4 \end{vmatrix} = 9, \ A_{21} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2, \ A_{31} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4, \ A_{12} = -\begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix} = 1, \ A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 2,$$

$$A_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -1, \ A_{13} = \begin{vmatrix} 1 & 3 \\ 5 & 3 \end{vmatrix} = -12, \ A_{23} = -\begin{vmatrix} 3 & 2` \\ 5 & 3 \end{vmatrix} = 1, \ A_{33} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7.$$

Teskari matritsa tuzamiz:

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -1`2 & 1 & 7 \end{pmatrix} = \begin{pmatrix} \frac{9}{5} & -\frac{2}{5} & -\frac{4}{5} \\ \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} \\ -\frac{12}{5} & \frac{1}{5} & \frac{7}{5} \end{pmatrix}.$$

$AA^{-1} = A^{-1}A = E$ ekanini tekshirish mumkin.

n ta noma'lumli *n* ta chiziqli tenglamalar sistemasini

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n \end{cases}$$

matritsa ko'rinishda

$$AX = B$$

kabi yozish mumkin, bunda

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}.$$

Agar A maxsusmas matritsa, ya'ni $\det A \neq 0$ bo'lsa, u holda bu sistemaning matritsa shaklidagi yechimi ushbu ko'rinishga ega bo'ladi:

$$X = A^{-1}B.$$

Masalan: Tenglamalar sistemasining echimini matrisa usuli yordamida yeching.

$$\begin{cases} 2x - 3y + z = -5 \\ x + 2y - 4z = -9 \\ 5x - 4y + 6z = 5 \end{cases}$$

Yechish. Tenglamalar sistemasi yordamida A matritsani tuzamiz

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} -5 \\ -9 \\ 5 \end{pmatrix}$$

Ushbu matritsaning determinantini va algebraik to'ldiruvchilarini hisoblaymiz

$$\Delta = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 24 + 60 - 4 - 10 - 32 + 18 = 56;$$

$$A_{11}=(-1)^2\begin{vmatrix}2 & -4\\ -4 & 6\end{vmatrix}=12-16=-4$$

$$A_{12}=(-1)^3\begin{vmatrix}1 & -4\\ 5 & 6\end{vmatrix}=-(6+20)=-26$$

$$A_{13}=(-1)^4\begin{vmatrix}1 & 2\\ 5 & -4\end{vmatrix}=-4-10=-14$$

$$A_{21}=(-1)^3\begin{vmatrix}-3 & 1\\ -4 & 6\end{vmatrix}=-(-18+4)=14$$

$$A_{22}=(-1)^4\begin{vmatrix}2 & 1\\ 5 & 6\end{vmatrix}=12-5=7$$

$$A_{23}=(-1)^5\begin{vmatrix}2 & -3\\ 5 & -4\end{vmatrix}=-(-8+15)=-7$$

$$A_{31}=(-1)^4\begin{vmatrix}-3 & 1\\ 2 & -4\end{vmatrix}=12-2=10$$

$$A_{32}=(-1)^5\begin{vmatrix}2 & 1\\ 1 & -4\end{vmatrix}=-(-8-1)=9$$

$$A_{33}=(-1)^6\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix}=4+3=7$$

Teskari matritsani tuzamiz

$$A^{-1}=\frac{1}{56}\begin{pmatrix}-4 & 14 & 10\\ -26 & 7 & 9\\ -14 & -7 & 7\end{pmatrix}$$

$X=A^{-1}B$ formulaga asosan noma'lumlarni topamiz

$$X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{56}\begin{pmatrix}-4 & 14 & 10\\ -26 & 7 & 9\\ -14 & -7 & 7\end{pmatrix}\begin{pmatrix}-5\\ -9\\ 5\end{pmatrix}=\begin{pmatrix}-4\cdot(-5)+14\cdot(-9)+10\cdot 5\\ -26\cdot(-5)+7\cdot(-9)+9\cdot 5\\ -14\cdot(-5)+(-7)\cdot(-9)+7\cdot 5\end{pmatrix}=$$

$$\begin{pmatrix}20-126+50\\ 130-63+45\\ 70+63+35\end{pmatrix}=\frac{1}{56}\begin{pmatrix}-56\\ 112\\ 168\end{pmatrix}=\begin{pmatrix}-\frac{56}{56}\\ \frac{112}{56}\\ \frac{168}{56}\end{pmatrix}=\begin{pmatrix}-1\\ 2\\ 3\end{pmatrix},\quad x=-1;\ y=2;\ z=3$$

J: (-1;2;3)

Matritsa minorining noldan farqli eng katta darajasi, matritsaning rangi deyiladi va r(A) kabi belgilanadi.

Masalan. Matritsaning rangini toping

$$A=\begin{pmatrix}2 & 0 & 4 & 0\\ 3 & 0 & 6 & 0\\ 1 & 0 & -3 & 0\end{pmatrix}$$

Yechish. Ma'lumki bu matritsaning barcha 3-tartibli minorlari nolga teng. Noldan farqli 2-tartibli minori bor

$$\begin{vmatrix} 3 & 6 \\ 1 & -3 \end{vmatrix} = -15 \neq 0$$

Demak r(A)=2.

Matritsa rangini xossalari:

1) Matritsani transponirlasak rangi o‘zgarmaydi.
2) Matritsaning noldan iborat qatorini o‘chirib tashlansa, matritsaning rangi o‘zgarmaydi.

7. Gauss usuli bilan tenglamalar sistemasini yechish.

Gaussning noma’lumlarni ketma-ket yo‘qotish usuli, chiziqli algebraik tenglamalar sistemasini yechish usullari ichida eng universal va eng samaralisidir. Soddalik uchun to‘rt noma`lumli chiziqli sistema uchun Gauss usulini ko‘rib chiqamiz.

Ushbu sistema berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1; \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2; \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3; \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4, \end{cases} \tag{5}$$

bu yerda x_i (i=1,4) – noma`lumlar, $a_{ij}$ ($j$=1,4) va $b_i$ ($i$=1,4)- ma`lum koeffitsiyentlar. qulaylik uchun $a_{15}= v_1$, $a_{25} = v_2$, $a_{35}= v_3$, $a_{45} = v_4$ deb olamiz.

Gauss usulining to‘liq tavsifiga o’tamiz. Birinchi qadamning yetakchi elementi deb ataladigan a_{11} koeffitsiyentni noldan farqli deb hisoblaymiz. (5) tenglamaning birinchi hamma hadlarini a_{11} ga bo‘lib, quyidagiga ega bo‘lamiz:

$$x_1+v_{12}x_2+v_{13}x_3+v_{14}x_4=v_{15} \tag{6}$$

bu yerda

$$v_{1j} = \frac{a_{1j}}{a_{11}}(j = 2,3,4,5).$$

(6) tenglikdan foydalanib (5) sistemaning ikkinchi, uchinchi va to‘rtinchi tenglamalaridan x_1 noma`lumni yo‘qotamiz. Buning uchun (6) tenglamani $a_{21}$, $a_{31}$ va $a_{41}$ ga ko‘paytirib natijani mos ravishda sistemaning ikkinchi, uchinchi va to‘rtinchi tenglamalaridan ayirish kerak. U holda uch noma`lumli quyidagi sistemaga ega bo‘lamiz:

$$\begin{cases} a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 + a_{24}^{(1)}x_4 = a_{25}^{(1)}; \\ a_{32}^{(1)}x_2 + a_{33}^{(1)}x_3 + a_{34}^{(1)}x_4 = a_{35}^{(1)}; \\ a_{42}^{(1)}x_2 + a_{43}^{(1)}x_3 + a_{44}^{(1)}x_4 = a_{45}^{(1)}, \end{cases} \quad (7)$$

bu yerda

$$a_{ij}^{(1)} = a_{ij} - a_{i1}v_{1j} \quad (i=2,3,4,\ j=2,3,4,5) \quad (8)$$

Endi shu sistemani o’zgartirish bilan shug’ullanamiz.

Ikkinchi qadamni bajarishga o‘tishdan oldin, ikkinchi qadamning yetakchi elementi deb ataladigan $a_{22}^{(1)}$ elementni noldan farqli deb faraz qilamiz (aks holda tenglamalarning o‘rnini tegishli ravishda almashtirish lozim). (8) sistemaning birinchi tenglamasini $a_{22}^{(1)}$ ga bo’lamiz, u holda

$$x_2 + b_{23}^{(1)}x_3 + b_{24}^{(1)}x_4 = v_{25}^{(1)} \quad (9)$$

bu yerda

$$b_{2j}^{(1)} = \frac{a_{2j}^{(1)}}{a_{22}^{(1)}} \qquad (j = 3,4,5)$$

Yuqoridagiga o‘xshash x_2 ni yo‘qotamiz, u holda

$$\begin{cases} a_{33}^{(2)}x_3 + a_{34}^{(2)}x_4 = a_{35}^{(2)}; \\ a_{43}^{(2)}x_3 + a_{44}^{(2)}x_4 = a_{45}^{(2)}, \end{cases} \quad (10)$$

sistemaga ega bo‘lamiz, bu yerda

$$a_{ij}^{(1)} = a_{ij}^{(1)} - a_{ij}^{(1)} b_{2j}^{(1)} \quad (i = 3,4; \;\; j = 3,4,5)$$

(10) ning birinchi tenglamasini a_{33} ga bo‘lamiz, u holda

$$x_3 + b_{34}^{(2)} x_4 = v_{35}^{(2)}$$

bo‘ladi, bu yerda

$$b_{34}^{(2)} = \frac{a_{34}^{(2)}}{a_{33}^{(2)}}, \quad b_{35}^{(2)} = \frac{a_{35}^{(2)}}{a_{33}^{(2)}}.$$

Bu tenglama yordamida (10) sistemaning ikkinchi tenglamasidan x_3 ni yo‘qotamiz. Quyidagi tenglamaga ega bo‘lamiz:

$$a_{44}^{(3)} x_4 = a_{45}^{(2)},$$

bu yerda

$$a_{4j}^{(3)} = a_{4j}^{(2)} - a_{43}^{(2)} b_{3j}^{(2)} \quad (j = 4,5).$$

Shunday qilib, (5) sistemani uchburchak matritsali o‘ziga teng kuchli bo‘lgan quyidagi sistemaga keltirdik:

$$\begin{cases} x_1 + b_{12}x_2 + b_{13}x_3 + b_{14}x_4 = b_{15}; \\ x_2 + b_{23}^{(1)} x_3 + b_{24}^{(1)} x_4 = b_{25}^{(1)}; \\ x_3 + a_{34}^{(2)} x_4 = b_{35}^{(2)}; \\ a_{44}^{(3)} x_4 = b_{45}^{(3)}. \end{cases}$$

Bu yerda ketma-ket quyidagilarni aniqlaymiz:

$$\begin{cases} x_4 = \dfrac{a_{45}^{(3)}}{a_{44}^{(3)}}; \\ x_3 = b_{35}^{(2)} - b_{34}^{(2)} x_4; \\ x_2 = b_{25}^{(1)} - b_{24}^{(1)} x_4 - b_{23}^{(1)} x_3; \\ x_1 = b_{15} - b_{14}x_4 - b_{13}x_3 - b_{12}x_2. \end{cases}$$

Ko‘rinib turibdiki barcha yechimlar topildi.

Masalan: Ushbu

$$\begin{cases} x+y+5z+2t=1, \\ x+y+3z+4t=-3, \\ 2x+3y+11z+5t=2, \\ 2x+y+3z+2t=-3 \end{cases}$$

chiziqli tenglamalar sistemasini Gauss usuli bilan yeching.

Yechish: Ikkinchi, uchinchi, to‘rtinchi tenlamalardan x larni yo‘qotamiz. Buning uchun birinchi tenglamani ketma-ket *-1, -2, -2* ga ko‘paytiramiz va mos ravishda ikkinchi, uchinchi, to‘rtinchi tenglamalar bilan qo‘shamiz. Natijada ushbu sistemaga ega bo‘lamiz:

$$\begin{cases} x+y+5z+2t=1, \\ 2z-2t=4, \\ y+z+t=0, \\ -y-7z-2t=-5, \end{cases}$$

yoki

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ y+7z+2t=5, \\ z-t=2. \end{cases}$$

Uchinchi tenglamadan ikkinchi tenglamani ayiramiz:

$$\begin{cases} x+y+5z+2t=1, \\ y+z+t=0, \\ 6z+t=5, \\ z-t=2, \end{cases}$$

So‘ngra to‘rtinchi tenglamani -6 ga ko‘paytirib, uchinchi tenglamaga qo‘shsak, uchburchakli sistema hosil bo‘0ladi:

$$\begin{cases} x+y+5z+2t=1, \\ \qquad y+z+t=0, \\ \qquad\qquad z-t=2, \\ \qquad\qquad\quad 7t=-7. \end{cases}$$

bundan,

$$t=-1,\ \ z=2+t=1,\ \ y=-z-t=0,\ \ \ x=1-y-5z-2t=-2.$$

J: $x=-2,\ \ \ y=0,\ \ \ z=1,\ \ \ t=-1$.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

Teskari matritsani toping va $A\,A^{-1}=E$ ekanligini isbotlang.

1. $A=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -1 & 7 \\ 6 & 3 & -5 \end{pmatrix}$ $\quad A^{-1}=?$ $\qquad$ J: $A^{-1}=\frac{1}{92}\begin{pmatrix} -16 & 11 & 25 \\ 52 & -7 & -41 \\ 12 & 9 & -13 \end{pmatrix}$

2. $A=\begin{pmatrix} 5 & 4 & -3 \\ 2 & -1 & 7 \\ 6 & 3 & -5 \end{pmatrix}$ $\quad A^{-1}=?$ $\qquad$ J: $A^{-1}=\frac{1}{12}\begin{pmatrix} -6 & -3 & 7 \\ 12 & 6 & -10 \\ 6 & 9 & -5 \end{pmatrix}$

3. $A=\begin{pmatrix} 5 & 2 & -4 \\ 3 & -1 & 7 \\ 8 & 5 & -3 \end{pmatrix}$ $\quad A^{-1}=?$ $\qquad$ J: $A^{-1}=\frac{1}{122}\begin{pmatrix} -32 & -14 & 10 \\ 65 & 17 & -47 \\ 23 & -9 & -11 \end{pmatrix}$

4. $A=\begin{pmatrix} 4 & 5 & -7 \\ 1 & -2 & 3 \\ 7 & 8 & -9 \end{pmatrix}$ $\quad A^{-1}=?$ $\qquad$ J: $A^{-1}=-\frac{1}{28}\begin{pmatrix} -6 & 11 & 1 \\ 30 & 13 & -19 \\ 22 & 3 & -13 \end{pmatrix}$

Tenglamalar sistemasini yeching:

5. $\begin{cases} 3x-y=4 \\ 6x-2x=8 \end{cases}$ *J: (1;-1)* $\qquad$ *6.* $\begin{cases} 2x+y=3 \\ 3x+2y=5 \end{cases}$ *J:(1;1;1)*

7. $\begin{cases} mx-ny=(m-n)^2 \\ 2x-y=n(m\neq 2n) \end{cases}$ *J:(m ;2m-n)* $\qquad$ *8.* $\begin{cases} 5x-y-z=5 \\ x+2y+3z=2 \\ 4x+3y+2z=3 \end{cases}$ *J:(1;-1;1)*

9. $\begin{cases} 3x+2y+2z=0 \\ 5x+2y+3z=0 \end{cases}$ *J: (2k;k;-4k)*

10. $\begin{cases} 3x+4y-z=8 \\ 2x+y+z=2 \\ 3x-y+2z=0 \end{cases}$ *J: (1;2;3)*

Tenglamalar sistemasini matritsa usuli bilan yeching.

13. $\begin{cases} 3x-y+z=1 \\ x+2y+4z=-1 \\ 5x+y+2z=4 \end{cases}$ J:(1;1;-1)

14. $\begin{cases} x+y+z=-1 \\ 2x-3y+4z=9 \\ 4x-11y+10z=25 \end{cases}$ J:(1;-1;1)

15. $\begin{cases} 2x-3y+z=2 \\ x+5y-4z+5=0 \\ 4x+y-3z+4=0 \end{cases}$ J:(5;6;10)

16. $\begin{cases} 2x-4y+3z=1 \\ x-2y+4z=3 \\ 3x-y+5z=2 \end{cases}$ J:(-1;0;1)

17. $\begin{cases} 2x-y+z=2 \\ 3x+2y-2z=-2 \\ x-2y+z=1 \end{cases}$ J:(2;-1;-3)

18. $\begin{cases} x+2y+3z=5 \\ 2x-y-z=1 \\ x+3y+4z=6 \end{cases}$ J:(1;-1;1)

19. $\begin{cases} 3x+4y+2z=9 \\ x-y+4z=4 \\ 5x+2y+10z=17 \end{cases}$ J:(1;1;1)

20. $\begin{cases} 2x-3y+z=0 \\ x+y+z=3 \\ 3x-2y+2z=3 \end{cases}$ J:(1;1;1)

Tenglamalar sistemasini Gauss usuli bilan yeching.

21. $\begin{cases} x+2y+z=4 \\ y+3z+t=4 \\ 4x+z+t=6 \\ x+y+5t=7 \end{cases}$ J:(1;1;1;1)

22. $\begin{cases} x+y-3z+2t=6 \\ x-2y-t=-6 \\ y+z+3t=16 \\ 2x-3y+2z=6 \end{cases}$ J:(1;1;1;1)

MUSTAQIL YECHISH UCHUN MASHQLAR.

A matritsa berilgan. A^{-1} teskari matrisani toping va $AA^{-1}=A^{-1}A=E$ ekanini tekshiring.

3.1. $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix}$ **3.2.** $\begin{pmatrix} 3 & 2 & -1 \\ 7 & 3 & 0 \\ 1 & 2 & 2 \end{pmatrix}$ **3.3.** $\begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$ **3.4.** $\begin{pmatrix} -2 & 3 & 3 \\ 4 & 5 & 1 \\ -3 & 4 & 0 \end{pmatrix}$

3.5. $\begin{pmatrix} 0 & 1 & -3 \\ 1 & -5 & 4 \\ 2 & 3 & 2 \end{pmatrix}$ **3.6.** $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$ **3.7.** $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix}$

3.8. $\begin{pmatrix} -1 & 8 & 1 \\ -1 & 5 & 5 \\ 0 & -1 & 3 \end{pmatrix}$ **3.9.** $\begin{pmatrix} 4 & -2 & 1 \\ -3 & 4 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ **3.10.** $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix}$

3.11. $\begin{pmatrix} 3 & -3 & 4 \\ -1 & -5 & -7 \\ 0 & -1 & 5 \end{pmatrix}$ **3.12.** $\begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix}$ **3.13.** $\begin{pmatrix} 1 & -1 & 8 \\ 1 & -5 & 5 \\ -2 & 3 & 10 \end{pmatrix}$

3.14. $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 1 & 3 \\ -7 & 0 & 2 \end{pmatrix}$ **3.15.** $\begin{pmatrix} 5 & -1 & 3 \\ 4 & -2 & 0 \\ 2 & -4 & 5 \end{pmatrix}$ **3.16.** $\begin{pmatrix} 1 & -3 & -2 \\ -2 & 1 & 3 \\ -2 & 4 & 4 \end{pmatrix}$

3.17. $\begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix}$ **3.18.** $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix}$ **3.19.** $\begin{pmatrix} 4 & 1 & 2 \\ 3 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$ **3.20.** $\begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix}$

Berilgan tenglamalar sistemasini birgalikda ekanligini tekshiring, agar birgalikda bo‘lsa, ularni:

a) Kramer qoidasidan foydalanib,

b) matritsa usuli bilan,

c) Gauss usuli bilan yeching.

3.21. $\begin{cases} 3x+2y+z=5, \\ 2x+3y+z=1, \\ 2x+y+3z=11. \end{cases}$

3.22. $\begin{cases} 4x-3y+2z=9, \\ 2x+5y-3z=4, \\ 5x+6y+2z=18. \end{cases}$

3.23. $\begin{cases} 2x-y-z=4, \\ 3x+4y-2z=11, \\ 3x-2y+4z=11. \end{cases}$

3.24. $\begin{cases} x+y-z=1, \\ 8x+3y-6z=2, \\ -4x-y+3z=-3. \end{cases}$

3.25. $\begin{cases} 7x-5y=31, \\ 4x+11z=-43, \\ 2x+3y+4z=-20. \end{cases}$

3.26. $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=20, \\ 3x-2y-5z=6. \end{cases}$

3.27. $\begin{cases} x+y+3z=-1, \\ 2x-y+2z=-4, \\ 4x+y+4z=-2. \end{cases}$

3.28. $\begin{cases} 3x+4y+2z=8, \\ 2x-y-3z=-1, \\ x+5y+z=-7. \end{cases}$

3.29. $\begin{cases} x-4y-2z=-7, \\ 3x+y-z=5, \\ -3x+5y+6z=7. \end{cases}$

3.30. $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=20, \\ 3x-y+z=0. \end{cases}$

3.31. $\begin{cases} x+5y+z=-2, \\ 2x-4y-3z=0, \\ 3x+4y+2z=3. \end{cases}$

3.32. $\begin{cases} 2x-3y+2z=-6, \\ 5x+8y-z=0, \\ x+2y+3z=6. \end{cases}$

3.33. $\begin{cases} x-4y-2z=0, \\ 3x-5y-6z=7, \\ 3x+y+z=6. \end{cases}$

3.34. $\begin{cases} 2x-y+5z=10, \\ 5x+2y-13z=21, \\ 3x-y+5z=12. \end{cases}$

3.35. $\begin{cases} 2x+y-5z=-1, \\ x+y-z=-2, \\ 4x-3y+z=13. \end{cases}$

3.36. $\begin{cases} 2x+3y+4z=-10, \\ 4x+11z=-29, \\ 7x-5y=7. \end{cases}$

3.37. $\begin{cases} 2x+7y-z=10, \\ 3x-5y+3z=-14, \\ x+2y+z=-1. \end{cases}$

3.38. $\begin{cases} 4x+y-3z=-6, \\ 8x+3y-6z=-15, \\ x+y-z=-4. \end{cases}$

3.39. $\begin{cases} 3x-2y-5z=-14, \\ x-2y+3z=0, \\ 2x+3y-4z=-10. \end{cases}$ **3.40.** $\begin{cases} 5x+6y-2z=-9, \\ 2x+5y-3z=-1, \\ 4x-3y+2z=-15. \end{cases}$

Quyidagi matritsaviy tenglamalarni yeching:

3.41. $X\begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$ 3.4**2.** $X\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}=\begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

3.43. $X\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}=\begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$ 3.4**4.** $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}X=\begin{pmatrix} 5 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$

3.45. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}X=\begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \\ 0 & -1 & 1 \end{pmatrix}$ 3.4**6.** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}X=\begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$

3.47. $\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}X=\begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$ 3.4**8.** $\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix}X=\begin{pmatrix} 0 & 0 \\ 1 & 5 \end{pmatrix}$

3.49. $\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}X=\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ 3.5**0.** $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}X=\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

3.51. $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}X=\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ 3.5**2.** $X\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

3.53. $\begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}X=\begin{pmatrix} 0 & -1 & 3 \\ 10 & 0 & 25 \\ -5 & 5 & 0 \end{pmatrix}$ 3.5**4.** $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}X=\begin{pmatrix} -1 & 7 \\ 3 & 5 \end{pmatrix}$

3.55. $X\begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$ 3.5**6.** $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -4 \end{pmatrix}\cdot X=\begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

3.57. $X\cdot\begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix}=\begin{pmatrix} 2 & 3 & 0 \\ 3 & 4 & -5 \\ 1 & -1 & 2 \end{pmatrix}$ **3.58.** $X\cdot\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix}=\begin{pmatrix} 0 & 2 & -3 \\ -1 & 2 & 3 \\ 1 & 0 & -1 \end{pmatrix}$

3.59. $\begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & -3 \\ 6 & -5 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

3.60. $X \cdot \begin{pmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ 6 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -3 & 0 \\ 7 & -4 & 3 \\ -2 & 0 & 1 \end{pmatrix}$

3.61. $X \cdot \begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 6 \\ -2 & 5 & -3 \\ 1 & 8 & -1 \end{pmatrix}$

3.62. $X \cdot \begin{pmatrix} 3 & -1 & 4 \\ 7 & 8 & -2 \\ 2 & -3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 5 \\ -2 & 6 & 3 \\ 1 & -5 & 0 \end{pmatrix}$

3.63. $\begin{pmatrix} -5 & 7 & -4 \\ 8 & 0 & -1 \\ 4 & -5 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{pmatrix}$

3.64. $\begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & -4 & 3 \\ 0 & 5 & 6 \\ 8 & 7 & -4 \end{pmatrix}$

3.65. $X \cdot \begin{pmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 7 \\ 3 & -5 & 9 \\ -2 & 4 & 3 \end{pmatrix}$

3.66. $\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 6 & 2 & 3 \\ 7 & -1 & -4 \\ -3 & 0 & 5 \end{pmatrix}$

3-BOB. **VEKTORLAR ALGEBRASI ELEMENTLARI**

Skalyar va vektor kattaliklar haqida tushuncha.

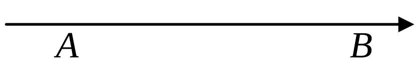
Vektor - geometriyaning asosiy tushunchalaridan biri. Vektor ham son (uzunlik), ham yo‘nalish bilan xarakterlanadi.

Vektor tushunchasi X1X asr nemis matematigi G. Grassman va irland matematigi U. Xamilton asarlarida kiritilgan. Hozirgi zamon matematikasi va uning tadbiqlarida bu tushuncha muhim rol o‘ynab, mexanika, nisbiylik nazariyasi, kvant fizika, matematik iqtisod va tabiatshunoslikning boshqa ko‘p bo‘limlarida qo‘llaniladi.

Skalyar miqdor deb, faqat son qiymati bilan aniqlanadigan kattaliklarga aytiladi. Masalan: uzunlik, vaqt, hajm, yuza va boshqalar. Shunday miqdorlar ham borki, ular o‘zlarining son qiymatlari bilan to‘la aniqlanmaydi; ularni to‘liq aniqlash uchun son qiymatlari bilan bir qatorda yo‘nalishlari ham berilgan bo‘lishi kerak. Masalan, harakat, kuch, tezlik, tezlanish kabi miqdorlar. To‘g‘ri chiziqda oddiy kesma bilan bir qatorda yo‘nalgan kesma, ya’ni bir uchi uning boshi, ikkinchi uchi uning oxiri hisoblangan kesmaga qaraladi. Bunday kesma vektor deyiladi.

Shunday qilib, vektor miqdor geometrik usulda ma’lum uzunlikdagi va aniq yo‘nalishdagi kesma yordamida tasvirlanadi:

$$A\vec{B} = \vec{a}$$



1-chizma

Boshi A nuqtada, oxiri B nuqtada bo‘lgan yo‘naltirilgan kesma *vektor* deb ataladi va u $\overrightarrow{AB}$ yoki $\vec{a}$ kabi belgilanadi.

3.1. MAVZU: VEKTORLAR USTIDA CHIZIQLI AMALLAR. BA'ZIS VEKTORLAR. VEKTORNI O'QDAGI PROEKSIYASI VA YO'NALTIRUVCHI KOSINUSLARI.

MAVZUGA OID NAZARIY MATERIALLAR

$\vec{a}_1, \vec{a}_2, ..., \vec{a}_n$ vektorlarning *chiziqli kombinatsiyasi* deb

$$\vec{a} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + ... + \lambda_n \vec{a}_n$$

formula bilan aniqlanuvchi $\vec{a}$ vektorga aytiladi, bunda $\lambda_1, \lambda_2, ..., \lambda_n$ - tayin sonlar

Agar $\vec{a}_1, ..., \vec{a}_n$ vektorlar sistemasi uchun kamida bittasi noldan farqli shunday $\lambda_1, ..., \lambda_n$ sonlar mavjud bo'lib, $\lambda_1 \vec{a}_1 + ... + \lambda_n \vec{a} = 0$ shart bajarilsa, u sistema *chiziqli bog'liq sistema* deyiladi.Agar yuqoridagi tenglik faqat $\lambda_1 = ... = \lambda_n = 0$ bo'lganda o'rinli bo'lsa, $\vec{a}_1, ..., \vec{a}_n$ vektorlar sistemasi *chiziqli erkli* deyiladi.

Ikkita kollinear vektorlar ham har doim chiziqli bog'liqdir. Shuningdek, uchta komplanar vektor har doim chiziqli bog'liq. Fazodagi ixtiyoriy to'rtta yoki undan ortiq vektorlar har doim chiziqli bog'liq.

n ta chiziqli bog'liqmas vektorlar sistemasi $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ berilgan bo'lib, agar ixtiyoriy $\vec{a}$ vektorni ularning chiziqli kombinatsiyasi, y`ani

$$\vec{a} = \lambda_1 \vec{e}_1 + ... + \lambda_n \vec{e}_n$$

shaklida ifodalash mumkin bo'l`sa, u holda berilgan sistema *bazis* deyiladi.

Bu tenglik $\vec{a}$ vektorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazis boyicha *yoyilmasi* deyiladi.

Fazoda chiziqli bog‘liq bo‘lmagan har qanday uchta $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ vektor bazis tashkil qiladi, shu sababli fazodagi har qanday $\lambda_1, \lambda_2, ..., \lambda_n$ $\vec{a}$ vektor shu bzis bo‘yicha yoyilishi mumkin:

$$\vec{a} = \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2 + \lambda_3 \vec{e}_3$$

$\lambda_1, \lambda_2, .\lambda_3$ sonlar $\vec{a}$ vektorning berilgan bazisdagi koordinatalari bo‘lib, quyidagicha yoziladi:

$$\vec{a} = \{\lambda_1, \lambda_2, \lambda_3\}$$

Agar bazisning vektorlari o‘zaro perpendikulyar va birlik uzunlikka ega bo‘lsa, bu bazis ortonormallangan bazis deyilib, u ortlar deb ataluvchi $\vec{i}, \vec{j}, \vec{k}$ vektorlar orqali belgilanadi.

Agar $\vec{i}, \vec{j}, \vec{k}$ mos ravishda *OX,OY,OZ* o‘qlari bo‘yicha yo‘naltirilgan ortlar bo‘lsa, u holda ixtiyoriy $\vec{a}$ vektorning $\vec{i}, \vec{j}, \vec{k}$ bazisdagi yoyilmasi quyidagicha ifodalanadi:

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \text{ yoki } \vec{a} = \{a_x; a_y; a_z\},$$

bunda $a_x; a_y; a_z$ - $\vec{a}$ vektorning koordinatalari.

Masalan. $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ vektorning koordinatalari $\{2; -3; 4\}$ bo‘ladi.

$\vec{a}$ vektorning uzunligi $|\vec{a}|$ kabi belgilanadi va quyidagi formula bilan hisoblanadi

$$|\vec{a}| = \sqrt{\vec{a}_x^{\,2} + \vec{a}_y^{\,2} + \vec{a}_z^{\,2}}$$

Masalan. $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ vektorning uzunligi quyidagicha topiladi.

$$\left|\vec{a}\right| = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29} \qquad \text{J: } \sqrt{29}$$

Boshlang‘ich va oxirgi nuqtalari ustma-ust tushadigan vektor nol-vektor deyiladi va $\vec{0}$ ga teng.

Uzunligi birga teng vektor *birlik vektor* deyiladi. $\vec{a}$ vektorning birlik vektori $\vec{a}^{\,0}$ kabi belgilanadi

$$\vec{a}^{\,0} = \frac{a_x}{\left|\vec{a}\right|} i + \frac{a_y}{\left|\vec{a}\right|} j + \frac{a_z}{\left|\vec{a}\right|} k$$

Masalan. $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ berilgan bo‘lsa, $\vec{a}^{\,0}$ vektor

$$\left|\vec{a}\right| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

$$\vec{a} = \frac{1}{3} i + \frac{2}{3} j + \frac{2}{3} k \text{ ga teng.}$$

Noldan farqli vektorlar bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotsa, bunday vektorlar *kollinear* vektorlar deyiladi (2-chizma).

Agar ikki vektor o‘zaro kollinear, bir xil yo‘nalgan va modullari teng bo‘lsa, bu vektorlar *teng vektorlar* deyiladi.

Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlarni *komplanar* vektorlar deyiladi.

$\vec{a}$ vektorning yo‘nalishi uning koordinata o‘qlari bilan hosil qilgan α, β, γ burchaklari bilan aniqlanadi.

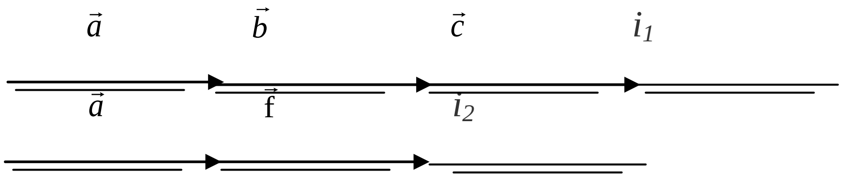
$\vec{a}$ vektorning yo‘naltiruvchi kosinuslari

$$cos\alpha = \frac{a_x}{\left|\vec{a}\right|} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \ \cos\beta = \frac{a_y}{\left|\vec{a}\right|} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \cos\gamma = \frac{a_z}{\left|\vec{a}\right|} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

formula bilan aniqlanadi va ular

$$coa^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

munosabat bilan bog‘langan.



2-chizma

Masalan. $\vec{a} = 20i + 30j - 60k$ vektorning yo‘naltiruvchi kosinuslari topilsin.

$$\left|\vec{a}\right| = \sqrt{20^2 + 30^2 + (-60)^2} = \sqrt{4900} == 70$$

$$cos\alpha = \frac{a_x}{\left|\vec{a}\right|} = \frac{20}{70} = \frac{2}{7}, \ \cos\beta = \frac{a_y}{\left|\vec{a}\right|} = \frac{30}{70} = \frac{3}{7}, \cos\gamma = \frac{a_z}{\left|\vec{a}\right|} = -\frac{60}{70} = -\frac{6}{7}$$

$coa^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ga ko‘ra

$$\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 = \frac{4}{49} + \frac{9}{49} + \frac{36}{49} = \frac{49}{49} = 1.$$

Vektorlar ustida amallar.

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$ va $\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$ vektorlar berilgan bo‘lsin. U holda

$$\vec{a} \pm \vec{b} = (a_x \pm b_x)\vec{i} + (a_y \pm b_y)\vec{j} + (a_z \pm b_z)\vec{k}$$

$$\lambda\vec{a} = \lambda a_x\vec{i} + \lambda a_y\vec{j} + \lambda a_z\vec{k}$$

Agar vektorning bosh va oxirgi nuqtalarining koordinatalari $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ be rilgan bo‘lsa, u holda $\vec{AB}$ vektorning ortlar bo‘yicha yoyilmasi

$$\vec{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

ko‘rinishda bo‘ladi.

Masalan. $A(1;3;2)$ va $B(5;8;-1)$ nuqtalar berilgan. $\vec{AB} = u$ vektor uning koordinatalari aniqlansin.

Yechish:

$$\vec{AB} = (5-1)\vec{i} + (8-3)\vec{j} + (-1-2)\vec{k} = 4\vec{i} + 5\vec{j} - 3\vec{k},$$ J: $\{4;5;-3\}$

A va *B* nuqtalar orasidagi masofa yoki $\vec{AB}$ vektorning uzunligi

$$\left|\vec{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

formula bilan hisoblaniladi.

Masalan: $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ vektorning uzunligi topilsin.

Yechish: $\left|\vec{AB}\right| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

J: 7.

$\vec{a} = \vec{AB}$ vektorning *l* o‘q bo‘yicha tashkil etuvchi (komponenti) deb, shu vektor boshi va oxirining proeksiyalarini birlashtiruvchi $\vec{A_1B_1}$ vektorga aytiladi.

$\vec{a} = \vec{AB}$ vektorning *l* o‘q yo‘nalishi bilan bir xil yoki bir xil emasligiga qarab, ”+” yoki ”-” ishora bilan olinadigan tashkil etuvchisining uzunligiga aytiladi.

$$pr_l \vec{AB} = \pm\left|\vec{A_1B_1}\right|$$

$\vec{a}$ vektorning l o‘qqa proeksiyasi a_1 deb belgilanadi, yani:

$$pr_l\,\vec{a} = a_1$$

$A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ berilgan bo‘lsa, u holda $\vec{AB}$ vektorning koordinata o‘qlaridagi proeksiyalari quyidagilardan iborat:

$$\left.\begin{aligned} pr_x\,\vec{AB} &= X = x_2 - x_1 \\ pr_y\,\vec{AB} &= Y = y_2 - y_1 \\ pr_z\,\vec{AB} &= Z = z_2 - z_1 \end{aligned}\right\}$$

Proeksiyalarning asosiy xossalari:

a) $pr_l\,\vec{a} = \left|\vec{a}\right|\cos\varphi$ yoki $a_l = \left|\vec{a}\right|\cos\varphi$

Bunda $\varphi - \vec{a}$ vektor bilan o‘q orasidagi burchak;

b) $pr_l\left(\vec{a} + \vec{b}\right) = pr_l\,\vec{a} + pr_l\,\vec{b}$ yoki $pr_l\left(\vec{a} + \vec{b}\right) = a_l + b_l$;

v) $pr_l \lambda\,\vec{a} = \lambda pr_l\,\vec{a}$ yoki $pr_l \lambda\,\vec{a} = \lambda a_l$.

Masalan. $\vec{a} = \{3;2;-5\}$ va $\vec{b} = \{2;-3;1\}$ vektorlar berilgan. $2\vec{a} - \vec{b}$ vektorning koordinata o‘qlaridagi proeksiyalari topilsin.

Yechish: $2\vec{a} - \vec{b} = \{2\cdot 3 - 2; 2\cdot 2 - (-3); 2\cdot(-5) - 1\} = \{4;7;-11\}$.

Fazodagi nuqtaning radius-vektori.

$\vec{OM} = \vec{r}$ radius-vektorning moduli yoki uzunligi ushbu:

$$\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$$

formula bilan aniqlanadi.

Masalan. *M(5;-3;4)* nuqtaning radius-vektorining uzunligi topilsin.

Yechish: $\vec{OM} = \vec{r} = \sqrt{5^2 + (-3)^2 + 4^2} = \sqrt{50} = 5\sqrt{2}.$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. $\vec{a}(3,2)$, $\vec{b}(5;1)$, $\vec{c}(-1,3)$ vektorlar berilgan $2\vec{a} + 3\vec{b} - \vec{c}$, $16\vec{a} + 5\vec{b} - 9\vec{c}$ vektorlarning koordinatalarini toping. J: (22;4), (82;10).

2. $\vec{a}(3,0,-2)$, $\vec{b}(1,2,-5)$, $\vec{c}(-1,1,1)$, $\vec{d}(8,4,1)$ vektorlar berilgan $-5\vec{a} + \vec{b} - 6\vec{c} + \vec{d}$, $3\vec{a} - \vec{b} - \vec{c} - \vec{d}$ vektorlarning koordinatalarini toping. J:(10;0;0)

3. *A(2;2;0)* va *B(0;-2;5)* nuqtalar berilgan. $\vec{AB} = u$ vektor yasalsin hamda uning uzunligi va yo‘naltiruvchi kosinuslari aniqlansin.

$$\text{J: } |A| = 3\sqrt{5};\ \cos\alpha = -\frac{2\sqrt{5}}{15};\cos\beta = -\frac{4\sqrt{5}}{15};\cos\gamma = \frac{\sqrt{5}}{3}$$

4. a) $\vec{a} = \{12;-15;-16\}$ vektorning yo‘naltiruvchi kosinuslarini toping.

$$\text{J: } |\vec{a}| = \sqrt{65};\ \cos\alpha = -\frac{2\sqrt{65}}{65};\cos\beta = -\frac{\sqrt{65}}{13};\cos\gamma = \frac{6\sqrt{65}}{65}$$

b) $\vec{a} = \{3;-2;6\}$ va $\vec{b} = \{-2;1;0\}$ vektorlar berilgan 1) $\vec{a} + \vec{b}$ 2) $\vec{a} - \vec{b}$ 3) $2\vec{a}$ vektorlarning koordinatalarini toping. J: 1) $\{1;-1;6\}$ 2) $\{5;-3;6\}$ 3) $\{6;-4;12\}$

5. Uchburchakning *A(-1;-2;4), B(-4;-2;0) va C(3;-2;1)* uchlari berilgan. Uning *B* uchidagi ichki burchagini toping. *J:* $\alpha = \frac{\pi}{4}$

6. To‘rtburchakning *A(4;-2;4), B(1;4;0), C(-4;1;1)* va *D(-5;5;1)* uchlari bo‘lsa, *AC* va *BD* diagonallarining perpendikulyarligini isbotlang.

7. $\vec{a} = \{1;2;3\}$ $\vec{b} = \{6;4;2\}$ vektorlar orasidagi burchakning kosinusini toping.

J: $\varphi = \arccos\left(\frac{2}{7}\right)$

3.2. MAVZU: IKKITA VEKTORNING SKALYAR KO‘PAYTMASI VA UNING XOSSALARI. IKKI VEKTOR ORASIDAGI BURCHAK.

MAVZUGA OID NAZARIY MATERIALLAR

Ikkita $\vec{a}$ va $\vec{b}$ vektorning *skalyar ko‘paytmasi* deb, $\vec{a}\cdot\vec{b}$ ko‘rinishda belgilanuvchi va shu vektorlar uzunliklari va ular orasidagi burchak kosinusi bilan ko‘paytmasiga teng bo‘lgan songa aytiladi:

$$\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot\cos\varphi$$

Skalyar ko‘paytmaning asosiy xossalari:

a) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (o‘rin almashtirish qonuni);

b) $\vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\,\vec{c}$ (taqsimot qonuni);

v) $(\lambda a)\cdot\vec{b}=\vec{a}=\vec{a}\left(\lambda\vec{b}\right)=\lambda\left(\vec{a}\cdot\vec{b}\right)$ (guruhlash qonuni);

g) agar $\vec{a}=0$,yoki $\vec{b}=0$, yoki $\vec{a}\perp\vec{b}$ bo‘lsa, $\vec{a}\cdot\vec{b}=0$ bo‘ladi (vektorlarning ortogonallik sharti);

d) $\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$ yoki $\vec{a}^{\,2}=\left|\vec{a}\right|^2$;

e) $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot pr_{\vec{a}}\vec{b}=\left|\vec{b}\,pr_{\vec{b}}\vec{a}\right|$;

Koordinata o‘qlari ortlarining skalyar ko‘paytmasi:

$$\vec{i}^{\,2}=1,\ \vec{j}^{\,2}=1,\ \vec{k}^{\,2}=1,\ \vec{i}\cdot\vec{j}=0,\ \vec{i}\cdot\vec{k}=0,\ \vec{j}\cdot\vec{k}=0,$$

$$\vec{a}=a_x\,\vec{i}+a_y\,\vec{j}+a_z\,\vec{k}\ va\ \vec{b}=b_x\,\vec{i}+b_y\,\vec{j}+b_z\,\vec{k}$$

vektorlar berilgan bo‘lsin. U holda:

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z;$$

$$\vec{a}^{\,2} = \left|\vec{a}\right|^2 = a_x^2 + a_y^2 + a_z^2$$

$\vec{a}$ va $\vec{b}$ vektorlar orasidagi φ burchak ushbu formula bo‘yicha hisoblanadi:

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\cdot\sqrt{b_x^2 + b_y^2 + b_z^2}}.$$

$\vec{a}$ va $\vec{b}$ vektorlarning perpendikulyarlik sharti:

$$\vec{a}\cdot\vec{b} = 0 \text{ yoki } a_x b_y + a_y b_y + a_z b_z = 0.$$

$\vec{F}$ kuch jismni l vector yo‘nalishida $\vec{BC}$ masofaga ko‘chirish natijasida bajargan ish ushbu formula bilan hisoblanadi:

$$A = \vec{F}\cdot\vec{BC} = \left|\vec{F}\right|\cdot\left|\vec{BC}\right|\cdot\cos\varphi,$$

bunda φ - ko‘chish yo‘nalishi $\vec{l}$ va $\vec{F}$ kuchning ta’sir etuvchi orasidagi burchak.

Masalan.:1) $\vec{a} = i + 3j + 3k$ va $\vec{b} = i + k$ vektorlarning skalyar ko‘paytmasi topilsin.

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z a_z = 1\cdot 1 + 3\cdot 0 + 3\cdot 1 = 1 + 3 = 4$$

2) $\vec{a} = -\vec{i} + \vec{j}$ ва $\vec{b} = \vec{i} - 2\vec{j} + 2\vec{k}$ vektorlar orasidagi burchak aniqlansin.

$$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a^2{}_x + a^2{}_y + a^2{}_z}\cdot\sqrt{b_x^2 + b_y^2 + b_z^2}} = \frac{-1-2}{\sqrt{2}\cdot\sqrt{9}} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\cos\alpha = -\frac{-1}{\sqrt{2}} \qquad \alpha = 135^0$$

3) *M* ning qanday qiymatida $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$ *ва* $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ vektorlar perpendikulyar bo‘ladi.

Yechish. Vektorlarning skalyar ko‘paytmasini topamiz:

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z a_z = m\cdot 4 + 3\cdot m - 28 = 7m - 28;$$

$\vec{a} \perp \vec{b}$ bo‘lsa, $\vec{a}\cdot\vec{b} = 0$ tengligidan *7m-28=0, m=4.* j: 4.

4) Agar $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3, \vec{a} \perp \vec{b}$ bo‘lsa, $\left(5\vec{a} + 3\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right)$ ni hisoblang.

Yechish.

$$\left(5\vec{a} + 3\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right) = 10\vec{a}^2 - 5\vec{a}\vec{b} + 6\vec{a}\vec{b} - 3\vec{b}^2 = 10\vec{a}^2 - 3\vec{b}^2 = 40 - 27 = 13.$$

3.3. MAVZU: IKKI VEKTORNING VEKTOR KO‘PAYTMASI VA UNING XOSSALARI.

MAVZUGA OID NAZARIY MATERIALLAR

$\vec{a}$ vektorning $\vec{b}$ vektorga vektor ko‘paytmasi deb $\vec{c} = \vec{a}\times\vec{b}$ ko‘rinishda belgilanuvchi va quyidagi shartlarni qanoatlantiruvchi $\vec{c}$ vektorga aytiladi:

a) $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga *perpendikulyar*:

b) $\vec{c}$ vektor uchidan qaralganda $\vec{a}$ vektordan $\vec{b}$ vektorga eng qisqa burilish soat mili yo‘nalishiga teskari yo‘nalishda kyzatiladi($\vec{a}$, $\vec{b}$, $\vec{c}$ vektorning bunday joylashuvining *o‘ng uchlik* deyiladi);

v) $\vec{c}$ vektorning moduli $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning S yuziga teng, ya'ni $|\vec{c}| = S = |\vec{a}||\vec{b}|\sin\varphi$ ($\varphi - \vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak).

Vektor ko'paytmaning asosiy xossalari:

a) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$;

b) $\left(\lambda\vec{a}\right)\times\vec{b} = \vec{a}\times\left(\lambda\vec{b}\right) = \lambda\left(\vec{a}\times\vec{b}\right)$;

v) $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$;

g) Agar $\vec{a} = \vec{0}$, yoki $\vec{b} = \vec{0}$, yoki $\vec{a} /\!/ \vec{b}$ bo'lsa, u holda $\vec{a}\times\vec{b} = \vec{0}$. Xususan $\vec{a}\times\vec{a} = \vec{0}$.

Koordinata o'qlari *ortlarining* vektor ko'paytmasi:

$$\vec{i}\times\vec{i} = \vec{0},\ \vec{j}\times\vec{j} = 0,\ \vec{k}\times\vec{k} = 0.$$
$$\vec{i}\times\vec{j} = \vec{k},\ \vec{j}\times\vec{k} = \vec{i},\ \vec{k}\times\vec{i} = \vec{j}.$$

Agar

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$
$$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

vektorlar koordinatalari bilan berilgan bo'lsa, u holda vektor ko'paytma quyidagicha topiladi:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Masalan: $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasini toping.

$$\vec{a} = 2j + k \quad va\ \vec{b} = \vec{i} + 2\vec{k}$$

$$\vec{a}\,x\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 2 & 1\\ 1 & 0 & 2\end{vmatrix}=4\vec{i}+\vec{j}-2\vec{k}\,.$$

Agar $\vec{a}$ va $\vec{b}$ vektorlar *kollinear* bo‘lsa, u holda

$$\frac{a_x}{b_x}=\frac{a_y}{b_y}=\frac{a_z}{b_z}\,.$$

$\vec{a}$ va $\vec{b}$ vektorlardan yasalgan *parallelogrammning yuzi*:

$$S=\left|\vec{a}\times\vec{b}\right|,$$

shu vektorlarda yasalgan *uchburchakning yuzi*:

$$S_{\Delta}=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

Jism A nuqtasiga qo‘yilgan $\vec{F}$ kuchning O nuqtaga nisbatan $\vec{M}$ *momenti*

$$\vec{M}=\vec{OA}\times\vec{F}$$

formula bilan hisoblanadi.

Masalan.1) $\vec{a}=2\vec{i}-3\vec{j}$ va $\vec{b}=3\vec{i}+4\vec{j}$ vektorlarga qurilgan parallelogrammning yuzini toping.

Yechish: $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning S yuzi shu vektorlar vektor ko‘paytmasining moduliga teng: $s=\left|\vec{a}\times\vec{b}\right|$.

$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasini topamiz

$$\vec a\,x\,\vec b=\begin{vmatrix}\vec i & \vec j & \vec k\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec i & \vec j & \vec k\\ 2 & -3 & 0\\ 3 & 0 & 4\end{vmatrix}=-12\vec i-8\vec j+9\vec k\,.$$

Demak, $S=\sqrt{(-12)^2+(-8)^2+9^2}=\sqrt{144+64+81}=17kv.$ birlik.

2) $\vec a=2j+k\quad va\ \vec b=\vec i+2\vec k$ vektorlardan yasalgan uchburchak yuzi topilsin.

Yechish:

$$\vec a\,x\,\vec b=\begin{vmatrix}\vec i & \vec j & \vec k\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}\vec i & \vec j & \vec k\\ 0 & 2 & 1\\ 1 & 0 & 2\end{vmatrix}=4\vec i+\vec j-2\vec k==4\vec i+\vec j-2\vec k$$

Demak ,uchburchak yuzi

$$S=\frac{|\vec a x\vec b|}{2}=\frac{\sqrt{16+1+4}}{2}=\frac{\sqrt{21}}{2}\text{j: } S=\frac{\sqrt{21}}{2}kv.birlik.$$

3) Uchlari $A(1;1;1), B(2;3;4)\ va\ C(4;3;2)$ nuqtalarda bo‘lgan uchburchak yuzasi hisoblansin.

Yechish. $\vec{AB}\ va\ \vec{AC}$ vektorlarni topamiz:

$$\vec{AB}=(2-1)\vec i+(3-1)\vec j+(4-1)\vec k=\vec i+2\vec j+3\vec k,$$
$$\vec{AC}=(4-1)\vec i+(3-1)\vec j+(2-1)\vec k=3\vec i+2\vec j+\vec k$$

$\vec{AB}\ va\ \vec{AC}$ vektorlardan yasalgan parallelogrammning yuzini yarmi uchburchakning yuziga teng, shuning uchun $\vec{AB}\ va\ \vec{AC}$ vektorlarning vektor ko‘paytmasini topamiz;

$$\vec{AB}\times\vec{AC}=\begin{vmatrix}\vec i & \vec j & \vec k\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=-4\vec i+8\vec j-4\vec k\,.$$

Bundan

$$S_{ABC}=\frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|=\frac{1}{2}\sqrt{16+64+16}=\sqrt{24}(kv.bir.)\quad \text{j:}\ \sqrt{24}kv.bir.$$

4) $\vec{a}+3\vec{b}\ va\ 3\vec{a}+\vec{b}$ vektorlardan yasalgan parallelogrammning yuzini hisoblang, agar $\left|\vec{a}\right|=\left|\vec{b}\right|=1, \left(\hat{a,b}\right)=30^0$ ga teng bo‘lsa.

Yechish. $\left(\vec{a}+3\vec{b}\right)\times\left(3\vec{a}+\vec{b}\right)=3\vec{a}\times\vec{b}+\vec{a}\times\vec{b}+9\vec{b}\times\vec{a}+3\vec{b}\times\vec{b}=$

$=3\cdot0+\vec{a}\times\vec{b}-9\vec{a}\times\vec{b}+3\cdot0=-8\vec{a}\times\vec{b}$

($\vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0,\ \vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$ ekanligidan) .

Demak,

$$S=8\left|\vec{a}\times\vec{b}\right|=8\cdot1\cdot1\cdot\sin30^0=4(kv.bir.)\ \text{j: 4 kv.birlik.}$$

3.4. MAVZU: UCHTA VEKTORNING ARALASH KO‘PAYTMASI VA UNING GEOMETRIK MA’NOSI. UCHTA VEKTORNING KOMPLANARLIK SHARTI.

MAVZUGA OID NAZARIY MATERIALLAR

Ta’rif. $\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasi deb, $\vec{a}$ vektorni $\vec{b}$ vektorga vektor ko’paytirishdan hosil bo’lgan $\vec{a}\times\vec{b}$ vektorni $\vec{c}$ vektorga skalyar ko’paytirib topilgan songa aytiladi va $\vec{a}\,\vec{b}\,\vec{c}$ kabi belgilanadi.

Agar $\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlar o‘zlarining koordinatalari bilan berilgan bo‘lsa, u holda aralash ko‘paytma quyidagicha ifodalanadi:

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\begin{vmatrix} a_ч & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}.$$

Aralash ko‘paytmaning xossalari.

a) $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=-\left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=-\left(\vec{c}\times\vec{b}\right)\cdot\vec{a}$;

b) $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{a}\,\vec{b}\,\vec{c}$;

c) $\vec{a}\,\vec{b}\,\vec{c}=\vec{b}\,\vec{c}\,\vec{a}=\vec{c}\,\vec{a}\,\vec{b}$;

d) agar vektorlardan kamida bittasi *nol vektor* yoki $\vec{a}$,$\vec{b}$, $\vec{c}$ vektorlar *komplanar* bo‘lsa, y holda $\vec{a}\,\vec{b}\,\vec{c}=0$ bo‘ladi. Yani $\vec{a},\vec{b},\vec{c}$ vektorlar *komplanar* bo‘lsa , u holda

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}=0.$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlardan yasalgan *parallelepipedning hajmi*:

$$V=\pm\vec{a}\,\vec{b}\,\vec{c}\begin{cases}+\,vektorlar\,o`ng\,bog`lam\,tashkil\,etadi,\\ -\,vektorlar\,chap\,bog`lam\,tashkil\,etadi.\end{cases}$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlardan yasalgan *piramidaning* hajmi:

$$V_{pir.}=\pm\frac{1}{6}\vec{a}\,\vec{b}\,\vec{c}$$

$\vec{a}$,$\vec{b}$ va $\vec{c}$ vektorlardan yasalgan *tetraedrning* hajmi:

$$V_{tetroed.}=\pm\frac{1}{3}\vec{a}\,\vec{b}\,\vec{c}$$

Masalan.1) Uchta vektorning aralash ko‘paytmasini toping.

$\vec{a}=2\vec{i}-3\vec{j}+4\vec{k}$; $\vec{b}=\vec{i}+4\vec{j}-5\vec{k}$ va $\vec{c}=3\vec{i}-2\vec{j}+6\vec{k}$.

Yechish:

$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}=\begin{vmatrix} 2 & -3 & 4 \\ 1 & 4 & -5 \\ 3 & -2 & 6 \end{vmatrix}=48-8+45-48-20+18=35. \qquad \text{J:35.}$$

2) Masalan. a $=-i+3j+2k, b=2i-3j-4k, c=-3i+12j+6k$ vektorlarning o‘zaro komplanar ekani ko‘rsatilsin.

Yechish:

$$abc=\begin{vmatrix} -1 & 3 & 2 \\ 2 & -3 & -4 \\ -3 & 12 & 6 \end{vmatrix}=18+48+36-18-48-36=0;$$

3) Uchlari $A(1;2;0), B(-1;2;1), C(0;-3;2)$ va $D(1;0;1)$ nuqtalarda bo‘lgan piramidaning hajmini hisoblang.

Yechish. Piramidaning A uchidan chiqqan qirralariga mos keluvchi vektorlarni topamiz:

$$\vec{AB}=\{-2;0;1\}, \vec{AC}=\{-1;-5;2\}, \vec{AD}=\{0;-2;1\}.$$

Piramidaning hajmi shu vektorlarga qurilgan parallelepiped hajmining $\frac{1}{6}$ qismiga teng bo‘lganligi sababli

$$V=\pm\frac{1}{6}\begin{vmatrix} -2 & 0 & 1 \\ -1 & -5 & 2 \\ 0 & -2 & 1 \end{vmatrix}=\frac{1}{6}\cdot 4=\frac{2}{3}kub.birlik.$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. Ushbu amallarni bajaring.

1. $\vec{i}\times\left(\vec{j}+\vec{k}\right)-\vec{j}\times\left(\vec{i}+\vec{k}\right)+\vec{k}\times\left(\vec{i}\times\vec{j}+\vec{k}\right)$ $J: 2(k-i)$

2. $\left(\vec{a}+\vec{b}+\vec{c}\right)\times\vec{c}+\left(\vec{a}+\vec{b}+\vec{c}\right)\times\vec{b}+\left(\vec{b}-\vec{c}\right)\times\vec{a}$ $J: 2a\times c$

3. $\left(2\vec{a}+\vec{b}\right)\times\left(\vec{c}-\vec{a}\right)+\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}\right)$ $J: a\times c$

4. $2\vec{i}\cdot\left(\vec{j}\times\vec{k}\right)+3\vec{j}\cdot\left(\vec{i}\times\vec{k}\right)+4\vec{k}\left(\vec{i}\times\vec{j}\right)$ $J:\ 3$

2. *A(1;2;0), B(3;0;-3), C(5;2;6)* nuqtalar berilgan *ABC* uchburchakning yuzini toping.

3. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\vec{a}\times\vec{b}$ ekani isbotlansin. J: 14 kv.bir.

4. Uchburchakning uchlari *A(1;-2;8), B(0;0;4), C(6;2;0)* bo‘lsin. Uning *BD* balandligi va yuzasi topilsin. J: $S=7\sqrt{5};\ |BD|=\dfrac{2\sqrt{21}}{3}$

5. $\vec{a}=\{1;1;0\}$, $\vec{b}=\{1;-2;2\}$ vektorlar orasidagi burchak topilsin. J: 135^0.

6. $\vec{a}=\{-1;3;2\}$ $\vec{b}=\{2;-3;4\}$ $\vec{c}=\{-3;12;6\}$ vektorlarni komplanarligini tekshiring. J: $\vec{a}\,\vec{b}\,\vec{c}=24$

7. *A(1;2;-1), B(0;1;5), C(-1;2;1)* va *D(2;1;3)* nuqtalar bir tekislikda yotishini isbotlang. J: Bir tekislikda yotadi, $\vec{a}\,\vec{b}\,\vec{c}=0$.

8. Uchlari *A(2;-1;1), B(5;54), C(-1;2;1)* va *D(2;1;3)* nuqtalarda bo‘lgan tetraedrning hajmini hisoblang. J: *V=12 kv.bir*.

9. $\vec{a}$ *={3;-1;-2}* va $\vec{b}$ *={1;2;-1}* vektorlar berilgan. 1) $\vec{a}\times\vec{b}$ 2) $\left(2\vec{a}+\vec{b}\right)\times\vec{b}$ vektor ko‘paytmaning koordinatalarini toping. J: *1) {5;1;7} 2) {10;2;14}.*

10. $\left(\vec{a}+\vec{b}\right)^2+\left(\vec{a}-\vec{b}\right)^2=2\left(\vec{a}^{\,2}+\vec{b}^{\,2}\right)$ ayniyatni isbotlang va uning geometric ma’nosini ifodalang.

11. 1.; $\vec{a} = \{2;-1;3\}$ $\vec{b} = \{-6;3;-9\}$ vektorlarning kollinearligini isbotlang va ularning uzunliklarini va yo'naltiruvchi kosinuslarini toping.

$$\text{J:1.} \left|\vec{a}\right| = \sqrt{14},\ \cos\alpha = \frac{\sqrt{14}}{7};\cos\beta = -\frac{\sqrt{14}}{14};\ \cos\gamma = \frac{3\sqrt{14}}{14}.$$

$$\left|\vec{b}\right| = \sqrt{126},\ \cos\alpha = -\frac{\sqrt{126}}{21};\cos\beta = \frac{\sqrt{126}}{42};\ \cos\gamma = -\frac{3\sqrt{126}}{42}.$$

12. *A(2;2;0)* va *B(0;-2;5)* nuqtalar berilgan. $\vec{AB} = u$ vektor yasalsin hamda uning uzunligi va yo'nalishi aniqlansin.

$$\text{J: } \vec{AB} = \{-2;-4;5\}, \left|\vec{AB}\right| = 3\sqrt{5},\ \cos\alpha = -\frac{2\sqrt{5}}{15};\ \cos\beta = -\frac{4\sqrt{5}}{15};\ \cos\gamma = \frac{\sqrt{5}}{3}$$

MUSTAQIL YECHISH UCHUN MASHQLAR.

A,B va C nuqtalarning koordinatalari berilgan.

a) $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak kosinusini;

b) $\alpha\vec{a} + \beta\vec{b}$ vektorning $\vec{a}$ vektor yo'nalishidagi proeksiyasini toping:

1.1. $A(9;10;1),\ B(7;6;-1),\ C(4;0;-4),$
$\vec{a} = 2\vec{AB} - 3\vec{AC}, \vec{b} = 4\vec{BC} + \vec{AC}; \alpha = 1, \beta = 2.$

1.2. $A(0;2;1),\ B(1;2;0),\ C(0;3;-1),$
$\vec{a} = 3\vec{AC} - 3\vec{BC}, \vec{b} = 2\vec{AB} + 5\vec{BC}; \alpha = -1, \beta = 2.$

1.3. $A(0;4;8),\ B(-5;4;-2),\ C(-1;4;1),$
$\vec{a} = \vec{AB} - 4\vec{AC}, \vec{b} = 4\vec{AC} + \vec{AB}; \alpha = -2, \beta = 3.$

1.4. $A(3;0;1),\ B(-2;3;2),\ C(1;1;-2),$
$\vec{a} = \vec{BC} - 3\vec{AB}, \vec{b} = 6\vec{BC} + 5\vec{AC}; \alpha = 2, \beta = -3.$

1.5. $A(4;1;-3),\ B(5;1;-2),\ C(-1;3;3),$
$\vec{a} = 4\,\vec{AC} - 2\vec{CB},\ \vec{b} = 7\,\vec{AB} + 5\vec{BC};\ \alpha = \beta = 3.$

1.6. $A(4;1;1),\ B(3;1;2),\ C(0;1;-2),$
$\vec{a} = 3\,\vec{BC} - 4\,\vec{CA},\ \vec{b} = 6\,\vec{BA} - \vec{AC};\ \alpha = 3, \beta = 2.$

1.7. $A(-3;4;-5),\ B(0;1;-2),\ C(-1;2;3),$
$\vec{a} = 4\,\vec{AB} - 3\,\vec{BC},\ \vec{b} = 2\,5\vec{CA} - 2\,\vec{BA};\ \alpha = -2, \beta = 5.$

1.8. $A(7;5;-2),\ B(6;0;0),\ C(7;2;2),$
$\vec{a} = \vec{AB} - 3\,\vec{BC},\ \vec{b} = 2\,\vec{CB} + 5\vec{AC};\ \alpha = -4, \beta = 2.$

1.9. $A(-3;-7;-3),\ B(-1;-3;-1),\ C(2;3;2),$
$\vec{a} = 2\vec{BC} - 5\,\vec{AB},\ \vec{b} = 5\,\vec{AC} - 5\vec{CB};\ \alpha = -3, \beta = 1.$

1.10. $A(2;-1;8),\ B(3;1;7),\ C(2;0;7),$
$\vec{a} = \vec{AB} - 3\,\vec{BC},\ \vec{b} = 2\,\vec{CB} - 2\,\vec{AC};\ \alpha = 5, \beta = 6.$

1.11. $A(-1;-1;8),\ B(4;-1;-2),\ C(0;-1;1),$
$\vec{a} = 6\vec{BC} - 3\,2\vec{AB},\ \vec{b} = 2\,\vec{AC} + 5\vec{AB};\ \alpha = -4, \beta = 3.$

1.12. $A(-2;4;-2),\ B(3;1;0),\ C(0;3;-4),$
$\vec{a} = 3\,\vec{AB} - 4\,\vec{AC},\ \vec{b} = 2\,\vec{BC} + 5\vec{CA};\ \alpha = 3, \beta = -6.$

1.13. $A(1;1;4),\ B(-2;1;5),\ C(-1;3;3),$
$\vec{a} = 4\,\vec{AC} - 2\,\vec{BC},\ \vec{b} = 2\,\vec{AC} + 3\vec{AB};\ \alpha = -5, \beta = 3.$

1.14. $A(4;2;6),\ B(2;2;8),\ C(-4;2;0),$
$\vec{a} = 3\,5\vec{AB} - 7\,\vec{AC},\ \vec{b} = 2\,\vec{BC} + 3\vec{BA};\ \alpha = 9, \beta = 12.$

1.15. $A(15;-12;0),\ B(6;-3;0),\ C(9;-6;3),$
$\vec{a} = \vec{AC} - 6\,\vec{BC},\ \vec{b} = \vec{AB} + 3\vec{BC};\ \alpha = -7, \beta = 6.$

1.16. $A(-1;-5;-2),\ B(0;-6;4),\ C(-1;-8;2),$
$\vec{a} = 3\,\vec{BC} + 5\,\vec{AB},\ \vec{b} = \vec{AC} -+ 3\vec{AB};\ \alpha = -3, \beta = 4.$

1.17. $A(-1;-10;-5),\ B(1;-6;-3),\ C(0;0;4),$
$\vec{a} = 2\,\vec{BC} - 3\,\vec{AC}, \vec{b} = 4\,\vec{AB} + 3\vec{BC}; \alpha = 4, \beta = -6.$

1.18. $A(-3;3;7),\ B(-2;3;6),\ C(-3;2;6),$
$\vec{a} = 4\,\vec{AB} - \vec{AC}, \vec{b} = 2\,\vec{BC} - 3\,\vec{BA}; \alpha = -3, \beta = 8.$

1.19. $A(2;-2;-8),\ B(5;-2;-4),\ C(1;-2;-1),$
$\vec{a} = 5\,\vec{AB} - 3\,\vec{BC}, \vec{b} = 4\,\vec{CA} + \vec{AB}; \alpha = -4, \beta = 1.$

1.20. $A(1;2;4),\ B(-4;1;-6),\ C(-1;1;2),$
$\vec{a} = 3\vec{CA} - 2\,\vec{BC}, \vec{b} = 2\,\vec{AB} + 4\vec{BC}; \alpha = 3, \beta = -5.$

1.21. $A(1;1;4),\ B(-2;5;1),\ C(-1;3;3),$
$\vec{a} = 3\,\vec{AB} + \vec{AC}, \vec{b} = 2\,\vec{BC} + -3\vec{AB}; \alpha = 3, \beta = -4.$

1.22. $A(0;1;-2),\ B(3;1;2),\ C(4;1;1),$
$\vec{a} = 2\,\vec{AC} - 3\,\vec{BA}, \vec{b} = 3\,\vec{BC} - 4\,\vec{AB}; \alpha = -2, \beta = 6.$

1.23. $A(6;-8;10),\ B(0;-2;4),\ C(2;-4;6),$
$\vec{a} = 3\,\vec{AB} + 6\,\vec{BC}, \vec{b} = 2\,\vec{AC} - 5\vec{BC}; \alpha = 2, \beta = 8.$

1.24. $A(0;3;2),\ B(-2;-1;0),\ C(-5;-7;-3),$
$\vec{a} = 5\,\vec{BC} - 2\,\vec{CA}, \vec{b} = 6\,\vec{AB} + 4\,\vec{AC}; \alpha = -2, \beta = 5.$

1.25. $A(-1;4;6),\ B(0;2;5),\ C(-1;3;5),$
$\vec{a} = 8\vec{AC} - 4\,\vec{AB}, \vec{b} = 2\,\vec{BC} - 6\,\vec{AB}; \alpha = -3, \beta = -4.$

1.26. $A(1;-2;3),\ B(4;-2;-1),\ C(0;-2;4),$
$\vec{a} = 3\,2\vec{AC} - 3\,\vec{BC}, \vec{b} = 3\,\vec{AB} - 4\,\vec{BC}; \alpha = 2, \beta = 1.$

1.27. $A(-1;1;1),\ B(-6;4;3),\ C(-3;2;-1),$
$\vec{a} = \vec{AB} - 3\,\vec{BC}, \vec{b} = \vec{AC} + \vec{BC}; \alpha = 4, \beta = -6.$

1.28. $A(1;1;4),\ B(-2;5;5),\ C(-1;3;3),$
$\vec{a} = 2\,\vec{AC} - 3\,\vec{BC}, \vec{b} = 2\,\vec{AB} + \vec{BC}; \alpha = -2, \beta = 6.$

1.29. $A(-3;-2;-1),\ B(-1;-2;0),\ C(0;-1;-1),$
$\vec{a} = 3\vec{BC} - 4\vec{BC}, \vec{b} = 2\vec{AC} + 3\vec{BC}; \alpha = -6, \beta = 4.$

1.30. $A(5;-4;3),\ B(2;-1;0),\ C(3;-2;1),$
$\vec{a} = \vec{BC} + \vec{AC}, \vec{b} = 2\vec{AB} - 3\vec{CA}; \alpha = -5, \beta = 3.$

ABCD piramidaning uchlari berilgan.

a) piramidani berilgan qirralari orasidagi burchak kosinusini toping;

b) piramidaning berilgan yog‘i yuzini toping:

2.1. $A(6;-4;1),\ B(6;3;-1),\ C(2;5;7),\ D(-4;-2;3);$
$a)\ AB\ va\ AC;\ b)\ ACD$

2.2. $A(6;4;-7),\ B(-5;-4;2),\ C(5;7;-4),\ D(4;2;3);$
$a)\ BC\ va\ BD;\ b)\ ACD$

2.3. $A(-2;87),\ B(6;-2;-3),\ C(8;2;-3),\ D(3;5;3);$
$a)\ CA\ va\ CD;\ b)\ BAD$

2.4. $A(4;4;3),\ B(2;-4;5),\ C(-1;3;-4),\ D(4;-7;-9);$
$a)\ DA\ va\ DB;\ b)\ DAC$

2.5. $A(-5;-3;2),\ B(4;-2;-4),\ C(5;7;2),\ D(1;3;4);$
$a)\ AB\ va\ AD;\ b)\ CBD$

2.6. $A(-5;6;4),\ B(-6;2;4),\ C(9;-5;3),\ D(7;2;-8);$
$a)\ BC\ va\ BA;\ b)\ DAC$

2.7. $A(1;-9;7),\ B(3;-5;1),\ C(-9;3;-5),\ D(2;4;7);$
$a)\ CB\ va\ CD;\ b)\ ABD$

2.8. $A(4;-2;9),\ B(3;5;-1),\ C(5;1;7),\ D(-6;-3;5);$
$a)\ DA\ va\ DC;\ b)\ ABC$

2.10. $A(2;-5;1),\ B(3;-6;-7),\ C(-9;-6;7),\ D(7;2;5);$
$a)\ BD\ va\ BA;\ b)\ CAD$

2.11. $A(2;-5;-3),\ B(9;7;3),\ C(8;7;1),\ D(-2;-1;7);$
$a)\ CA\ va\ CB;\ \ b)\ ABD$

2.12. $A(67;4;3),\ B(0;-4;8),\ C(-3;1;5),\ D(-5;-6;-7);$
$a)\ DB\ va\ DC;\ \ b)\ ABC$

2.13. $A(-9;2;6),\ B(-7;2;3),\ C(5;-6;-4),\ D(4;-4;5);$
$a)\ AB\ va\ AC;\ \ b)\ DBC$

2.14. $A(-3;0;4),\ B(8;-6;5),\ C(4;-4;-3),\ D(6;3;5);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

2.15. $A(-3;8;2),\ B(-8;2;4),\ C(3;-7;5),\ D(5;4;-6);$
$a)\ CA\ va\ CD;\ \ b)\ BCD$

2.16. $A(5;-3;9),\ B(8;-5;1),\ C(-7;5;-3),\ D(4;2;5);$
$a)\ DA\ va\ DC;\ \ b)\ BAC$

2.17. $A(5;-1;6),\ B(-6;7;5),\ C(2;5;7),\ D(2;1;3);$
$a)\ AC\ va\ AD;\ \ b)\ BCD$

2.18. $A(1;2;3),\ B(3;-3;2),\ C(7;-5;4),\ D(-3;-7;-4);$
$a)\ BD\ va\ BA;\ \ b)\ CAD$

2.19. $A(4;-3;1),\ B(0;-3;5),\ C(-3;-2;1),\ D(9;4;7);$
$a)\ CA\ va\ CB;\ \ b)\ ABD$

2.20. $A(5;-4;-2),\ B(7;5;1),\ C(3;2;-4),\ D(-2;-5;3);$
$a)\ DB\ va\ DC;\ \ b)\ ABC$

2.21. $A(-7;2;3),\ B(0;-2;6),\ C(-1;3;7),\ D(-3;-4;-5);$
$a)\ AB\ va\ AD;\ \ b)\ CBD$

2.22. $A(-7;6;4),\ B(-4;1;1),\ C(3;-2;6),\ D(6;-2;3);$
$a)\ BC\ va\ BA;\ \ b)\ ACD$

2.23. $A(4;1;5),\ B(5;-3;2),\ C(3;-5;-4),\ D(8;5;7);$
$a)\ DA\ va\ DC;\ \ b)\ ABD$

2.24. $A(-5;4;2),\ B(-4;6;2),\ C(1;-5;3),\ D(3;6;-4);$
$a)\ DB\ va\ DC;\ \ b)\ BAC$

2.25. $A(3;-5;6;),\ B(6;-3;4),\ C(-5;3;-2),\ D(2;4;3);$
$a)\ AB\ va\ AC;\ \ b)\ DBC$

2.26. $A(4;-2;8),\ B(-2;2;3),\ C(6;4;1),\ D(-4;;-3;-5);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

2.27. $A(-3;2;4;),\ B(-2;5;3),\ C(6;4;1),\ D(4;;-2;-3);$
$a)\ CA\ va\ CD;\ \ b)\ BAD$

2.28. $A(-4;4;3),\ B(4;-3;-2),\ C(6;4;-1),\ D(1;3;1);$
$a)\ DA\ va\ DB;\ \ b)\ CAB$

2.29. $A(2;2;1),\ B(4;-2;3),\ C(-3;5;-2),\ D(6;;5;-7);$
$a)\ AC\ va\ AD;\ \ b)\ BCD$

2.30. $A(-3;-6;3),\ B(6;-3;-2),\ C(1;2;1),\ D(5;4;-3);$
$a)\ BC\ va\ BD;\ \ b)\ ACD$

Piramidaning uchlari A,B,C,D berilgan.

a) ko‘rsatilgan yoq yuzini;b) piramidaning *l* qirrasi va berilgan ikkita uchidan o‘tuvchi kesim yuzini; v) piramidaning hajmini hisoblang:

3.1. $A(5;-4;3),\ B(2;-1;0),\ C(3;-2;1),\ D(0;2;1)$
$a)\ ABC;\ b)\ l = AD,\ B\ va\ C.$

3.2. $A(0;1;2),\ B(1;-2;2),\ C(-1;2;1),\ D(2;0;1)$
$a)\ BCD;\ b)\ l = BA,\ D\ va\ C.$

3.3. $A(-5;-4;3),\ B(6;-1;2),\ C(1;0;1),\ D(0;2;1)$
$a)\ ACD;\ b)\ l = CB,\ A\ va\ D.$

3.4. $A(2;-1;1),\ B(-3;0;-6),\ C(-5;3;-2),\ D(-1;10;3)$
$a)\ ABD;\ b)\ l = CD,\ A\ va\ B.$

3.5. $A(1;-3;7),\ B(-1;0;3),\ C(-4;-2;1),\ D(4;2;-1)$
$a)\ ABD;\ b)\ l = BD,\ A\ va\ C.$

3.6. $A(-4;1;3),\ B(5;-1;2),\ C(2;1;-4),\ D(1;-3;0)$
$a)\ BCD;\ b)\ l = AC,\ A\ va\ C.$

3.7. $A(5;3;-4),\ B(1;0;3),\ C(2;-1;4),\ D(0;3;1)$
$a)\ ACD;\ b)\ l = AB, C\ va\ D.$

3.8. $A(3;7;-4),\ B-4(-4;1;3),\ C(2;3;0),\ D(-1;-1;-2)$
$a)\ ABD;\ b)\ l = BC, A\ va\ D.$

3.9. $A(-8;2;-5),\ B(-1;-3;0),\ C(-4;1;2),\ D(6;-5;-3)$
$a)\ ABC;\ b)\ l = BC,\ C\ va\ D.$

3.10. $A(7;-10;-3),\ B(3;-3;-1),\ C(0;-6;5),\ D(-3;-4;2)$
$a)\ BCD;\ b)\ l = AD, B\ va\ C.$

3.11. $A(-3;6;-4),\ B(1;0;-1),\ C(1;2;2),\ D(6;3;1)$
$a)\ ACD;\ b)\ l = BD, A\ va\ C.$

3.12. $A(-4;2;-5),\ B(8;5;-10),\ C(0;-3;2),\ D(6;2;-4)$
$a)\ ABD;\ b)\ l = AC, B\ va\ D.$

3.13. $A(1;2;-4),\ B(1;1;3),\ C(-2;-1;7),\ D(4;2;7)$
$a)\ ABC;\ b)\ l = AD, B\ va\ C.$

3.14. $A(6;-3;-6),\ B(2;-3;-7),\ C(2;5;-1),\ D(4;1;2)$
$a)\ BCD;\ b)\ l = AB, C\ va\ D.$

3.15. $A(7;6;-10),\ B(-3;6;3),\ C(-3;0;-6),\ D(2;-5;-1)$
$a)\ ACD;\ b)\ l = CB, A\ va\ D.$

3.16. $A(3;-6;-1),\ B(-9;-5;1),\ C(5;3;-2),\ D(-1;-1;0)$
$a)\ ABD;\ b)\ l = CD, A\ va\ B.$

3.17. $A(1;1;-1),\ B(4;2;1), C(0;5;2),\ D(0;2;5)$
$a)\ ABC;\ b)\ l = BD, A\ va\ C.$

3.18. $A(-7;9;-10),\ B(-6;0;5),\ C(1;2;1),\ D(-2;-1;2)$
$a)\ BCD;\ b)\ l = AC, B\ va\ D.$

3.19. $A(6;-4;1),\ B(-4;-8;4),\ C(1;7;-1),\ D(-4;0;-2)$
$a)\ ACD;\ b)\ l = AB, C\ va\ D.$

3.20. $A(-1;2;-2),\ B(-3;-6;-2),\ C(2;-3;-5),\ D(5;4;14)$
$a)\ ABD;\ b)\ l = CB,\ A\ va\ D.$

3.21. $A(-9;4;8),\ B(6;2;5),\ C(-3;0;3),\ D(0;2;1)$
$a)\ ABC;\ b)\ l = CD,\ A\ va\ B.$

3.22. $A(5;2;-4),\ B(1;2;3),\ C(-1;2;1),\ D(2;-1;2)$
$a)\ BCD;\ b)\ l = AD,\ B\ va\ C.$

3.23. $A(-2;0;-1),\ B(4;-2;2),\ C(3;1;-1),\ D(2;1;1)$
$a)\ ACD;\ b)\ l = BD,\ A\ va\ C.$

3.24. $A(-3;5;7),\ B(7;3;6;),\ C(-2;1;4),\ D(1;3;2)$
$a)\ ABD;\ b)\ l = AC,\ B\ va\ D.$

3.25. $A(-8;9;5),\ B(1;2;3),\ C(2;3;1),\ D(-1;1;1)$
$a)\ ABD;\ b)\ l = AD,\ B\ va\ C.$

3.26. $A(-12;8;-4),\ B(3;7;-2),\ C(3;6;-3),\ D(-7;5;1)$
$a)\ BCD;\ b)\ l = AB,\ C\ va\ D.$

3.27. $A(4;5;2),\ B(0;-2;-5),\ C(-4;5;1),\ D(-7;4;-3)$
$a)\ ACD;\ b)\ l = CB,\ A\ va\ D.$

3.28. $A(5;4;3),\ B(-2;1;2),\ C(0;-1;4),\ D(-3;2;-1)$
$a)\ ABD;\ b)\ l = CD,\ A\ va\ D.$

3.29. $A(-6;2;8),\ B(1;-5;0),\ C(0;1;-2),\ D(3;-1;4)$
$a)\ ABC;\ b)\ l = BD,\ A\ va\ C.$

3.30. $A(-4;-2;2),\ B(-1;1;2),\ C(3;0;-2),\ D(1;-1;1)$
$a)\ BCD;\ b)\ l = AC,\ B\ va\ D.$

4-BOB. TEKISLIKDA ANALITIK GEOMETRIYA

4.1. MAVZU: TEKISLIKDAGI TO‘G‘RI CHIZIQ TENGLAMALARI

MAVZUGA OID NAZARIY MATERIALLAR

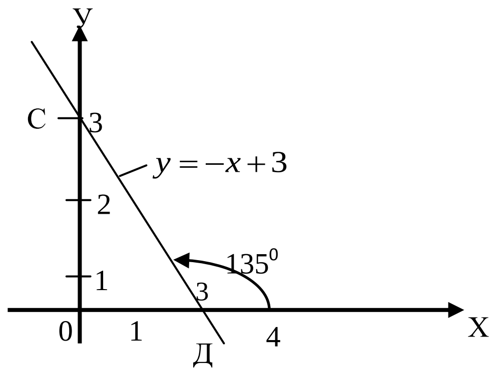
1. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi.

$$y = kx + b \qquad (1)$$

Bu yerda $k = tg\varphi$ - to‘g‘ri chiziqning burchak koeffitsiyenti; φ –to‘g‘ri chiziqning og‘sh burchagi; b - to‘g‘ri chiziqning OY o‘qida ajratgan kesmasi.

Masalan: *OY* o‘qi bilan $\varphi = 135^0$ burchak tashkil qiluvchi va *OY* o‘qini *(0;3)* nuqtada kesib o‘tuvchi to‘g‘ri chiziq tenglamasi tuzilsin va grafigi yasalsin.

Yechish: $k = tg(135^0) = -1,\ b = 3.$ (1)-formuladan $y = -x + 3$ ni topamiz. *x=0* bo‘lsa *y=3*, *y=0* bo‘lsa $x = 3$



$Д(3;\ 0)$
$C(0;3)$

1-chizma

2. To‘g‘ri chiziqning umumiy tenglamasi

$$Ax + By + C = 0, \quad A^2 + B^2 \neq 0 \qquad (2)$$

a) *C=0*; $A \neq 0; B \neq 0$ bo‘lsa, *Ax+By=0* to‘g‘ri chiziq koordinata boshidan o‘tadi;

b) $A = 0; B \neq 0; C \neq 0$ bo‘lsa , *By+C=0* to‘g‘ri chiziq *OX* o‘qiga parallel;

v) $B = 0; A \neq 0; C \neq 0$ bo‘lsa, *Ax+C=0* to‘g‘ri chiziq *OY* o‘qiga parallel;

g) $B = C = 0; A \neq 0$ bo‘lsa, Ax=0 to‘g‘ri chiziq *OY* o‘qidan iborat bo‘ladi

d) $A = C = 0; B \neq 0$ bo‘lsa, By=0 to‘g‘ri chiziq OX o‘qidan iborat bo‘ladi.

3. To‘g‘ri chiziqning kesmalarga nisbatan tenglamasi.

$Ax+By+C=0,$ tenglamada C ni tenglamaning o‘ng tomoniga o‘tkazaylik, ya’ni $Ax+By=-C.$ Bundan $-\frac{A}{C}x-\frac{B}{C}y=1$ ni hosil qilish mumkin. Bu yerda $-C/A=m$ va $-C/B=n$ deb belgilasak $\frac{x}{m}+\frac{y}{n}=1$ ni hosil qilamiz.

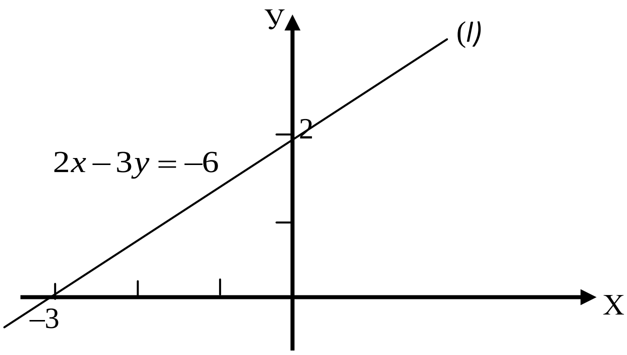
Masalan. $2x-3y+6=0$ to‘g‘ri chiziq tenglamasini kesmalarga nisbatan yozing va yasang.

Yechish:

$$2x-3y=-6 \quad \Big|:(-6)$$

$$-\frac{2}{6}x+\frac{3}{6}y=1$$

$$\frac{x}{-3}+\frac{y}{2}=1$$



2-chizma

4. Ikki to‘g‘ri chiziq orasidagi burchak.

Tenglamalari bilan berilgan l_1 va l_2 to‘g‘ri chiziqlarni olaylik:

$$l_1 : y=k_1x + b_1$$

$$l_2 : y=k_2x + b_2$$

To‘g‘ri chiziqlar orasidagi burchak:

$$\mathrm{tg}\varphi=\frac{k_1-k_2}{1+k_1*k_2} \qquad (3)$$

To‘g‘ri chiziqlarning parallellik sharti:

$$k_1=k_2 \qquad (4)$$

To‘g‘ri chiziqlarning o‘zaro perpendikulyarlik sharti:

$$k_2 = -\frac{1}{k_1} \qquad (5)$$

Masalan. $y = 2x + 1$ va $x - y - 2 = 0$ to‘g‘ri chiziqlar orasidagi burchakni toping.

Yechish:

l_1: $y=2x+1$, $k_1=2$

l_2: $x-y-2=0$.

l_2: $y=x-2$, $k_2=1$

Burchakni topamiz:

$$tg\varphi = \frac{2-1}{1+2*1} = \frac{1}{3} \qquad \varphi = arctg\frac{1}{3} \approx 18{,}5^0$$

5. Berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqlar tenglamasi

$$y-y_o=k(x-x_0) \qquad (6)$$

bu yerda x_0, y_0 – berilgan nuqtaning koordinatalari.

Masalan. $y=3x-4$ to‘g‘ri chiziqqa perpendikulyar bo‘lib $M(2;-3)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

Yechish. Izlanayotgan to‘g‘ri chiziqning burchak koeffitsiyentini to‘g‘ri chiziqlarning perpendikulyarlik (5) shartidan foydalanib topamiz:

Berilgan to‘g‘ri chiziqning burchak koeffitsiyenti $k_1=3$ ga tengligidan izlanayotgan to‘g‘ri chiziqning burchak koeffitsiyenti $k_2 = -\frac{1}{3}$ bo‘ladi.

Ularni (6) tenglamaga qo‘yamiz:

$$y+3= -\frac{1}{3}(x-2)$$

$$3y+9=-x+2$$

$$x+3y+7=0$$

Javob: $x+3y+7=0$

6. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.

$M_1(x_1;y_1)$ va $M_2(x_2;y_2)$ nuqtalar orqali o’tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (7)$$

Masalan. $M_1(4;\ -2)$ va $M_2(3;\ -1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

Yechish. Berilgan nuqtalarning koordinatalarini (7) tenglamaga qo‘yamiz:

$$\frac{y+2}{-1+2}=\frac{x-4}{3-4}, \qquad \frac{y+2}{1}=\frac{x-4}{-1},$$

Bundan $y=-x+2$. Javob: $y=-x+2$.

7. To‘g‘ri chiziqlar orasidagi burchaklar bissektrisalari tenglamasi.

$A_1x+B_1y+C=0$ va $A_2x+B_2y+C=0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasi formulasi quyidagicha.

$$\frac{A_1x+B_1y+C}{\sqrt{A_1^2+B_1^2}}=\pm\frac{A_2x+B_2y+C}{\sqrt{A_2^2+B_2^2}}$$

Masalan: $x+y-5=0$ va $7x-y-19=0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasini tuzing.

Yechish:
$$\frac{x+y-5}{\sqrt{1+1}}=\pm\frac{7x-y-19}{\sqrt{49+1}}$$

bundan, $5(x+y-5)\pm(7x-y-19)=0$

$5(x+y-5)+(7x-y-19)=0$,$3x+y-11=0$, $5(x+y-5)-(7x-y-19)=0$, $x-3y+3=0$.

8. To‘g‘ri chiziqning normal tenglamasi.

$$Ax+By+C=0 \qquad (8)$$

$$\mu = \pm \frac{1}{\sqrt{A^2 + B^2}}$$

To‘g‘ri chiziqning umumiy (8) tenglamasini normallashtirish uchun bu tenglamani $\mu = \pm \frac{1}{\sqrt{A^2 + B^2}}$ soniga ko‘paytirish yetarli bo‘lib, uning ishorasini tenglamadagi ozod had C ning ishorasiga qarama-qarshi qilib olish lozim.

Masalan. *12x-5y-65=0* to‘g‘ri chiziqning normal tenglamasi tuzilsin.

Yechish:

$$\mu = \frac{1}{\sqrt{12^2 + (-5)^2}} = \frac{1}{13}$$

$$\frac{12}{13}x - \frac{5}{13}y - 5 = 0$$

$$\cos\varphi = \frac{12}{13}, \sin\varphi = -\frac{5}{13}, p = 5$$

9. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa.

$M(x_0; y_0)$ nuqtadan $Ax+By+C=0$ to‘g‘ri chiziqqacha bo‘lgan masofa *(d)* ushbu formula yordamida topiladi:

$$d = \left| \frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}} \right|$$

Masalan. *M(3; -1)* nuqtadan *3x+4y-10=0* to‘g‘ri chiziqqacha bo‘lgan masofani toping.

Yechish:

$$d = \left| \frac{3 \cdot 3 + 4 \cdot (-1) - 10}{\sqrt{3^2 + 4^2}} \right| = \left| \frac{9 - 4 - 10}{\sqrt{25}} \right| = \left| \frac{-5}{5} \right| = 1$$

10. Ikki to‘g‘ri chiziqning kesishish nuqtasi .

$$\begin{cases} A_1 x + B_1 y = C_1 \\ A_2 x + B_2 y = C_2 \end{cases}$$

$$x=\frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} \quad \text{va} \quad y=\frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1-6. Nuqtalar orasidagi masofani toping.

1. $(1;1)$,$(4;5)$ J:5 2. $(1;-3)$,$(5;7)$ J: $2\sqrt{29}$ 3. $(6;-2)$, $(-1;3)$ J: $\sqrt{74}$

4. $(1;-6)$,$(-1;-3)$ J: $\sqrt{13}$ 5. $(2;5)$,$(4;-7)$ J: $2\sqrt{37}$ 6. $(a;b)$, $(b;a)$ J: $\sqrt{2}\,(a-b)$

7. $A(-1;3)$, $B(3;11)$ va $C(5;15)$ nuqtalar berilgan.

$|AB|+|BC|=|AC|$ ayniyatni isbotlang.

8. Funksiyalarning burchak koeffitsiyentli tenglamasi tuzilsin:

a) $x+3y=0$, J: $y=-\frac{1}{3}x$ b) $2x-5y=0$, J: $y=\frac{2}{5}x$ c) $y-2+x=0$, J: $y=2-x$

d) $2x-3y+6=0$ J: $y=\frac{2}{3}x+2$ *e)* $3x-4y=12$ J: $y=\frac{3}{4}x-3$

f) $4x+5y=10$ J: $y=2-\frac{4}{5}x$

9. Burchak koeffitsiyenti $k=\frac{2}{5}$; *OY* o‘qini *b=4* kesmada kesib o‘tuvchi to‘g‘ri chiziq tenglamasi tuzilsin. J: $2x-5y+20=0$

10. *OY* o‘qidan $b=4$ kesma ajratib *OX* o‘qi bilan 135^0 burchak tashkil etuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing. J: $x+y-4=0$

11. *OY* o‘qidan $b=-2$ kesma ajratib *OX* o‘qi bilan 60^0 burchak tashkil etuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing. J: $\sqrt{3}x-y-2=0$

12. Koordinatalar boshidan o‘tib, *OY* o‘qi bilan: 1). 45^0, 2). 120^0, 3). 60^0, 4). 90^0 burchak tashkil etuvchi to‘g‘ri chiziqlarni yasang va ularning tenglamalarini yozing.

J:1) *y=x*, 2) $y=-\sqrt{3}x$ 3) $y=\sqrt{3}x$ 4) $x=0$

13. 1) $3x+5y+15=0$; 2) $3x+2y=0$; 3) $y=-2$; 4) $\frac{x}{4}+\frac{y}{4}=1$ to‘g‘ri chiziqlar uchun k va b parametrlarni aniqlang.

J: 1) $k=-\frac{3}{5};b=-3.$ 2) $k=-\frac{3}{2};b=0.$ 3) $k=0;b=-2.$ 4) $k=-1;b=4.$

14. *A* (2; 3) nuqtadan o `tib, *OX* o‘qi bilan 45^0 burchak hosil qiluvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing. J: $x-y+1=0$

15. *5x + 2y + 6 = 0* va *x + y – 6 = 0* to‘g‘ri chiziqlarning burchak koeffitsiyentli tenglamasi tuzilsin. J: $y=-\frac{5}{2}x-3$; $y=6-x$

16. Abssissa o‘qidan kesgan kesmasi 3 ga, ordinata o‘qidan kesgan kesmasi 1 ga teng bo‘lgan to‘g‘ri chiziq tenglamasining burchak koeffitsiyentini toping .

J: $y=1-\frac{1}{3}x$

17. Abssissa o‘qini 1, ordinata o‘qini -2 nuqtada kesib o‘tadigan to‘g‘ri chiziq tenglamasi tuzilsin. J: $y=2x-2$

18. Quyidagi chiziqlarni *OY* o‘qi bilan kesishgan nuqtaning koordinatasi aniqlansin:

a) $x+3y=0$ b) $2x-5y=0$ c) $y=-2$ d) $2x-3y+6=0$ e) $3x-4y=12$ f) $4x+5y=10$

J: *a)y=0;x=0 b)y=0;x=0 c) y=-2 d) y=2 e)y=-3;x=0 f) y=2;x=0*

19. $5x - 2y + 6 = 0$ va $x + y - 6 = 0$ to‘g‘ri chiziq tenglamalarini burchak koeffitsiyentini toping . J: $y = \frac{5}{2}x + 3; y = 6 - x$

20. *A (-1;4)* nuqtadan o‘tib , *OX* o‘qi bilan 45^0 li burchak tashkil qilgan to‘g‘ri chiziq tenglamasi tuzilsin . J: $y = x + 5$

21. *A (2;3)* nuqtadan va *OY* o‘qidan $b = 6$ kesma kesuvchi to‘g‘ri chiziq tenglamasi tuzilsin . J: $y = \frac{3}{2}x + 6$

22. Quyidagi funksiyalarni grafigini yasang.

a) $y = 3$ b) $y = -2$ c) $|y| = 1$

23. 1) $4x + 3y - 12 = 0$; to‘g‘ri chiziqning kesmalarga nisbatan tenglamasini yozing va ularni yasang. J: 1) $\frac{x}{3} + \frac{y}{4} = 1$

24. 1) $2x - 3y - 6 = 0$; 2) $3x - 2y + 4 = 0$ to‘g‘ri chiziq tenglamalarini, kesmalar bo‘yicha tenglamasiga keltiring. J: 1) $\frac{x}{3} - \frac{y}{2} = 1$ $\frac{x}{4/3} - \frac{y}{2} = -1$

25. $Ax + 5y - 40 = 0$ to‘g‘ri chiziq A ning qanday qiymatlarida koordinata o‘qlaridan bir xil kesmalar ajratadi. J: A= *5*

26. $y = \frac{1}{2} \cdot x + 4$ to‘g‘ri chiziq berilgan. Uning koordinata o‘qlari bilan kesishish nuqtalarini toping. J: *(0;4), (-8;0)*

27. To‘g‘ri chiziq *OX* o‘qini *A(-6;0)* nuqtada, *OY* o‘qini *B(0;7)* nuqtada kesib o‘tadi. Bu to‘g‘ri chiziqning kesmalarga nisbatan tenglamasini tuzing.

J: $\frac{x}{-6} + \frac{y}{7} = 1$

28. *A (-2;3)* nuqtadan va *OY* o‘qidan *b = 6* kesma kesuvchi to‘g‘ri chiziq tenglamasi tuzilsin. J: $3x-2y+12=0$

29. $3x-5y+19=0$ va $10x+6y-50=0$ to‘g‘ri chiziqlar perpendikulyar ekanligi isbotlansin. J: $k_1=\frac{3}{5};\ k_2=-\frac{5}{3}$

30. *5x-y+7=0* va 2*x-3y+1=0* to‘g‘ri chiziqlar orasidagi burchakni aniqlang.

J: $\alpha=\frac{\pi}{4}$

31. 1) $3x-15y+16=0$, 2) $3x+15y-8=0$,

3) $6x-30y+13=0$, 4) $30x+6y+7=0$

To‘g‘ri chiziqlardan qaysilari perpendikulyar va qaysilari parallel.

32. Quyidagi to‘g‘ri chiziqlar orasidagi burchaklarni toping:

1) $\begin{cases} y=2x-3 \\ y=\frac{1}{2}x+1 \end{cases}$; 2) $\begin{cases} 5x-y+7=0 \\ 2x-3y+1=0 \end{cases}$ 3) $\begin{cases} 2x+y=0 \\ y=3x-4 \end{cases}$; 4) $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$

33. $A(4;\ 5)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing va ulardan $2x-3y+6=0$ to‘g‘ri chiziqqa perpendikulyar va parallel bo‘lganlarini ajrating. J: $3x+2y-22=0$;$2x-3y+7=0$

34. Uchburchak tomonlari

$$x+y=4,\ 3x-y=0,\ x-3y-8=0$$

tenglamalar bilan berilgan. Uning burchaklarini va yuzasini hisoblang.

J: $tgA=\frac{4}{3};tgB=tgC=2, S=16$

35. Uchlari $P(-4;\ 0)$, $Q(0;\ 4)$ va $R(2;\ \ 2)$ nuqtalarda bo‘lgan uchburchak medianalarining tenglamalarini tuzing .

J: $(M_1P)\ 3x-5y+12=0;\ (M_2R)\ x-3y+4=0; (M_3Q)\ x+y+4=0$

36. To‘g‘ri chiziqlar:

$$\begin{cases} \frac{x}{4}-\frac{y}{5}=1 \\ \frac{x}{2}+\frac{y}{18}=1 \end{cases}$$ orasidagi burchakni toping. J: $\alpha = arctg\frac{31}{49}$

37. $y=-\frac{2}{5}\cdot x+3$; $y=\frac{3}{7}\cdot x+\frac{2}{7}$ to‘g‘ri chiziqlar orasidagi burchakni toping. J: $\alpha = 45^0$

38. $x-y+3=0$ to‘g‘ri chiziqqa koordinatalar boshidan tushirilgan perpendikulyarning uzunligini va uning *OX* o‘qi bilan tashkil qilgan burchagini toping. J: $\alpha=\frac{\pi}{4};\ \ p=\sqrt{3}$

39. *A(2;1)* nuqtadan o‘tib *y=3x-4* to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing. J: $3x-y-5=0$

40. *A(5;-4)* nuqtadan o‘tuvchi va *3x+2y-7=0* to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing. J: $2x-3y-22=0$

41. Uchburchak uchlarining koordinatalari berilgan. *A(-3;-1), B(2;1),C(3;5)*

Uning *B* uchidan tushirilgan balandlik tenglamasini tuzing va balandligining uzunligini toping. J: $d=\frac{3\sqrt{2}}{2};\ x+y-3=0$

42. $(5;2)$ nuqtadan o‘tib $4x+6y+5=0$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin. J: $2x+3y-16=0$

43. $\left(\frac{1}{2};-\frac{2}{3}\right)$ nuqtadan o‘tib $4x-8y-1=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin. J: $6x+3y-1=0$

44. Uchburchak uchlarining koordinatalari berilgan: $A(1;0), B(3;6) va C(8;2)$. A uchidan tushirilgan mediana tenglamasi tuzilsin. J: $8x-9y-8=0$

45. $A(1;1)$, $B(7;4)$, $C(5;10)$ va $D(-1;7)$ nuqtalar parallelogram uchlari ekanligini ko‘rsating. J: $|AB|=|CD|; |BC|=|AD|$.

46. $(4;5)$ nuqtadan o‘tib OY o‘qiga parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin. J: $x=4$

47. Uchburchak uchlarining koordinatalari berilgan. *A(12;-4) ,B(0;5)* va C*(-12;-11).* Uning tomonlarining tenglamalarini tuzing.

J: $(AB) 3x+4y-20=0, (AC) 7x-24y-180=0, (BC) 4x-3y+15=0$

48. *A(1;2)* va *B(4;3)* nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing hamda bu to‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalarini aniqlang.

J: $x - 3y + 5 = 0; \quad (0; 5/3)$

49. *x-y-4=0* va *2x-11y+37=0* to‘g‘ri chiziqlarning kesishish nuqtasidan hamda koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing. J: $5x-9y=0$

50. Uchburchak uchlarining koordinatalari berilgan: *A(-3;-1),B(5;3), C(6;-4)* . Uning *C* uchidan o‘tkazilgan medianasining tenglamasini tuzing.
J: $x+y-2=0$

51. *4x-3y-12=0* va *3x+4y-12=0* to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini tuzing. J: $x-7y=0; 7x+y-24=0$

52. *3x-4y-20=0* va *8x-6y-5=0* to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini tuzing. J: $2x+2y+35=0; 14x-14y-24=0$

53. *A(2;5)* nuqtadan *6x+8y-6=0* to‘g‘ri chiziqqacha bo‘lgan masofani toping.

J: $\frac{23}{5}$

54. Ushbu

1) $5x+12y-26=0$, 2) $3x-4y+10=0$,3) $y=3x+5$, 4) $2x+2y+7=0$

to‘g‘ri chiziq tenglamalarini normal ko‘rinishga keltiring.

J: 1) $\frac{5}{13}x+\frac{12}{13}y-2=0$, 2) $-\frac{3}{5}x+\frac{4}{5}y-2=0$,
3) $-\frac{3\sqrt{10}}{10}x+\frac{\sqrt{10}}{10}y-\frac{\sqrt{10}}{2}=0$, 4) $-\frac{\sqrt{2}}{2}x-\frac{\sqrt{2}}{2}y-\frac{7\sqrt{2}}{4}=0$

55. Ushbu 1) $\frac{2}{5}x+\frac{3}{4}y-6=0$, 2) $\frac{12}{13}x-\frac{5}{13}y-7=0$.

3) $\frac{3}{5}x+\frac{3}{4}y-2=0$, 4) $\frac{1}{3}x+\frac{2}{3}y-4=0$

to‘g‘ri chiziq tenglamalaridan qaysilari normal ko‘rinishda? J: 2;4.

56. *R(3;-4)* nuqta koordinatalar boshidan to‘g‘ri chiziqqa tushirilgan perpendikulyarning asosi. To‘g‘ri chiziqning normal tenglamasini tuzing.

J: $\frac{3}{5}x-\frac{4}{5}y-5=0$

57. Uchlari $P(0;\ 5)$, $Q(-3;\ 1)$va $R(-1;\ -2)$nuqtalarda bo‘lgan uchburchakning R nuqtasidan o‘tkazilgan balandligining uzunligini toping. J: $h=\frac{17}{5}$

58. *M(1;2)* nuqtadan 20*x-21y-58=0* to‘g‘ri chiziqqacha bo‘lgan masofani toping.

J: $\frac{80}{29}$

59. Uchburchakning tomonlari tenglamalari berilgan: *x+3y-7=0(AB),*

4x-y-2=0(BC),6x+8y-35=0 (AC). *B* uchidan tushirilgan balandlik uzunligi topilsin.

J: $\frac{13}{10}$

60. $3x+y-3\sqrt{10}=0$ va $6x+2y+5\sqrt{10}=0$ to‘g‘ri chiziqlar orasidagi masofa topilsin.

J: $\frac{11}{2}$

61. $M(4;-1)$ nuqtadan hamda $x-3y+2=0$ va $y-4=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing. J: $5x-6y-26=0$

62. $3x-y+5=0$ va $2x+3y+1=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi hamda $7x-3y+5=0$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing. J: $77x-33y+133=0$

63. $3x-y=0$ va $x+4y-2=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tib, $2x+7y=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

J: $91x-26y-2=0$

64. Trapetsiya asoslarining tenglamalari $3x-4y-15=0$, $3x-4y-35=0$ berilgan. Trapetsiyaning balandligini toping. J:4

65. ΔABC: $A(1;-2)$, $B(7;1)$, $C(3;7)$ uchlari berilgan bo‘lsa: a) BC - tomon tenglamasi? b) A- uchidan tushirilgan balandlik tenglamasi tuzilsin? c) A- uchidan o‘tuvchi va BC tomonga parallel bo‘lgan to‘g‘ri chiziq tenglamasi tuzilsin?

J: $a)\,3x+2y-23=0;\ b)\,2x-3y-8=0;\ c)\,3x+2y+1=0$

4.2. MAVZU: TEKISLIKDA IKKINCHI DARAJALI (TARTIBLI) CHIZIQLAR.

MAVZUGA OID NAZARIY MATERIALLAR

Tekislikdagi ikkinchi tartibli chiziqning umumiy tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0 \qquad (1)$$

Bu yerda A,B,C,D,E,F lar o‘zgarmas koeffitsiyentlar bo’lib, A,B,C lardan kamida bittasi noldan farqli bo‘lishi zarur, aks holda to‘gri chiziqqa ega bo‘lamiz.

1. Aylana tenglamasi.

1-**Ta’rif.** Bir nuqtadan bir xil masofada yotgan nuqtalarning geometrik o‘rniga aylana deyiladi.

$$(x-a)^2+(y-b)^2=R^2 \qquad (2)$$

Bunda qavslarni ochib

$x^2+y^2-2ax-2by+(a^2+b^2-R^2)=0$ ni hosil qilamiz. Agarda $-2a=2D;$ $-2b=2E;$

$a^2+b^2-R^2=F$ deb belgilash kiritsak, aylana tenglamasi:

$$x^2+y^2+2Dx+2Ey+F=0$$

Markazi koordinata boshida radiusi R ga teng aylana tenglamasi quyidagicha bo‘ladi:

$$x^2+y^2=R^2 \qquad (3)$$

Markazi ixtiyoriy $C(a;b)$ nuqtada, radiusi R ga teng bo‘lgan aylana tenglamasi:

$$(x-a)^2+(y-b)^2=R^2$$

Masalan.1) Markazi $C(2:-3)$ nuqtada, radiusi $R=4$ ga teng aylana tenglamasi yozilsin.

Yechish:

$$(x-2)^2+(y+3)^2=4^2$$

$$x^2-4x+4+y^2+6y+9=16$$

$$x^2+y^2-4x+6y-3=0$$

2) $x^2+y^2-6x+8y=0$ aylananing markazi va radiusi topilsin.

Yechish:

$$x^2+y^2-6x+8y=0 \Leftrightarrow (x^2-6x)+(y^2+8y)=0$$

$$(x^2-6x+9)+(y^2+8y+16)-9-16=0$$

$$(x-3)^2+(y+4)^2=5^2$$

Demak, aylana markazi $M(3;-4)$ va radiusi esa $R=5$

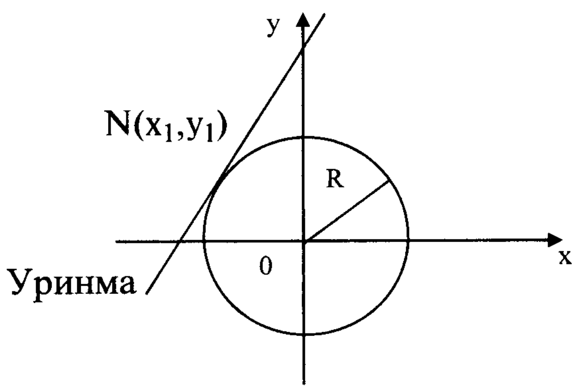
$N\ (x_1;y_1)$ nuqta aylananing biror nuqtasi bo‘lsa, u holda bu nuqtadan aylanaga o‘tkazilgan urinma tenglamasi

$$(x-a)(x_1-a)+(y-b)(y_1-b)=R^2$$

yoki

$$x \cdot x_1 + y \cdot y_1 = R^2 \qquad (4)$$

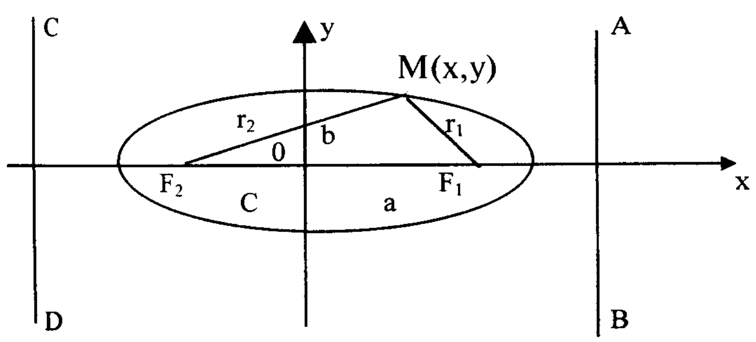
dan iborat bo‘ladi.



1-chizma

2.Ellips

2-Ta'rif. Ellips deb, har bir nuqtasidan berilgan ikki F_1 va F_2 nuqtagacha (fokuslargacha) bo'lan masofalarning yig'indisi o'zgarmas $2a$ miqdorga teng nuqtalarning geometrik o'rniga aytiladi.



$F_1(c,o)$:
$F_2(-C:O)$
$d(F_1,F_2)=2c$

2-chizma

Ellipsning *kanonik* tenglamasi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (5)$$

$$a^2=b^2+c^2$$

Bu yerda a – ellipsning katta yarim o'qi , $a=\sqrt{b^2+c^2}$.

b – ellipsning kichik yarim o'qi, $b=\sqrt{a^2-c^2}$ bo'lib, ular ellipsning fokusi $c=\sqrt{a^2-b^2}$ dan topiladi.

Ellipsning ekssentrisiteti

$$\varepsilon=\frac{2c}{2a}\Rightarrow \qquad \varepsilon=\frac{c}{a} \qquad (6)$$

bunda $0\le \varepsilon <1$

Ekssentrisitet ellipsning cho'ziqligi darajasini xarakterlaydi.

Ellipsning ixtiyoriy nuqtasidan (F_1 va F_2) fokuslarigacha bo‘lgan masofalar uning fokal radius-vektorlari (r_1 va r_2) deyiladi.

Ixtiyoriy *M (x,y)* nuqta uchun

$$r_1 = a - \varepsilon x, \qquad r_2 = a + \varepsilon x, \qquad r_1 + r_2 = 2a. \qquad (7)$$

Ellipsning kichik o‘qiga parallel bo‘lgan va undan $\frac{a}{\varepsilon}$ masofadan o‘tgan ikki to‘g‘ri chiziq ellipsning direktrisalari deyiladi:

$$x = -\frac{a}{\varepsilon} \text{ va } x = \frac{a}{\varepsilon} \qquad (8)$$

Ellipsning ixtiyoriy *M (x,y)* nuqtasiga o‘tkazilgan urinma tenglamasi

$$\frac{x \cdot x_1}{a^2} + \frac{y \cdot y_1}{b^2} = 1 \qquad (9)$$

ko‘rinishda bo‘ladi.

Masalan. 1) Katta o‘qi 10 ga teng va ekssentrisiteti ε=0,8 ga teng bo‘lgan ellipsning sodda tenglamasini tuzing.

Yechish:

$$2a=10 \Rightarrow a=5$$

(6)-formuladan $c=\varepsilon\, a=4$. (5)- formulalardan $b^2=a^2-c^2=5^2-4^2=25-16=9 \Leftrightarrow b=3.$

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1 \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1 \text{ ni hosil qilamiz.}$$

2) $4x^2+9y^{2=}16$ ellipsning katta va kichik yarim o‘qlarini, fokuslarini hamda ekssentrisitetini toping.

Yechish:

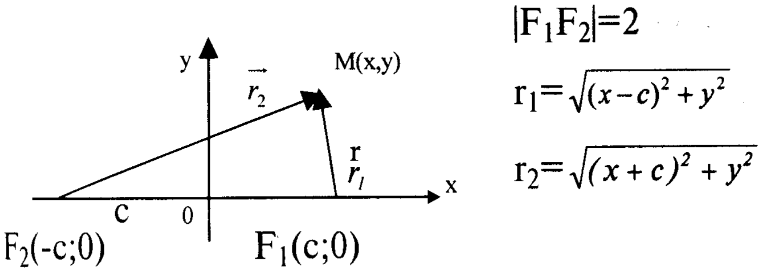
$$\frac{4x^2}{16}+\frac{9y^2}{16}=1 \Rightarrow \frac{x^2}{4}+\frac{y^2}{\frac{16}{9}}=1.$$

Bu yerda $a^2=4 \Rightarrow a=2,$ $b^2=\frac{16}{9} \Rightarrow b=\frac{4}{3},\ a^2=b^2+c^2$ ekanligidan

$c^2=a^2-b^2=4-\frac{16}{9}=\frac{36-16}{9}=\frac{20}{9},\quad c=\sqrt{\frac{20}{9}}=\frac{2\sqrt{5}}{3},\qquad \varepsilon=\frac{2\sqrt{5}}{3}\cdot\frac{1}{4}=\frac{\sqrt{5}}{6}.$

3.Giperbola.

3-Ta'rif. Giperbo'la deb shunday nuqtalarning geometric o'rniga aytiladiki, ularning har biridan berilgan ikki F_1 va F_2 nuqtagacha (fokuslargacha) bo'lgan masofalar ayirmasining absolyut qiymati o'zgarmas $2a$ miqdordan iboratdir.

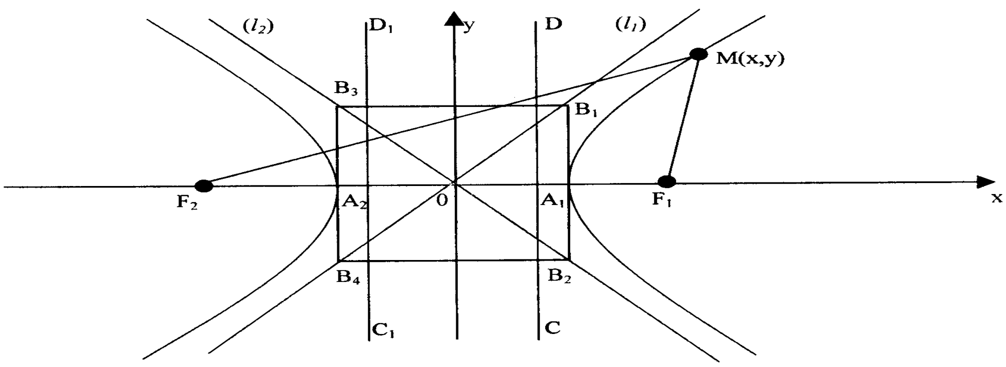


3-chizma

Giperbolaning kanonik tenglamasi.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (1)$$

$$b^2=c^2-a^2$$



4-chizma

A_1 *(a:0)* va A_2 *(-a:0)* nuqtalar giperbolaning uchlari deyiladi.

$[A_1A_2]$ kesmaga giperbolaning haqiqiy o‘qi deyiladi.

$[B_1B_2]$ kesmaga giperbolaning mavhum o‘qi deyiladi.

a - haqiqiy yarim o‘q, *в* – mavhum yarim o‘q deyiladi.

Mos ravishda

$$y=\pm\frac{в}{a}\ x \qquad (2)$$

formula bilan aniqlanuvchi ikki (l_1) va (l_2) to‘g‘ri chiziqlarga asimptotalar deyiladi. Formula

$$\varepsilon=\frac{c}{a}$$

bilan aniqlanuvchi kattalikka giperbolaning ekssentrisiteti deyiladi. $c>a$ bo‘lganligidan ε>1. Agar ε birga yaqin bo‘lsa, giperbola tarmoqlari shuncha siqiq va ε birdan qancha katta bo‘lsa, giperbola tarmoqlari shuncha yoyiq joylashgan bo‘ladi.

Giperbolaning fokal radiuslari :

$$x<0 \text{ da } \left.\begin{matrix} r_1=a-\varepsilon x \\ r_2=-a-\varepsilon x \end{matrix}\right\} \text{(chap tarmoq uchun).} \qquad (3)$$

$$x>0 \text{ da } \left.\begin{matrix} r_1=-a+\varepsilon x \\ r_2=a+\varepsilon x \end{matrix}\right\} \text{(o‘ng tarmoq uchun).} \qquad (3^1)$$

Giperbolaning direktrisalari tenglamasi:

$$x=\frac{a}{\varepsilon} \text{ va } x=-\frac{a}{\varepsilon} \qquad (4)$$

Yarim o‘qlari teng (*a*=*в*) bo‘lgan giperbolaga teng tomonli giperbola deyiladi va

$$x^2-y^2=a^2$$

formula bilan ifodalanadi.

Giperbolaning $(x_1;y_1)$ nuqtasiga o‘tkazilgan urinmaning tenglamasi:

$$\frac{x\cdot x_1}{a^2}-\frac{y\cdot y_1}{b^2}=1$$

Masalan. Fokuslari orasidagi masofa $2c=8$ bo‘lgan, uchlari orasidagi masofa $2a=6$ bo‘lgan giperbolaning kanonik tenglamasi tuzilsin.

Yechish:

$$2c=8\Rightarrow c=4$$

$$2a=6\Rightarrow a=3$$
$$b=\sqrt{c^2-a^2}=\sqrt{4^2-3^2}=\sqrt{16-9}=\sqrt{7}\Rightarrow b^2=7$$

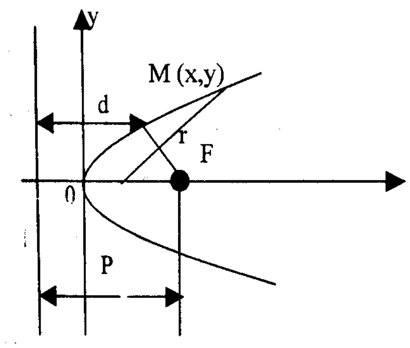
(1) formuladan $\frac{x^2}{9}-\frac{y^2}{7}=1$ hosil qilamiz.

Parabola

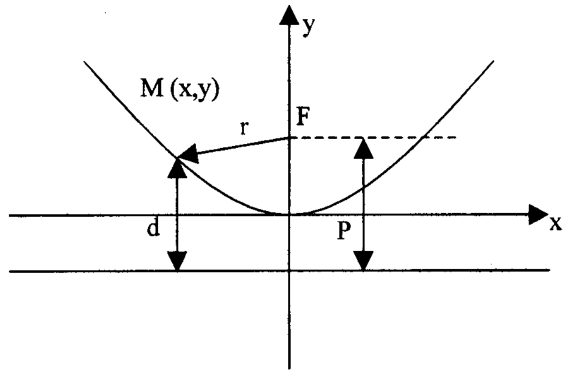
4-Ta’rif. Berilgan nuqtadan (fokusdan) va berilgan to‘g’ri chiziqdan (direktrisadan) bir xil uzoqlikda bo‘lgan nuqtalarning geometric o‘rni parabola deyiladi.

Parabola koordinata boshidan o‘tib *OX* o‘qiga simmetrik bo‘lsa uning tenglamasi:

$$y^2=2px \qquad (1)$$



5-cnizma



6-chizma

Direktrisa tenglamasi:

$$x = -\frac{P}{2} \qquad (2)$$

Parabolaning fokusi koordinatasi:

$$F\left(\frac{P}{2};0\right)$$

Parabolaning *M(x,y)* nuqtasining fokal radiusi

$$r = x + \frac{P}{2} \qquad (3)$$

Agar parabola koordinata boshidan o‘tib *OY* o‘qiga simmetrik bo‘lsa uning tenglamasi:

$$x^2=2py \qquad (4)$$

Uning direktrisasi tenglamasi:

$$y = -\frac{P}{2}, \qquad (5)$$

Fokus koordinatasi $F\left(0;\frac{P}{2}\right)$ nuqtada bo’lgan, *M(x,y)* nuqtasining fokal radiusi:

$$r = y + \frac{P}{2}$$

$A(x_1,y_1)$ nuqtasiga o‘tkazilgan urinma tenglamalari mos ravishda

$$yy_1=p(x+x_1) \text{ va } xx_1=p(y+y_1)$$

Masalan. $y=\frac{1}{4}x^2$ parabola fokusining koordinatalarini toping va direktrisasining tenglamasini tuzing.

Yechish: $y=\frac{1}{4}x^2$ ni kanonik ko‘rinishda yozamiz:

$y=\frac{1}{4}x^2 \Rightarrow x^2=4y$ dan $2p=4 \Rightarrow p=2$ ekanligi kelib chiqadi. Direktrisa tenglamasini $y=-\frac{P}{2}$ dan topamiz. $y=-\frac{P}{2}=-\frac{2}{2}=-1 \Rightarrow y=-1$ bo‘ladi. Parabola fokusining koordinatasi: $F\left(0;\frac{P}{2}\right)$ yoki $F(0;1)$.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. Aylana tenglamasi tuzilsin:

a) Markazi $(3;-1)$, radiusi 5; b) Markazi $(-2;-8)$, radiusi 10.

c) Markazi $(-1;5)$ va $(-4;-6)$ nuqtadan o‘tuvchi aylana tenglamasi tuzilsin.

J: $a) x^2+y^2-6x+2y-15=0;$ $b) x^2+y^2+4x+16y+32=0$ $c) x^2+y^2+2x-10y-104=0$

2. Aylana markazi va radiusi topilsin:

a) $x^2+y^2-4x+10y+13=0$ b) $x^2+y^2+6y+2=0$ c) $x^2+y^2+x=0$

d) $16x^2+16y^2+8x+32y+1=0$ e) $2x^2+2y^2-x+y=1$

J: $a) O(2;-5), r=4;$ $b)$ $O(0;3); r=\sqrt{7};$ $c) O(-\frac{1}{2};0), r=\frac{1}{2};$
$d)$ $O(-\frac{1}{4};-1), r=1;$ $e) O(\frac{1}{4};-\frac{1}{4}), r=\frac{\sqrt{10}}{4}$

2. Radiusi 3 ga, markazi $(2;-5)$ nuqtada bo‘lgan chiziq tenglamasi tuzilsin. :

J: $x^2+y^2-4x+10y+20=0$

4. $x^2+y^2+2x-6y+7=0$ aylananing radiusi va markazi aniqlansin.

J: $C(-1;3), r=\sqrt{3}$

5. $N\ (7;\ -2)$ nuqtadan o‘tib, markazi $C\ (3;\ -5)$ nuqtada bo‘lgan aylana tenglamasini yozing. J: $x^2+y^2-6x+10y+9=0$

6. $M\ (4;\ 2)$ va $N\ (12;\ 8)$ nuqtalar berilgan. Diametri MN kesmadan iborat bo‘lgan aylana tenglamasini yozing. J: $x^2+y^2-16x-10y+64=0$

7. $x^2+y^2-4x+8y-16=0$, va $x^2+y^2+8x+12y-14=0$ aylanalar markazlaridan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing. J: $x-3y-14=0$

8. Giperbolaning parametrlarini aniqlang va grafigini yasang.

a) $16x^2-25y^2=400$, b) $y^2-x^2=1$ c) $9x^2-25y^2=225$.

J: $a)\ a=5, b=4, c=\sqrt{4};\ b)\ a=1, b=1, c=\sqrt{2}; c)\ a=5, b=3, c=\sqrt{34}$

9. $144x^2-25y^2=3600$, 2) $9x^2-y^2=144$ giperbolalarning o‘qlarini uzunliklarini, fokuslarini va eksentrisiteti topilsin.

J: 1) $a=5, b=12, c=13, \varepsilon=\frac{13}{5}>1$; 2) $a=4, b=12,\ c=4\sqrt{10}$, $\varepsilon=\sqrt{10}$

10. Quyidagilar berilganda giperbolaning kanonik tenglamasini yozing:

1) fokuslari orasidagi masofa 10, ekssentrisitet $5/3$;

2) haqiqiy yarim o‘qi $\sqrt{20}$ va giperbola $N\ (-10;\ 4)$ nuqtadan o‘tadi;

3) fokuslar orasidagi masofa 10, uchlari orasidagi masofa 4.

J: 1) $\frac{x^2}{9}-\frac{y^2}{16}=1$; 2) $\frac{x^2}{20}-\frac{y^2}{4}=1$; 3) $\frac{x^2}{4}-\frac{y^2}{21}=1$

11. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ giperbolada abssissasi 3 ga teng nuqta olingan. Bu nuqtaning fokal radiuslarini toping. $r_1 = 2; r_2 = 8$

12. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ ellips berilgan. Uchlari ellipsning fokuslarida, fokuslari esa uning uchlarida bo‘lgan giperbola tenglamasini yozing va uni yasang.J: $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

13. $x^2 - 4y^2 = 16$ giperbolani va uning asimptotalarini yasang. Fokuslarini, ekssentrisitetini va asimptotalari orasidagi burchakni toping.

J: $y = \pm\dfrac{1}{2}x,\ c = 2\sqrt{5}, \varepsilon = \dfrac{\sqrt{5}}{2}, \varphi = arctg\left(\dfrac{4}{3}\right).$

14. $M_1\left(10; -\sqrt{5}\right)$ nuqta $\dfrac{x^2}{80} - \dfrac{y^2}{20} = 1$ giperbolada yotadi. M_1 nuqtaning fokal radiuslari yotgan to‘g‘ri chiziqlarning tenglamasini yozing.

J: $x = 0; \sqrt{5}x + 20 + 10\sqrt{5} = 0$

15. $\dfrac{x^2}{64} - \dfrac{y^2}{36} = 1$ giperbolaning o‘ng fokusidan 4,5 birlik masofada yotuvchi nuqtalarni toping. J: $x^2 + y^2 - 20x - 79.,75 = 0$

16. $2x - y - 10 = 0$ to‘g‘ri chiziq va $\dfrac{x^2}{20} - \dfrac{y^2}{5} = 1$ giperbolaning kesishish nuqtalarini toping. J: (6; 14/3)

17. Fokuslari orasidagi masofa 10 ga teng va mavhum yarim o‘qi 3 ga teng bo‘lgan giperbolaning kanonik tenglamasini tuzing. J: $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

18. Parabolalarning grafigi chizilsin:

a) $y = -x^2$, b) $x = -2y^2$, c) $x = y^2 - 1$, d) $y = x^2 - 6x + 13$, e) $x = 4 - y^2$, f) $y = x^2 + 2$,

19. $y^2 = 6x$ parabola berilgan. Uning p parametrini, direktrisa tenglamasini toping.

J: $p = 3,\ x = -\frac{3}{2}$

20. Koordinatalar boshidan va $N(6;\ 3)$ nuqtadan o‘tib, OY o‘qiga simmetrik bo‘lgan parabola tenglamasini yozing va uni yasang. J: $x^2 = 12y$

MUSTAQIL YECHISH UCHUN MASHQLAR.

1. ABC uchburchak uchlarining koordinatalari berilgan.

a) uchidan o‘tkazilgan mediana tenglamasini tuzing va uning uzunligini toping;

b) B uchidan o‘tkazilgan balandlik tenglamasini tuzing va shu balandlik uzunligini toping;

v) B burchak bissektrisasi tenglamasini tuzing va uning uzunligini toping.

1.1. $A(4;1), B(0;-2), C(-5;10)$

1.2. $A(-7;3), B(5;-2), C(8;2)$

1.3. $A(5;-1), B(1;-4), C(-4;8)$

1.4. $A(-14;6), B(-2;1), C(1;5)$

1.5. $A(6;0), B(2;-3), C(-3;9)$

1.6. $A(-9;2), B(3;-3), C(6;1)$

1.7. $A(7;-4), B(3;-7), C(-2;5)$

1.8. $A(-8;4), B(4;-1), C(7;3)$

1.9. $A(3;-3), B(-1;-6), C(-6;6)$

1.10. $A(-6;5), B(6;0), C(9;4)$

1.11. $A(4;11), B(-1;-1), C(7;5)$

1.12. $A(3;13), B(-2;1), C(6;7)$

1.13. $A(7;11), B(2;-1), C(10;5)$

1.14. $A(6;13), B(1;1), C(9;7)$

1.15. $A(4;14), B(-1;2), C(7;8)$

1.16. $A(6;10), B(1;-25), C(9;4)$

1.17. $A(4;13), B(-1;1), C(7;7)$

1.18. $A(6;11), B(1;-1), C(9;5)$

1.19. $A(4;10), B(-1;-2), C(7;4)$

1.20. $A(-4;10), B(1;7), C(0;4)$

1.21. $A(-10;-1), B(-6;-4), C(6;1)$

1.22. $A(18;8), B(12;0), C(0;5)$

1.23. $A(-6;-3), B(-6;-2), C(10;-1)$

1.24. $A(14;10), B(8;2), C(-4;7)$

1.25. $A(-2;-1), B(2;-4), C(14;1)$

1.26. $A(8;7), B(2;-4), C(14;1)$

1.27. $A(1;0), B(5;-3), C(17;2)$

1.28. $A(20;2), B(14;-6), C(26;-1)$

1.29. $A(-1;7), B(3;4), C(15;9)$

1.30. $A(7;6), B(1;2), C(-11;3)$

2. *ABC* uchburchakning uchlari berilgan. Quyidagilarni toping:

a) *AB,BC,AC* tomon tenglamasini tuzing va $|AB|, |BC|, |AC|$ tomon uzunliklarini toping.

b) *C* uchidan *AB* tomonga tushirilgan balandlik tenglamasini;

c) *A* uchidan *BC* tomonga tushirilgan mediana tenglamasini va balandlik uzunligini, mediana uzunligini;

g) “b” va “c” bandlarda topilgan balandlik va mediananing kesishish nuqtasi topilsin;

d) *C* nuqtadan o‘tuvchi *AB* tomonga parallel to‘g‘ri chiziq tenglamasini;

e) *C* uchidan *AB* to‘g‘ri chiziqqacha bo‘lgan masofani toping.

1.31. $A(4;-5), B(6;9), C(-4;-1)$

1.32. $A(1;-3), B(-5;4), C(-2;10)$

1.33. $A(1;8), B(-5;-4), C(-1;-3)$

1.34. $A(6;-4), B(-8;3), C(-2;-7)$.

1.35. $A(2;3), B(-4;-7), C(2;0)$

1.36. $A(-4;-8), B(4;1), C(0;7)$

1.37. $A(4;-2), B(7;0), C(-3;1)$

1.38. $A(4;1), B(-2;8), C(1;-5)$

1.39. $A(4;0), B(1;-3), C(5;2)$

1.40. $A(7;10), B(1;3), C(4;-2)$

1.41. $A(11;-3), B(-1;-3), C(7;1)$

1.42. $A(5;9), B(4;-1), C(0;1)$

1.43. $A(7;3), B(1;7), C(-2;1)$

1.44. $A(6;-4), B(-8;3), C(-2;-7)$

1.45. $A(2;6), B(6;-6), C(2;-4)$

1.46. $A(10;1), B(3;7), C(-3;4)$

1.47. $A(8;3), B(2;8), C(-4;-4)$

1.48. $A(7;7), B(-7;5), C(-3;-3)$

1.49. $A(3;-3), B(4;3), C(-6;1)$

1.50. $A(6;2), B(-6;8), C(2;-4)$

1.51. $A(7;5), B(-4;0), C(2;-5)$

1.52. $A(8;-1), B(2;6), C(-4;4)$

1.53. $A(-5;0), B(2;-6), C(8;-3)$

1.54. $A(1;-4), B(-1;10), C(-0;6)$

1.55. $A(-3;7), B(-1;3), C(2;-4)$

1.56. $A(10;4), B(-4;6), C(-1;3)$

1.57. $A(2;-6), B(3;11), C(-1;3)$

1.58. $A(-5;5), B(4;-7), C(-2;-7)$

3. Chiziq tenglamasini kanonik ko‘rinishga keltiring va uning shaklini chizing.

2.1. $x^2 + y^2 - 16x + 4y - 13 = 0$

2.2. $x^2 + y^2 - 6x + 4y - 3 = 0$

2.3. $x^2 + y^2 - 14x + 6y - 6 = 0$

2.4. $x^2 + y^2 + 16x - 17 = 0$

2.5. $x^2 + y^2 - 10x + 4y - 7 = 0$

2.6. $x^2 + y^2 - 18x + 6y - 10 = 0$

2.7. $x^2 + y^2 - 14x + 6y - 6 = 0$

2.8. $x^2 + y^2 - 22x + 8y + 16 = 0$

2.9. $x^2 + y^2 - 18x - 6y - 10 = 0$

2.10. $x^2 + y^2 + 18x + 6y - 10 = 0$

2.11. $x^2 + y^2 - 2x + 2y - 8 = 0$

2.12. $x^2 + y^2 - 28x + 6y - 20 = 0$

2.13. $x^2 + y^2 + 28x + 6y - 20 = 0$

2.14. $x^2 + y^2 - 28x + 6y - 20 = 0$

2.15. $x^2 + y^2 - 18x + 2y + 18 = 0$

2.16. $x^2 + y^2 - 18x + 6y - 31 = 0$

2.17. $x^2 + y^2 - 4x + 2y - 11 = 0$

2.18. $x^2 + y^2 - 10x + 4y - 20 = 0$

2.19. $9x^2 + 4y^2 = 36$

2.20. $2x^2 + 3y^2 = 6$

2.21. $4x^2 + 3y^2 = 12$

2.22. $3x^2 + 2y^2 = 6$

2.23. $5x^2 + 4y^2 = 20$

2.24. $8x^2 + 5y^2 = 40$

2.25. $3x^2 + y^2 = 30$

2.26. $7x^2 + 5y^2 = 35$

2.27. $x^2 + 4y^2 = 4$

2.28. $3x^2 + 5y^2 = 15$

2.29. $2x^2 + 6y^2 = 12$

2.30. $7x^2 + 4y^2 = 28$

2.31. $4x^2 + 7y^2 = 28$

2.32. $2x^2 + 17y^2 = 34$

2.33. $2x^2 + 4y^2 = 24$

2.34. $2x^2 + 4y^2 = 16$

5.Chiziq tenglamasini kanonik ko‘rinishga keltiring va uning shaklini chizing.

2.49. $9x^2-4y^2=36$ 2.50. $2x^2-3y^2=6$ 2.51. $4x^2-3y^2=12$

2.52. $3x^2-2y^2=6$ 2.53. $5x^2-4y^2=20$ 2.54. $8x^2-5y^2=40$

2.55. $3x^2-y^2=30$ 2.56. $7x^2-5y^2=35$ 2.57. $x^2-4y^2=4$

2.58. $3x^2-5y^2=15$ 2.59. $2x^2-6y^2=12$ 2.60. $7x^2-4y^2=28$

2.61. $4x^2-7y^2=28$ 2.62. $2x^2-17y^2=34$ 2.63. $2x^2-4y^2=24$

2.64. $y^2=4x-3$ 2.69. $y^2=4x$

2.65. $4x^2+5y=0$ 2.70 $6y^2-x=0$

2.66. $x^2+4y=0$ 2.71. $5y^2-x=0$

2.67. $2y^2+5y=0$ 2.72. $6y-x^2=0$

2.68. $y^2=5x-7$ 2.73. $6y^2-x=8$

6. Chiziq tenglamasini kanonik ko‘rinishga keltiring va uning shaklini chizing.

Quyidagilar ma’lum:*A,B* – egri chiziqda yotuvchi nuqtalar;

a- katta yarim o‘q(yoki haqiqiy yarim o‘q);

b- kichik(yoki mavhum) yarim o‘q; ε - ekssentrisitet;

$y=\pm kx$ giperbola asimptotalari tenglamasi; D- egri chiziq direktrisasi;

2c- fokus masofasi.

a) ellipsning; b) giperbolaning; v)parabolaning kanonik tenglamasini tuzing

2.74. a) $a=9, \varepsilon=\frac{\sqrt{17}}{9}$; b) $b=7; F(-\sqrt{130};0)$; c) simmetriya o‘qi $OY, A(-4;32)$

2.75. a) $b = 3, F(-\sqrt{55};0);$ b) $a = 8, \varepsilon = \frac{5}{4};$ c) D:*x=3*

2.76. a) $A\left(5;\frac{5}{6}\sqrt{11}\right), B\left(-4;\frac{5\sqrt{5}}{3}\right);$ b) $k = \frac{2}{7}; \varepsilon = \frac{\sqrt{53}}{7};$ v) D: $y = -4$

2.77. a) $\varepsilon = \frac{4}{5}, A\left(-4;\frac{9}{5}\right);$ b) $A\left(-5;\frac{9}{4}\right) va B\left(\frac{20}{3};-4\right);$ v) simmetriya o‘qi *OX*, $A(-6;10)$.

2.78. a) $2a = 18, \varepsilon = \frac{\sqrt{77}}{9};$ b) $k = \frac{6}{7}; c = \sqrt{85};$ v) D: *x=-3*

2.79. a) $b = 5; \varepsilon = \frac{2\sqrt{6}}{7};$ b) $k = \frac{4}{7}; 2a = 14;$ v) D:*x=-3*

2.80. a) $a = 6, \varepsilon = \frac{7\sqrt{3}}{2};$ b) $b = 1, F(-\sqrt{17};0);$ v) simmetriya o‘qi *OY*, $A(-4;-10)$

2.81. a) $b = 4, F(-3;0);$ b) $a = 3, \varepsilon = \frac{\sqrt{13}}{3};$ v) D: *x=8.*

2.82. a) $A(-3\sqrt{5};4) va \quad B(6;-2\sqrt{5});$ b) $k = \frac{5}{9}, \varepsilon = \frac{\sqrt{106}}{9}$; v) D: *y=-16*

2.83. a) $\varepsilon = \frac{\sqrt{39}}{8}; A\left(-4;\frac{5\sqrt{3}}{2}\right);$ b) $A\left(-6;\frac{7\sqrt{7}}{4}\right) \; va \; B\left(\frac{16\sqrt{6}}{7};5\right);$ v) simmetriya o‘qi *OX*,

A(-3;6).

2.84. a) $2a = 12, \varepsilon = \frac{\sqrt{5}}{3};$2 b) $k = \frac{1}{3}; 2c = 4\sqrt{10};$ v) D: *x=8.*

2.85. a) $b = 2, \varepsilon = \frac{\sqrt{3}}{2}$; b) $k = \frac{1}{3}, 2a = 18;$ v) D: *x=-5.*

2.86. a) $a = 9, \varepsilon = \frac{\sqrt{65}}{9};$ b) $b = 4, F(-4\sqrt{5};0);$ v) simmetriya o‘qi *OY*, *A (-3;4).*

2.87. a) $b = 2, F(-2\sqrt{15};0);$ b) $a = 5, \varepsilon = \frac{\sqrt{29}}{5};$ v) D: $x = \frac{5}{8}.$

2.88. a) $A\left(-3;\frac{6}{7}\sqrt{10}\right) va B\left(\frac{7}{3}\sqrt{5};-2\right)$; b) $k=\frac{1}{3};\varepsilon=\frac{\sqrt{10}}{3}$; v) D: $y=-\frac{3}{8}$

2.89. a) $\varepsilon=\frac{4\sqrt{2}}{9};A\left(6;-\frac{7\sqrt{5}}{3}\right)$; b) $A\left(-\frac{9\sqrt{5}}{2};4\right) va B\left(3;-\frac{8\sqrt{10}}{3}\right)$;

v) simmetriya o‘qi *OX*, *A(-3;8)*.

2.90. a) $2a=16,\varepsilon=\frac{\sqrt{7}}{4}$; b) $k=\frac{3}{8},2c=2\sqrt{73}$; v) D: $y=6$

2.91. a) $b=2;\varepsilon=\frac{3\sqrt{5}}{7}$; b) $k=\frac{5}{6},2a=12$; v) D: $x=-\frac{5}{9}$

2.92. a) $a=4,\varepsilon=\frac{\sqrt{7}}{4}$; b) $b=3,F(-\sqrt{34};0)$; v) simmetriya o‘qi *OY*, *A(-3;-4)*.

2.93. a) $b=6,F\left(\sqrt{13};0\right)$; b) $a=9,\varepsilon=\frac{\sqrt{85}}{9}$; v) D: $x=6$.

5-BOB. MATEMATIK ANALIZGA KIRISH.

Dunyodagi narsalar harakat, o‘zgaruvchan miqdorlar va ular orasidagi bog‘lanishdan iborat. Shuning uchun ham, miqdoriy munosabatlarni tavsiflashda sonlar va arifmetika qay darajada zarur bo‘lsa, o‘zgaruvchi miqdorlarni tavsiflash va o‘rganish shu darajada zarurdir.

O‘zgaruvchi miqdorlar va ular o‘zaro qanday bog‘liqligini tasvirlovchi til va matematik metodlar asosini aynan matematik analiz tashkil qiladi.

5.1 MAVZU: BIR O‘ZGARUVCHILI FUNKSIYA.

MAVZUGA OID NAZARIY MATERIALLAR

1. O‘zgaruvchi va o‘zgarmas miqdorlar.

Har xil sonli qiymatlarni qabul qiladigan miqdorning o‘zini *o‘zgaruvchi miqdor* deb ataladi.

Qaralayotgan sharoitda o‘zini sonli qiymatlarini o‘zgartirmaydigan miqdor *o‘zgarmas miqdor* deb ataladi.

Bir miqdorning o‘zi vaziyatga qarab o‘zgaruvchi yoki o‘zgarmas bo‘lishi ham mumkin. Masalan, tekis harakat qilayotgan jismning tezligi o‘zgarmas, erkin tushayotgan jismning tezligi esa o‘zgaruvchidir.

O‘zgarmas miqdorlar $a,b,c,...$ harflar bilan belgilanadi. O‘zgaruvchi

miqdorlar esa x, y, z,. harflar bilan belgilanadi. O‘zgaruvchi miqdorlarning barcha son qiymatlari to‘plami shu o‘zgaruvchining *o‘zgarish sohasi* deyiladi.

2. Funksiya tushunchasi.

1-Ta’rif. Agar x miqdorning D sohadagi har bir qiymatiga biror qonun yoki qoida bo‘yicha y ning biror E sohadagi aniq bir qiymati mos qo‘yilsa, y o‘zgaruvchi miqdor x o‘zgaruvchi miqdorning *funksiyasi* deb ataladi.

O‘zgaruvchi x miqdor *erkli o‘zgaruvchi* yoki *argument*, y miqdor esa *erksiz o‘zgaruvchi* yoki *funksiya* deb ataladi.

y ning x o‘zgaruvchining funksiyasi ekanligi ramziy tarzda $y = f(x)$

ko‘rinishda yoziladi . $y = f(x)$ funksiyaning argument x ni aniq x_0 qiymatidagi xususiy qiymati $f(x_0)$ yoki $y|_{x=x_0}$ kabi belgilanadi.

Masalan, agar $f(x) = 2x + 3$ bo‘lsa $f(1) = 2 \cdot 1 + 3 = 5$, $f(0) = 2 \cdot 0 + 3 = 3$, $f(2) = 2 \cdot 2 + 3 = 7$.

2-Ta’rif. Argument x ning $f(x)$ funksiya ma’noga ega bo‘ladigan qiymatlari to‘plami funksiyaning *aniqlanish sohasi* deb ataladi va *D*(*f*) orqali belgilanadi.

Masalan $f(x) = \frac{1}{x-2}$ funksiya *x=2* nuqtadan boshqa *x* ning barcha qiymatlarida aniqlangan. *x=2* da kasrning maxraji nolga aylanib, funksiya ma’nosini yo‘qotadi, chunki hech bir sonni nolga bo‘lib bo‘lmaydi.

Demak, $D\left(\frac{1}{x-2}\right) = (-\infty;\ 2) \cup (2;\ +\infty)$

3-Ta’rif. $f(x)$ funksiyaning qabul qiladigan qiymatlari to‘plami uning *o‘zgarish sohasi* yoki *qiymatlar to‘plami* deb ataladi va $E(f)$ orqali belgilanadi.

Masalan. *E*(*sinx*)=[-1;1], *E*(*tgx*)=(-∞; ∞).

x va *y* o‘zgaruvchi orasidagi moslik formula orqali ifodalanganda funksiya *analitik usulda* berilgan deb ataladi.

Masalan: $y = x^2$, $y = \frac{1}{x^2 - 1}$, $y = \frac{2^x}{x+1}$.

3. Asosiy elementar funksiyalar,ularning aniqlanish va o‘zgarish sohalari, grafigi.

1. $y = C$-o‘zgarmas(konstanta) funksiya, bunda C –o‘zgarmas son.

2. $y = x^n$-darajali funksiya, bunda n noldan farqli son.

3. $y = a^x$ -ko‘rsatkichli funksiya ($a > 0,\ a \neq 1$).

4. $y = \ell og_a x$ -logarifmik funksiya ($a > 0,\ a \neq 1$).

5. *y=sinx*, *y=cosx*, *y=tgx*, *y=ctgx* -trigonometrik funksiyalar.

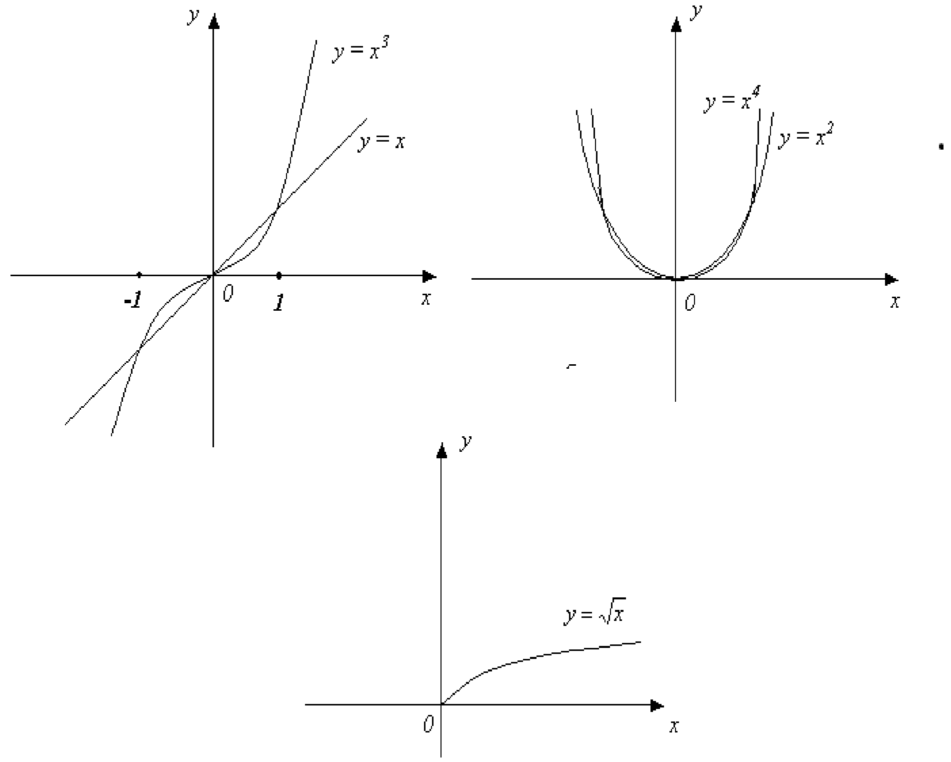
6. *y=arcsinx*, *y=arccosx*, *y=arctgx*, *y=arcctgx* -teskari trigonometrik funksiyalar.

Bu funksiyalar *asosiy elementar funksiyalar* deb ataladi. Shu funksiyalarning aniqlanish va o‘zgarish sohalarini hamda grafiklari bilan tanishamiz.

1. $y = C$ -o‘zgarmas funksiya butun sonlar o‘qida aniqlangan bo‘lib uning qiymatlari sohasi birgina C sondan iborat, ya’ni $D(C) = (-\infty;+\infty)$, $E(C) = C$. Bu funksiyaning grafigi *0X* o‘qiga parallel to‘g‘ri chiziqdan iborat ekanligi aytib o‘tilgan edi.

2. $y = x^n$ -darajali funksiyaning aniqlanish va o‘zgarish sohalari hamda grafigi

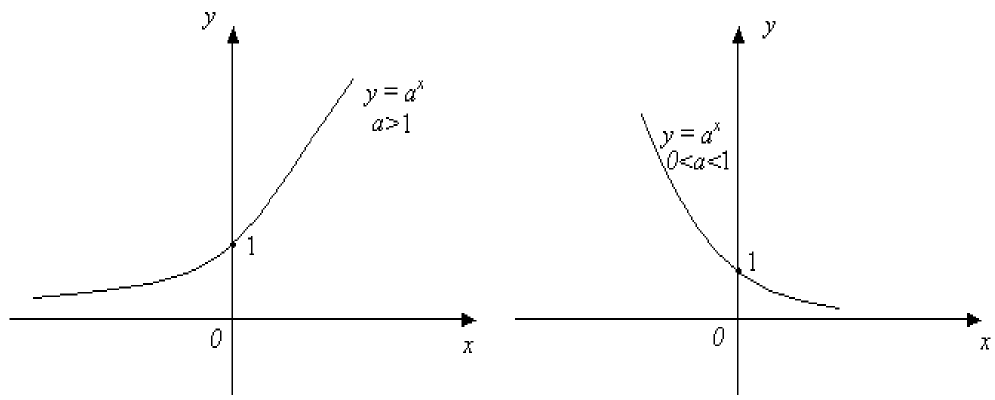
n ko‘rsatkichga bog‘liq. Masalan, $D(x) = D(x^2) = D(x^3) = (-\infty;+\infty)$, $E(x) = (-\infty;+\infty)$, $E(x^2) = [0;+\infty)$, $D(\sqrt{x}) = [0;+\infty)$, $E(\sqrt{x}) = [0;+\infty)$



1-chizma.

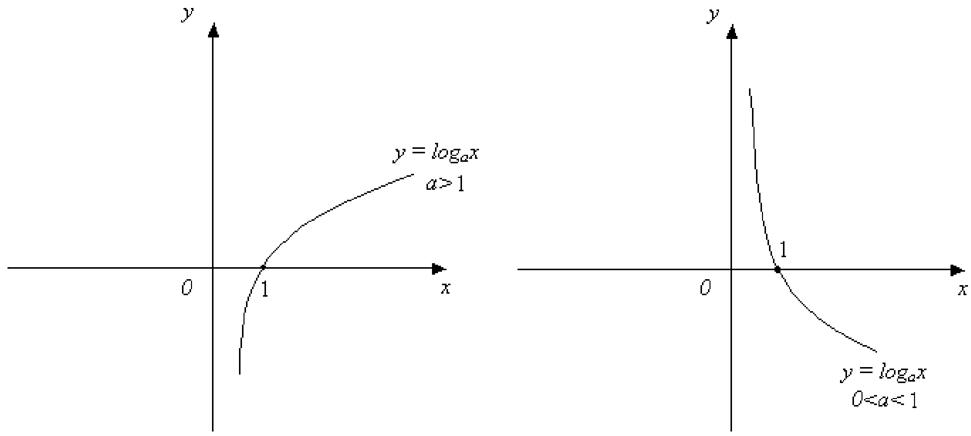
Ba’zi bir darajali funksiyalarning grafiklari 1-chizmada keltirilgan.

3. $y = a^x$ ko‘rsatkichli funksiya ($a > 0,\ a \neq 1$) uchun: $D(a^x) = (-\infty;+\infty)$, $E(a^x) = (0;+\infty)$. Ko‘rsatkichli funksiyaning grafigi 2-chizmada tasvirlangan.



2-chizma.

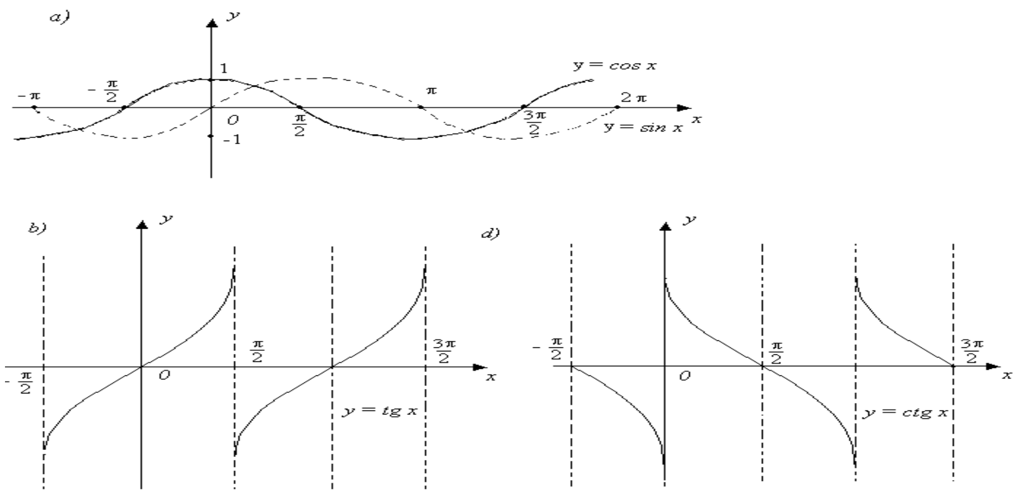
4. $y = \ell og_a x$ logarifmik funksiya ($a > 0,\ a \neq 1$) uchun: $D(\ell og_a x) = (0;+\infty)$, $E(\ell og_a x) = (-\infty;+\infty)$. Logarifmik funksiyaning grafigi chizmada tasvirlangan.



3-chizma.

5. *y=sinx*, *y=cosx*, *y=tgx*, *y=ctgx*-trigonometrik funksiyalar uchun: $D(\sin x)=D(\cos x)=(-\infty;+\infty)$, $E(\sin x)=E(\cos x)=[-1;\,1]$. $y=tgx$ funksiya son o‘qining $(2k+1)\frac{\pi}{2}$ ko‘rinishdagi nuqtalaridan farqli, $y=ctgx$ funksiya esa son o‘qining $x=k\pi$ (*k*-butun son) ko‘rinishdagi nuqtalaridan farqli barcha nuqtalarida aniqlangan. $E(tgx)=E(ctgx)=(-\infty;+\infty)$. Trigonometrik funksiyalarning grafiklari chizmada tasvirlangan.

6. *y=arcsinx*, *y=arccosx*, *y=arctgx*, *y=arcctgx* -teskari trigonometrik funksiyalarning aniqlanish, o‘zgarish sohalari hamda ularning grafiklari bilan keyinroq tanishamiz.



4-chizma.

Asosiy elementar funksiyalar va murakkab funksiya tushunchalaridan foydalanib *elementar funksiya* tushunchasiga ta'rif beramiz.

4-Ta'rif. *Elementar funksiya* deb asosiy elementar funksiyalardan chekli sondagi arifmetik amallar va ulardan olingan murakkab funksiyalardan tuzilgan funksiyaga aytiladi. Asosiy elementar funksiyalarning o'zlari ham elementar funksiyalar sinfiga tegishli.

Masalan. $y = \ell g(1+\sin^2 x)$, $y = 3^{tg(\sin x)}$, $y = \dfrac{x^3 + x^2 + x + 1}{x^2 - 2x + 3}$, $y = \sin^2 3^x$ funksiyalarning barchasi elementar funksiyalardir.

4. Funksiyaning juft va toqligi, davriyligi.

5-Ta'rif. Agar $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli barcha x lar uchun $f(-x) = f(x)$ tenglik bajarilsa $f(x)$ funksiya *juft funksiya* deb ataladi. Agar har bir $x \in D(f)$ uchun $f(-x) = -f(x)$ tenglik bajarilsa $f(x)$ funksiya *toq funksiya* deb ataladi. Juft funksiyaning grafigi *0Y* o'qqa nisbatan, toq funksiyaning grafigi koordinatalar boshiga nisbatan simmetrik bo'ladi.

Masalan. *y=x*, $y = x^3$, *y=sinx* funksiyalar toq $y = x^2, y = x^4$, *y=cosx* funksiyalar juft funksiyalardir. $y = a^x, y = \ell og_a x$ funksiyalar juft ham emas toq ham emas.

Ko'rinib turibdiki, juft funksiyaning ham, toq funksiyaning ham aniqlanish sohasi koordinatalar boshiga nisbatan simmetrik bo'ladi.

6-Ta'rif. Agar o'zgarmas $T \neq 0$ son mavjud bo'lib $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli barcha x lar uchun $f(x \pm T) = f(x)$ tenglik bajarilsa, $f(x)$ funksiya *davriy funksiya* deb ataladi. $f(x \pm T) = f(x)$ tenglikni qanoatlantiruvchi musbat T sonlarning eng kichigi T_0 $f(x)$ funksiya *davri* deb ataladi.

Masalan. *y=sinx*, *y=cosx* funksiyalar davri 2π ga teng davriy funksiyalar, *y=tgx*, *y=ctgx* funksiyalar esa davri π ga teng davriy funksiyalardir.

7-Ta'rif. Agar x ning shu kesmaga tegishli ixtiyoriy ikkita x_1 va x_2 qiymatlari uchun $x_1 < x_2$ bo'lganda $f(x_1) < f(x_2)$ (yoki $f(x_1) > f(x_2)$) tengsizlik o'rinli bo'lsa, $f(x)$ funksiya $[a;\, b]$ kesmada *o'suvchi* (yoki *kamayuvchi*) deyiladi.

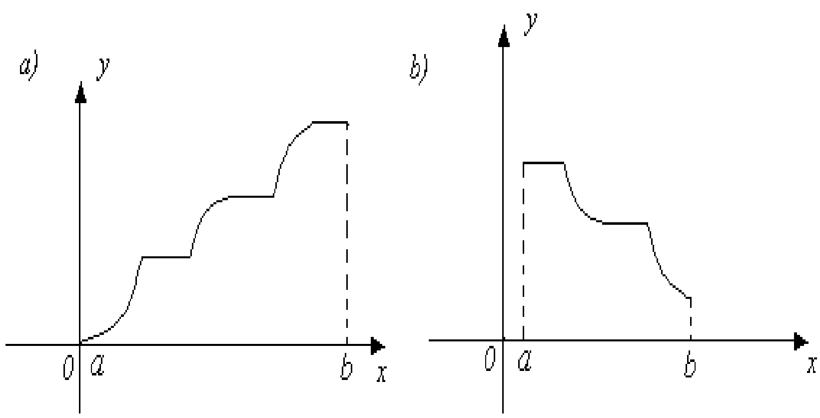
Boshqacha aytganda argumentning katta qiymatiga funksiyaning katta qiymati mos kelsa funksiya o'suvchi deyilar ekan.

Masalan. $y = \sqrt{x}$ (1-chizma) funksiya $[0;\, +\infty)$ da, $y = a^x$ $(a > 1)$ (2-chizma) funksiya $(-\infty;\, +\infty)$da, $y = \ell og_a x$ $(a > 1)$ (3(a)-chizma) funksiya $(0;\, +\infty)$da, *y=sinx* (4-chizma) funksiya $\left[-\frac{\pi}{2};\, \frac{\pi}{2}\right]$ kesmada, *y=cosx* (4-chizma) funksiya $[-\pi;\, 0]$ kesmada o'suvchi.

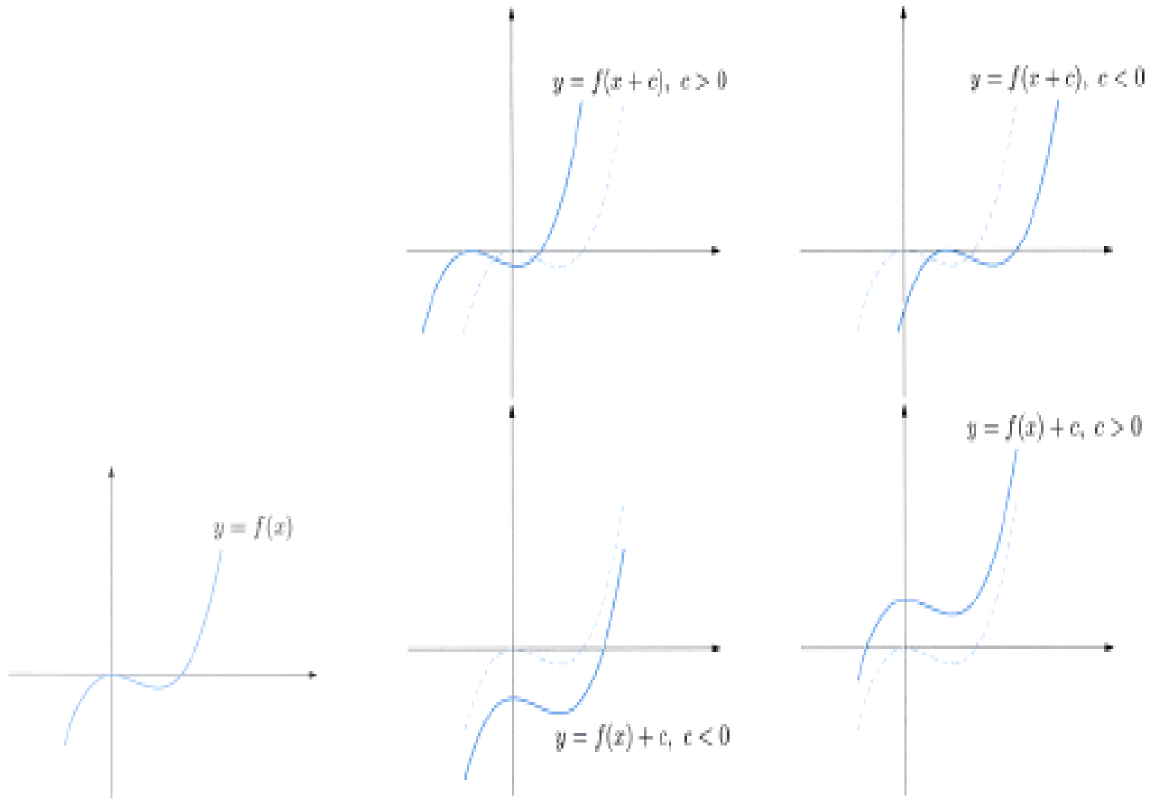
Argumentning katta qiymatiga funksiyaning kichik qiymati mos kelganda funksiya kamayuvchi deyilar ekan.

Masalan. $y = x^2$, $y = x^4$ funksiyalar (1-chizma) $(-\infty;\, 0)$ oraliqda, $y = a^x$ $(0 < a < 1)$ funksiya $(-\infty;\, +\infty)$ oraliqda, $y = \ell og_a x$ $(0 < a < 1)$ (3-chizma) funksiya $(0;\, +\infty)$ oraliqda, *y=cosx* (4-chizma) funksiya $[0;\, \pi]$kesmada kamayuvchi.

Masalan, 5(a)-chizmada kamaymaydigan, 5(b)-chizmada o'smaydigan funksiyalarning grafiklari tasvirlangan.



5-chizma.



6-chizma.

Berilgan funksiyaga va uning argumentiga o‘zgarmas c sonini qo‘shish natijasida funksiya grafigining o‘zgarishlari 6-chizmada tasvirlangan.

5.2. MAVZU: SONLI KETMA-KETLIKLAR.

LIMIT TUSHUNCHASI.

MAVZUGA OID NAZARIY MATERIALLAR

1. Sonli ketma-ketliklar

1-Ta‘rif. Natural sonlar to‘plamida aniqlangan $x_n = f(n)$, $n \in N$ funksiyaning qiymatlari to‘plami *sonli ketma-ketliklar* deb ataladi.

Demak

$$x_1,\ x_2,\ \dots,\ x_n. \text{ yoki } f(1),\ f(2),\dots,\ f(n)$$

sonli ketma-ketlikni tashkil etadi. Sonli ketma-ketlik $\{x_n\}$ yoki $\{f(n)\}$ orqali belgilanadi. Ketma-ketlikning n-hadi $x_n = f(n)$ uning *umumiy hadi* deb ataladi.

Ketma-ketlikning umumiy hadi ma'lum bo'lsa u berilgan hisoblanadi.

Masalan.1) $x_n = \frac{1}{2^n}$ funksiya $\{x_n\} = \left\{\frac{1}{2^n}\right\} = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ..., \frac{1}{2^n}, ...\right\}$ ketma-ketlikni beradi.

2) $x_n = 2n$ funksiya $\{x_n\} = \{2n\} = \{2, 4, 6, ..., 2n, ...\}$ketma-ketlikni beradi.

3) $x_n = 1 + (-1)^n$ funksiya $\{x_n\} = \{1 + (-1)^n\} = \{0,\ 2,\ 0,\ 2,, ..., 1 + (-1)^n, ...\}$ ketma-ketlikni beradi.

4) $x_n = \frac{1}{n}$ funksiya $\{x_n\} = \left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{n}, ...\right\}$ketma-ketlikni beradi.

Barcha misollarda *n* natural son, ya'ni $n \in N$.

Agar istalgan *n* natural son uchun $x_n < x_{n+1}$ tengsizlik bajarilsa, $\{x_n\}$*monoton o'suvchi ketma-ketlik* deb ataladi.

Agar istalgan *n* natural son uchun $x_n > x_{n+1}$ tengsizlik bajarilsa, $\{x_n\}$*monoton kamayuvchi ketma-ketlik* deb ataladi.

Agar istalgan *n* natural son uchun $x_n \geq x_{n+1}$ tengsizlik bajarilsa, $\{x_n\}$ *o'smaydigan ketma-ketlik* deb ataladi.

Agar istalgan *n* natural son uchun $x_n \leq x_{n+1}$ tengsizlik bajarilsa, $\{x_n\}$ *kamaymaydigan ketma-ketlik* deb ataladi.

Masalan. 1) $\{x_n\} = \{n^2\} = \{1, 4, 9, 16, ..., n^2, ...\}$ -o'suvchi, quyidan chegaralangan ketma-ketlik.

2) $\{x_n\} = \{1 - 2n\} = \{-1, -3, -5, ...\}$ -kamayuvchi, yuqoridan chegaralangan ketma-ketlik.

3) $\{x_n\}=\left\{\frac{n}{n+1}\right\}=\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},....,\frac{n}{n+1},...\right\}$-o‘suvchi, chegaralangan ketma-ketlik.

2. Ketma-ketlikning limiti.

2-Ta‘rif. Agar istalgancha kichik $\varepsilon>0$ son uchun shunday $N=N(\varepsilon)$ natural son mavjud bo‘lsaki, undan katta barcha *n* lar uchun $|x_n-a|<\varepsilon$ tengsizlik bajarilsa, *a* o‘zgarmas son $\{x_n\}$ ketma-ketlikning *limiti* deb ataladi va $\lim_{n\to\infty}x_n=a$ yoki $x_n\to a$ kabi belgilanadi.

Agar $\{x_n\}$ ketma-ketlikning limiti *a* chekli son bo‘lsa *ketma-ketlik yaqinlashuvchi***,** aks holda ya’ni *a* mavjud bo‘lmasa yoki ∞ bo‘lsa u *uzoqlashuvchi ketma-ketlik* deb ataladi.

Masalan. $\{x_n\}=\left\{\frac{n}{n+1}\right\}$ ketma-ketlikning limiti 1 ga teng ekanligi ko‘rsatilsin.

Ixtiyoriy $\varepsilon>0$ sonni olib $|x_n-a|=\left|\frac{n}{n+1}-1\right|<\varepsilon$ yoki $\left|-\frac{1}{n}\right|<\varepsilon$ tengsizlikni tuzamiz. Biroq *n>0*, shuning uchun $\frac{1}{n}<\varepsilon$ yoki $n>\frac{1}{\varepsilon}$.

Bundan ko‘rinadiki, $N=N(\varepsilon)$ sifatida $\frac{1}{\varepsilon}$ natural bo‘lganda shu sonni o‘zini, u natural son bo‘lmaganda $\left[\frac{1}{\varepsilon}\right]+1$ sonni (*[x]*- *x* ning butun qismi) olinsa, u holda *n* ning $n>N(\varepsilon)$ tengsizlikni qanoatlantiradigan barcha qiymatlari uchun

$\left|\frac{1}{n}\right|<\varepsilon$ yoki $\left|\frac{n}{n+1}-1\right|<\varepsilon$ tengsizlik bajariladi. Bu esa $\lim_{n\to\infty}\frac{n}{n+1}=1$ ekanini bildiradi.

3. Funksiyaning nuqtadagi limiti.

3-Ta‘rif. Argument *x* ning *a* dan farqli va unga yaqinlashuvchi barcha $\{x_n\}$ ketma-ketliklar uchun $y=f(x)$ funksiyaning shu ketma-ketlik nuqtalaridagi qiymatlaridan tuzilgan $\{f(x_n)\}$ ketma-ketlik b songa yaqinlashsa, b son $y = f(x)$ funksiyaning $x=a$ nuqtadagi (yoki $x \to a$ dagi) limiti deb ataladi va $\lim_{x\to a} f(x) = b$ yoki $x \to a$ da $f(x) \to b$ ko‘rinishda yoziladi.

Endi funksiya limitining Koshi ta‘rifi deb yuritiluvchi ta‘rifini keltiramiz.

4-Ta‘rif. Agar ixtiyoriy musbat $\varepsilon > 0$ son uchun shunday $\delta > 0$ -son topilsaki, x ning $|x-x_0| < \delta$ tengsizlikni qanoatlantiruvchi barcha $x \in R$, $x \neq x_0$ qiymatlarida $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, A soniga f(x) funksiyaning $x \to x_0$ intilgandagi limiti deyiladi va $\lim_{x\to x_0} f(x) = A$ kabi yoziladi.

5-**Ta‘rif.** Agar ixtiyoriy musbat $\varepsilon > 0$ son uchun shunday $\delta > 0$ -son topilsaki, x ning $x_0 < x < x_0+\delta$ ($x_0 - \delta < x < x_0$) tengsizlikni qanoatlantiruvchi barcha $x \in R$, $x \neq x_0$ qiymatlarida $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, A soniga f(x) funksiyaning x_0 nuqtadagi o’ng (chap) limiti deyiladi va $\lim_{x\to x_0+0} f(x) = A$ yoki f(x+0)=A ($\lim_{x\to x_0-0} f(x) = A$ yoki f(x- 0)=A) kabi yoziladi.

f(x) funksiyaning x_0 nuqtadagi o’ng va chap limitlari bir tomonlama limitlar deyiladi. Agar f(x) funksiyaning x_0 nuqtadagi o’ng va chap limitlari mavjud va ular o’zaro teng, ya’ni $f(x_0+0) = f(x_0-0) = A$ bo’lsa, f(x) funksiyaning x_0 nuqtadagi limiti mavjud va $\lim_{x\to x_0} f(x) = A$ bo’ladi.

Masalan.1) $\lim_{x\to 5} \frac{x^2-25}{x^2-5x} = 2$ ekanini tarifdan foydalanib isbotlang.

$f(x) = \frac{x^2-25}{x^2-5x}$ funksiyani x=5 nuqtaning biror atrofida, masalan *(4;6)* intervalda qaraylik. Ixtiyoriy $\varepsilon > 0$ sonni olib $|f(x) - b| < \varepsilon$ ni $x \neq 5$ deb quyidagicha o‘zgartiramiz:

$$\left|\frac{x^2-25}{x^2-5x} - 2\right| = \left|\frac{(x-5)(x+5)}{x(x-5)} - 2\right| = \left|\frac{x+5}{x} - 2\right| = \left|\frac{5-x}{x}\right| = \frac{|5-x|}{|x|}.$$

$x>4$ ekanini hisobga olsak $|x|=x>4$ bo‘lib $\left|\frac{x^2-25}{x^2-5x}-2\right|<\frac{|5-x|}{4}$ kelib chiqadi. Bundan ko‘rinib turibdiki, $\delta=4\varepsilon$ deb olsak, u holda $0<|x-5|<\delta$ tengsizlikni qanoatlantiradigan barcha $x\in(4;6)$ uchun

$$\left|\frac{x^2-25}{x^2-5x}-2\right|<\frac{\delta}{4}=\varepsilon$$

tengsizlik bajariladi. Bundan 2 soni $f(x)=\frac{x^2-25}{x^2-5x}$ funksiyaning x=5 nuqtadagi limiti bo‘lishi kelib chiqadi.

2) $\lim\limits_{x\to 2}\frac{1}{x-2}=\infty$ ekani isbotlansin.

$f(x)=\frac{1}{x-2}$ funksiyani qaraylik. Ixtiyoriy $M>0$ sonni olsak, $|f(x)|=\left|\frac{1}{x-2}\right|>M$ tengsizlik $|x-2|<\frac{1}{M}$ bo‘lganda bajarilishi ko‘rinib turibdi. Agar $\delta=\frac{1}{M}$ deb olinsa, $|x-2|<\delta$ tengsizlikni qanoatlantiradigan barcha x lar uchun $\left|\frac{1}{x-2}\right|>\frac{1}{\delta}=M$ yoki $\left|\frac{1}{x-2}\right|>M$ tengsizlik bajariladi. Bu esa $x\to 2$ da $f(x)=\frac{1}{x-2}$ funksiya cheksizlikka intilishini bildiradi, ya’ni $\lim\limits_{x\to 2}\frac{1}{x-2}=\infty$

3) $\lim\limits_{x\to\infty}\frac{x+1}{x}=1$ ekani isbotlansin.

$f(x)=\frac{x+1}{x}$ funksiyani qaraylik. Istalgan $\varepsilon<0$ sonni olsak $|f(x)-b|=\left|\frac{x+1}{x}-1\right|=\left|\frac{x+1-x}{x}\right|=\frac{1}{|x|}$ bo‘lib $N=\frac{2}{\varepsilon}$ desak, barcha $|x|>N$ uchun $\left|\frac{x+1}{x}-1\right|<\frac{2}{N}=\varepsilon$ tengsizlik o‘rinli bo‘ladi. Bundan 1 soni $f(x)=\frac{x+1}{x}$ funksiyaning $x\to\infty$ dagi limiti bo‘lishi ayon bo‘ladi.

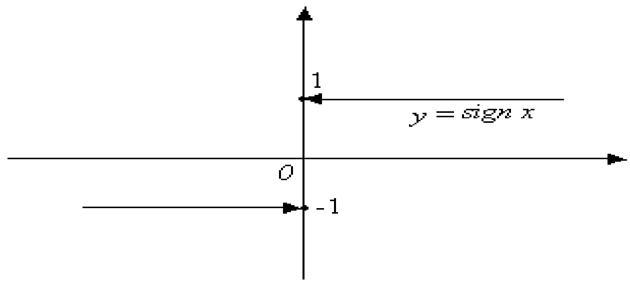
4) $\lim_{x\to\infty} x^2 = \infty$ ekani isbotlansin.

$f(x) = x^2$ funksiyani qaraylik. Istalgan M>0 sonni olib $|f(x)| > M$ tengsizlikni tuzamiz. x^2 >M, bundan $|x| > \sqrt{M}$ kelib chiqadi. $N = \sqrt{M}$ deb olinsa, $|x| > N$ tengsizlikni qanoatlantiradigan barcha x lar uchun $x^2 > N^2 = M$ tengsizlik bajariladi. Bu $\lim_{x\to\infty} x^2 = \infty$ ekanini bildiradi.

5)

$$f(x) = signx = \begin{cases} 1, & agar \quad x > 0 \quad bo'lsa, \\ 0, & agar \quad x = 0 \quad bo'lsa, \\ -1, & agar \quad x < 0 \quad bo'lsa \end{cases}$$

funksiya $x=a$ nuqtada limitga ega emas, chunki $f(-0)$=-1, $f(+0)$=1 va $f(-0) \neq f(+0)$ (7-chizma). Bu funksiya *0* dan farqli istalgan nuqtada limitga ega.



7-chizma.

4. Limitlar haqida asosiy teoremalar. Ajoyib limitlar.

Cheksiz kichik funksiyalarni taqqoslash.

1-Teorema. Chekli sondagi limitga ega funksiyalar algebraik yig‘indisining limiti qo‘shiluvchi funksiyalar limitlarining algebraik yig‘indisiga teng, ya’ni

$$\ell im(u_1(x) + u_2(x) + ... + u_n(x)) = \ell im\, u_1(x) + \ell im\, u_2(x) + ... + \ell im\, u_n(x)$$

Masalan. 1) $\ell im_{x\to 2} \frac{x^2-4}{x-2} = \ell im_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \ell im_{x\to 2}(x+2) = \ell im_{x\to 2} x + \ell im_{x\to 2} 2 = 2 + 2 = 4$.

2) $\lim\limits_{x\to\infty}\frac{x^4-5x^2}{x^4}=\lim\limits_{x\to\infty}\left(\frac{x^4}{x^4}-\frac{5x^2}{x^4}\right)=\lim\limits_{x\to\infty}\left(1-\frac{5}{x^2}\right)=\lim\limits_{x\to\infty}1-\lim\limits_{x\to\infty}\frac{5}{x^2}=1-0=1$.

2-Teorema. Chekli sondagi limitga ega funksiyalar ko‘paytmasining limiti shu funksiyalar limitlarining ko‘paytmasiga teng, ya’ni

$$\lim(u_1(x)\cdot u_2(x)\cdot\ldots\cdot u_n(x))=\lim u_1(x)\cdot\lim u_2(x)\cdot\ldots\cdot\lim u_n(x).$$

Masalan.1) $\lim\limits_{x\to2}(x+3)(x-4)=\lim\limits_{x\to2}(x+3)\lim\limits_{x\to2}(x-4)=[\lim\limits_{x\to2}x+\lim\limits_{x\to2}3]\cdot[\lim\limits_{x\to2}x-\lim\limits_{x\to2}4]=$

$=(2+3)(2-4)=5\cdot(-2)=-10$.

2) $\lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)\left(2-\frac{1}{x^2}\right)=\lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)\lim\limits_{x\to\infty}\left(2-\frac{1}{x^2}\right)=(1-0)(2+0)=2$

3) $\lim\limits_{x\to-1}7x^2=7\lim\limits_{x\to-1}x^2=7\cdot(-1)^2=7$.

3-Teorema. Ikkita limitga ega funksiya bo‘linmasining limiti maxrajning limitin noldan farqli bo‘lganda, shu funksiyalar limitlarining bo‘linmasiga teng, ya’ni agar $\lim v\neq0$ bo‘lsa, $\lim\frac{u}{v}=\frac{\lim u}{\lim v}$ bo‘ladi.

Masalan.1) $\lim\limits_{x\to2}\frac{2x+3}{3x+1}$ ni toping.

Yechish. $\lim\limits_{x\to2}(3x+1)=3\cdot2+1=7\neq0$. Shuning uchun:

$$\lim\limits_{x\to2}\frac{2x+3}{3x+1}=\frac{\lim\limits_{x\to2}(2x+3)}{\lim\limits_{x\to2}(3x+1)}=\frac{2\cdot2+3}{3\cdot2+1}=\frac{7}{7}=1.$$

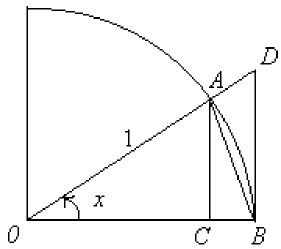
2) $\lim\limits_{x\to3}\frac{x+1}{x-3}$ ni toping.

Yechish. $\lim\limits_{x\to 3}(x-3)=3-3=0$ bo‘lgani uchun teoremani qo‘llab bo‘lmaydi. Suratning limiti $\lim\limits_{x\to 3}(x+1)=3+1=4\neq 0$ bo‘lgani uchun berilgan ifodaning teskarisining limitini topamiz: $\lim\limits_{x\to 3}\frac{x-3}{x+1}=\frac{\lim\limits_{x\to 3}(x-3)}{\lim\limits_{x\to 3}(x+1)}=\frac{3-3}{3+1}=\frac{0}{4}=0$

Bundan $\lim\limits_{x\to 3}\frac{x+1}{x-3}=\infty$ kelib chiqadi, chunki cheksiz kichik funksiyaga teskari funksiya cheksiz katta funksiya bo‘ladi.

3) $\lim\limits_{x\to 0}\sin x=0$ isbotlansin.

Radiusi 1 ga teng aylanani qaraymiz. 8-chizmadan: $x>0$ bo’lsa $\frac{AC}{OA}=\sin x$; $AC=\sin x$, $\breve{AB}=x$ (markaziy burchak o‘zi tiralgan yoy bilan o‘lchanadi), AC< $\breve{AB}$ yoki $\sin x<x$ ekani ayon bo‘ladi. $x<0$ bo‘lganda $|\sin x|<|x|$ bo‘lishi ravshan.



8-chizma.

Shunday qilib $x>0$ uchun $0<\sin x<x$ va $x<0$ uchun $0<|\sin x|<|x|$ tengsizliklarga ega bo‘ldik.

$\lim\limits_{x\to 0}0=\lim\limits_{x\to 0}x=0$ ekanligini hisobga olsak $\lim\limits_{x\to 0}\sin x=0$ ekanligi kelib chiqadi.

4). $\lim\limits_{x\to 0}\sin\frac{x}{2}=0$ isbotlansin.

$0<\left|\sin\frac{x}{2}\right|<|\sin x|$ ekani ravshan. $\lim\limits_{x\to 0}0=\lim\limits_{x\to 0}|\sin x|=0$ bo‘lgani uchun teoremaga binoan $\lim\limits_{x\to 0}\left|\sin\frac{x}{2}\right|=0$ yoki $\lim\limits_{x\to 0}\sin\frac{x}{2}=0$ kelib chiqadi.

5). $\lim\limits_{x\to 0}\cos x=1$ ekanligi isbotlansin.

$2\sin^2\frac{x}{2}=1-\cos x$ yoki $\cos x=1-2\sin^2\frac{x}{2}$ ekanligini e`tiborga olsak

$\lim\limits_{x\to 0} \cos x = \lim\limits_{x\to 0}\left(1-2\sin^2\frac{x}{2}\right) = 1-2\lim\limits_{x\to 0}\sin^2\frac{x}{2} = 1-2\cdot 0^2 = 1$ hosil bo‘ladi.

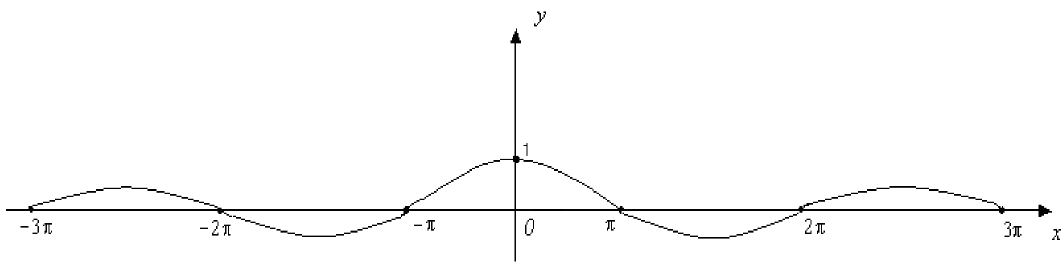
5. Birinchi ajoyib limit.

$\frac{\sin x}{x}$ funksiya faqat $x=0$ nuqtada aniqlanmagan, chunki bu nuqtada kasrning surati ham, maxraji ham 0 ga aylanadi, ya’ni $\frac{0}{0}$ ko‘rinishga ega bo‘ladi. Shu funksiyaning $x\to 0$ dagi limitini topamiz. Bu limit *birinchi ajoyib limit* deb ataladi.

4-Teorema. $\frac{\sin x}{x}$ funksiya $x\to 0$ da 1 ga teng limitga ega.

$$\lim_{x\to 0}\frac{\sin x}{x}=1.$$

$y=\frac{\sin x}{x}$ funksiyaning grafigi 9-chizmada tasvirlangan.



9-chizma

Masalan.1) $\lim\limits_{x\to 0}\frac{tg\,x}{x}=\lim\limits_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x}=\lim\limits_{x\to 0}\frac{\sin x}{x}\frac{1}{\cos x}=\lim\limits_{x\to 0}\frac{\sin x}{x}\lim\limits_{x\to 0}\frac{1}{\cos x}=1\cdot\frac{1}{1}=1.$

2) $\lim\limits_{x\to 0}\frac{\sin mx}{x}=\lim\limits_{x\to 0} m\cdot\frac{\sin mx}{mx}=m\lim\limits_{x\to 0}\frac{\sin mx}{mx}=m\cdot 1=m$. ($m$-o‘zgarmas son).

3). $\lim\limits_{x\to 0}\frac{\sin\alpha x}{\sin\beta x}=\lim\limits_{x\to 0}\frac{\frac{\sin\alpha x}{x}}{\frac{\sin\beta x}{x}}=\frac{\lim\limits_{x\to 0}\frac{\sin\alpha x}{x}}{\lim\limits_{x\to 0}\frac{\sin\beta x}{x}}=\frac{\alpha}{\beta}.$

6. Ikkinchi ajoyib limit.

Ushbu $\{x_n\}=\left\{\left(1+\frac{1}{n}\right)^n\right\}$ sonli ketma-ketlikni qaraymiz, bunda *n*-natural son.

5-Teorema. Umumiy hadi $x_n=\left(1+\frac{1}{n}\right)^n$ bo‘lgan ketma-ketlik $n\to\infty$ da

2 bilan 3 orasida yotadigan e ($e=2{,}7182818284\,...$) limitga ega.

6-Teorema. $\left(1+\frac{1}{x}\right)^x$ funksiya $x\to\infty$ da *e* songa teng limitga ega:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

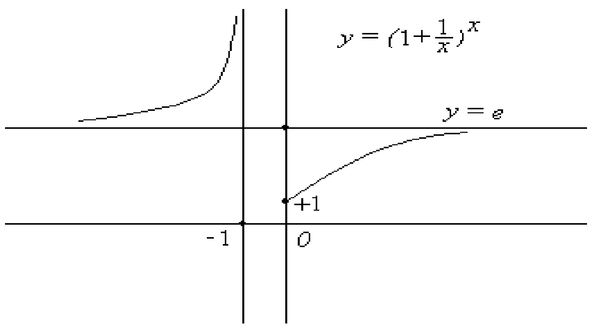
Bu limit *ikkinchi ajoyib limit* deb yuritiladi.

Agar bu tenglikda $\frac{1}{x}=\alpha$ deb faraz qilinsa, u holda $x\to\infty$ da $\alpha\to 0$ $(\alpha\neq 0)$ va

$$\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e$$

tenglikni hosil qilamiz. Bu ikkinchi ajoyib limitning yana bir ko‘rinishi

$y=\left(1+\frac{1}{x}\right)^x$ funksiyaning grafigi 10-chizmada tasvirlangan.



10-chizma

Masalan.1) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+8}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^8=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^8=e(1+0)^8=e$.

2) $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$ topilsin.

Yechish. $x=3t$ desak, $x\to\infty$ da $t\to\infty$ va

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{3t}=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}\left(1+\frac{1}{t}\right)^{t}\left(1+\frac{1}{t}\right)^{t}=$$

$$=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}=e\cdot e\cdot e=e^{3}$$ bo‘ladi.

3) $\lim\limits_{x\to\infty}\left(\frac{x+4}{x+1}\right)^{x+2}=\lim\limits_{x\to\infty}\left(\frac{x+1+3}{x+1}\right)^{x+1+1}=\lim\limits_{x\to\infty}\left(1+\frac{3}{x+1}\right)^{(x+1)+1}=$

$$=\lim_{y\to\infty}\left(1+\frac{3}{y}\right)^{y+1}=\lim_{y\to\infty}\left(1+\frac{3}{y}\right)^{y}\cdot\lim_{y\to\infty}\left(1+\frac{3}{y}\right)^{1}=e^{3}\cdot 1=e^{3}.$$

4) $\lim\limits_{x\to 0}\frac{\sin 5x}{x}=\lim\limits_{x\to 0}\frac{5x}{x}=5.$

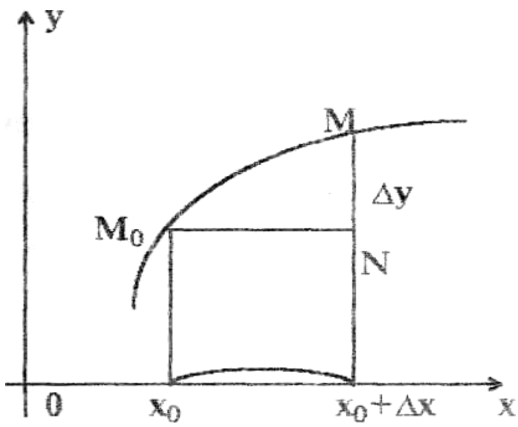
5) $\lim\limits_{x\to 0}\frac{tg\,5x}{\sin 7x}=\lim\limits_{x\to 0}\frac{5x}{7x}=\frac{5}{7}.$

7. Funksiyalarning uzluksizligi.

Faraz qilaylik; $y=f(x)$ funksiya biror x_0 qiymatda aniqlangan $y_0=f(x_0)$ bo‘lsin. Agar x biror musbat yoki manfiy (farqi yo‘q) Δx orttirma olsa va $x=x_0+\Delta x$ qiymatga ega bo‘lib qolsa, funksiya ham orttirma oladi. Ya'ni

$y_0+\Delta y=f(x_0+\Delta x)$ bo'ladi. Bundan funksiyaning orttirmasi $\Delta y=f(x_o+\Delta x)-f(x_0)$ formula bilan ifodalanadi.

4-**Ta‘rif.** Agar f(x) funksiya x_0 nuqtada chekli limitga ega bo’lib, bu limit funksiyaning shu nuqtadagi qiymatiga teng, yani $\lim_{x\to x_0} f(x)=f(x_0)$ bo’lsa, u holda f(x) funksiya x_0 nuqtada uzluksiz deyiladi.



11-chizma

Agar $\Delta y = f(x_o+\Delta x) - f(x_0)$ funksiya orttirmasining Δx argument orttirmasi $\Delta x \to 0$ nolga intilgandagi limiti nolga teng bo'lsa, ya'ni

$$\lim_{\Delta x \to 0} \Delta y = 0 \qquad (6)$$

yoki

$$\lim_{\Delta x \to 0} [f(x_0 + \Delta x) - f(x_0)] = 0 \qquad (7)$$

bo'lsa, $\boldsymbol{x=x_0}$ qiymatda (yoki $\boldsymbol{x}_0$ nuqtada) funksiya uzluksiz deyiladi.

Masalan. $y=x^2$ funksiyaning ixtiyoriy x_0 nuqtada uzluksizligini isbotlaymiz. Haqiqatan,

$$y_0=x_0^2, \quad y_0+\Delta y = (x_0+\Delta x)^2;$$

$$\Delta y_0 = x_0^2+2x_0\Delta x+\Delta x^2-x_0^2 = 2x_0\Delta x+\Delta x^2$$

x istalgan usul bilan nolga intilganda

$$\lim_{\Delta x \to 0} \Delta y = \lim_{\Delta x \to 0} (2x_0\Delta x + \Delta x^2) = 2x \lim_{\Delta x \to 0} \Delta x + \lim_{\Delta x \to 0} \Delta x \cdot \lim_{\Delta x \to 0} \Delta x = 0$$

7-Teorema. Agar $f_1(x)$ va $f_2(x)$ funksiyalar x nuqtada uzluksiz bo'lsa, unda $f(x)=f_1(x)+f_2(x)$ yig'indi ham $\boldsymbol{x}$ nuqtada uzluksiz funksiyadir.

Natija:

a) Ikki uzluksiz funksiyaning ko‘paytmasi uzluksiz funksiyadir.

b) Ikki uzluksiz funksiyaning bo‘linmasi, agar qaralayotgan nuqtada maxraj nolga aylanmasa, uzluksiz funksiyadir.

b) Agar $x=x_0$ da $U=(ų(x))$ uzluksiz va $U_0= ų(x_0)$ nuqtada funksiya uzluksiz bo‘lsa, murakkab funksiya ham x nuqtada uzluksizdir.

5-Ta‘rif. Agar $y=f(x)$ funksiya biror (a,b) intervalning har bir nuqtasida uzluksiz bo‘lsa, funksiya shu intervalda uzluksiz deyiladi, bu erda $a<b$.

Agar funksiya $x=a$ da ham aniqlangan va bunda $\lim\limits_{x\to a+0} f(x) = f(a)$ bo‘lsa, u holda $f(x)$ funksiya $x=a$ nuqtada o‘ngdan uzluksiz deyiladi.

Agar $\lim\limits_{x\to b-0} f(x) = f(b)$ bajarilsa, u holda $f(x)$ funksiya $x=b$ nuqtada chapdan uzluksiz deyiladi.

Agar biror $x=x_o$ nuqtada $y=f(x)$ funksiya uchun uzluksiz shartlaridan hech bo‘lmaganda bittasi bajarilmasa, ya’ni agar $x=x_o$ da funksiya aniqlanmagan bo'lsa, yoki $\lim\limits_{x\to x_0} f(x)$ limit mavjud bo‘lmasa, yoki $x\to x_o$ intilish ixtiyoriy bo‘lsayu o‘ngda va chapda turgan ifodalar mavjud bo‘lsada, $\lim\limits_{x\to x_0} f(x) \neq f(x)$ bo‘lsa,u holda $y=f(x)$ funksiya $x=x_0$ nuqtada uzilgan bo‘ladi. Bu holda $x=x_0$ nuqta funksiyasining uzilish nuqtasi deyiladi.

Masalan. $y=\dfrac{1}{x}$ funksiya $x=0$ bo‘lganda uzilishga ega. Haqiqatan $x=0$ da funksiya aniqlanmagan. Bu funksiya istalgan $x \neq 0$ da uzluksizdir.

8-Teorema. Barcha elementar funksiyalar o‘zlarining aniqlanish sohalarida uzluksizdirlar.

Masalan.1) $y=x^2$ funksiya istalgan **x** nuqtada uzluksiz va shuning uchun

$$\lim_{x\to x_0} x^2 = x_0^2\ , \quad \lim_{x\to 3} x^2 = 3^2 = 9.$$

2) $\ell im_{x\to\frac{\pi}{2}} 4^{\sin x}$ topilsin.

Yechish. $4^{\sin x}$ murakkab funksiya $x=\dfrac{\pi}{2}$ nuqtada uzluksiz bo‘lgani uchun

$\ell im_{x\to\frac{\pi}{2}} 4^{\sin x} = 4^{\sin\frac{\pi}{2}} = 4^1 = 4$ bo‘ladi.

3) $\ell im_{x\to 0} \dfrac{a^x-1}{x}$ topilsin.

Yechish. Bu yerda $\dfrac{0}{0}$ ko‘rinishdagi aniqmaslikka egamiz. $a^x-1=t$ almashtirish olamiz. U holda $a^x=1+t$, $\quad x=\ell og_a(1+t)$ bo‘lib $x\to 0$ da $t\to 0$ da va

$$\ell im_{x\to 0}\frac{a^x-1}{x} = \ell im_{t\to 0}\frac{t}{\ell og_a(1+t)} = \ell im_{t\to 0}\frac{1}{\frac{1}{t}\ell og_a(1+t)} = \frac{1}{\ell im_{t\to 0}\ell og_a(1+t)^{\frac{1}{t}}} = \frac{1}{\ell og_a e} = \ell og_e a = \ell na$$

bo‘ladi. Xususiy holda $\ell im_{x\to 0}\dfrac{e^x-1}{x} = \ell ne = 1$ kelib chiqadi, ya’ni $x\to 0$ да $e^x-1\sim x$

4) $\ell im_{x\to 0}\dfrac{(1+x)^p-1}{x}$ topilsin.

Yechish. Bu yerda $\dfrac{0}{0}$ aniqmaslikka egamiz. $(1+x)^p-1=y$ almashtirish olamiz. U holda $(1+x)^p=1+y$, yoki buni e asosga ko‘ra logarifmlasak $p\ell n(1+x)=\ell n(1+y)$ bo‘ladi. $x\to 0$ da $y\to 0$. Demak,

$$\ell im_{x\to 0}\frac{(1+x)^p-1}{x} = \ell im_{x\to 0}\frac{y}{x} = \ell im_{x\to 0}\frac{p\ell n(1+x)}{x}\cdot\frac{y}{\ell n(1+y)} = p\,\ell im_{x\to 0}\frac{\ell n(1+x)}{x}\cdot\ell im_{y\to 0}\frac{y}{\ell n(1+y)} = p\cdot 1\cdot 1 = p.$$

Shunday qilib, $\ell im_{x\to 0}\dfrac{(1+x)^p-1}{x}=p$ formulaga ega bo‘ldik.

Endi asosiy elementar funksiyalarning aniqlanish sohalarining chetlaridagi limitlari hamda ajoyib limitlar jadvalini keltiramiz.

1) $x = a$ nuqtada uzluksiz $y = f(x)$ funksiya uchun $\ell im_{x\to a} f(x) = f(a)$ bo‘ladi.

2) $\ell im_{x\to+\infty} e^x = +\infty$, $\ell im_{x\to-\infty} e^x = 0$.

3) $a > 1$ bo‘lganda $\ell im_{x\to+\infty} a^x = +\infty$, $\ell im_{x\to-\infty} a^x = 0$ bo‘ladi.

4) $0 < a < 1$ bo‘lganda $\ell im_{x\to+\infty} a^x = 0$, $\ell im_{x\to-\infty} a^x = +\infty$ bo‘ladi.

5) $\alpha > 0$ bo‘lganda $\ell im_{x\to+\infty} x^\alpha = +\infty$, $\alpha < 0$ bo‘lganda $\ell im_{x\to+\infty} x^\alpha = 0$ bo‘ladi;

6) $\ell im_{x\to+\infty} \ell nx = +\infty$, $\ell im_{x\to+0} \ell nx = -\infty$.

6) $a > 1$ bo‘lganda $\ell im_{x\to+\infty} \ell og_a x = +\infty$, $\ell im_{x\to+0} \ell og_a x = -\infty$

7) $0 < a < 1$ bo‘lganda $\ell im_{x\to+\infty} \ell og_a x = -\infty$, $\ell im_{x\to+0} \ell og_a x = +\infty$.

8) $\ell im_{x\to\frac{\pi}{2}-0} tgx = +\infty$, $\ell im_{x\to\frac{\pi}{2}+0} tgx = -\infty$.

9) $\ell im_{x\to+\infty} arctgx = \frac{\pi}{2}$, $\ell im_{x\to-\infty} arctgx = -\frac{\pi}{2}$.

10) $\ell im_{x\to 0} \frac{\sin x}{x} = 1$.

11) $\ell im_{x\to\pm\infty} \left(1 + \frac{1}{x}\right)^x = e$.

12) $\ell im_{x\to 0} \frac{a^x - 1}{x} = \ell n a$

12) $\ell im_{x\to 0} \frac{e^x - 1}{x} = 1$

13) $\ell im_{x\to 0} \frac{(1+x)^p - 1}{x} = p$

14) $\ell im_{x\to 0} \frac{\ell og_a(1+x)}{x} = \frac{1}{\ell n a}$

15) $\ell im_{x\to 0} \frac{\ell n(1+x)}{x} = 1$.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. $f(x) = x^2 - 3x + 4$ funksiya berilgan. *f*(1), *f*(0), *f*(-1), topilsin.

J:f(1)=2, f(0)=4,f(-1)=8.

2. $f(x)=x^2+4$ funksiya berilgan.

Quyidagi qiymatlar topilsin: a) $f(5)$ b) $f(\sqrt{3})$ d) $f(a+1)$ e) $f(a^2)$ f) $f(2a)$.

J:a) $a)\, f(5)=29, b)\, f(\sqrt{3})=7, d)\, f(a+1)=a^2+2a+5,\ e)\, f(a^2)=a^4+4,\ f)\, f(2a)=4(a^2+1)$

3. $f(x)=\dfrac{x+1}{2+3x}$ bo‘lsa $f\left(\dfrac{1}{x}\right)$ va $\dfrac{1}{f(x)}$ topilsin. J: $f\left(\dfrac{1}{x}\right)=\dfrac{x+1}{2+3}$; $\dfrac{1}{f(x)}=\dfrac{2+3x}{x+1}$

4. Quyidagi funksiyalarning aniqlanish sohasi topilsin.

a) $\sqrt{9-x^2}$, b) $\sqrt[3]{x+a}-\sqrt[5]{b-x}$, c) $\dfrac{x^2+3x+1}{x+1}$, f) $\ell g(x^2+5x+6)$, g) $y=4^x$

J: $a)\ 9-x^2\geq 0,\ |x|\leq 3,\ b)\ x\in(-\infty;+\infty),\ c)\ x+1\neq 0,\ x\neq 1,\ f)\ x\in(-\infty;-3)\cup(-2;\infty)$

5. Quyidagi funksiyalarning grafiklari yasalsin.

a) $y=2x-3$, b) $y=\dfrac{1}{4}x^2-1$, c) $y=x-x^2$, d) $y=x^2+2x-3$, e) $y=\ell og_2\dfrac{1}{x}$

6. a) $y=8$, b) $y=x^2$, d) $y=\sin x$, e) $y=\cos 5x^2$, f) $y=7^x$, g), $y=\ell gx$ funksiyalardan qaysi biri murakkab funksiya. J: e

7. Quyidagi funksiyalardan qaysi birlari juft funksiya. a) $y=x^2-2x+1$, b) $y=x^3-1$, d) $y=\sin^2 x+\cos x$, e) $y=\sqrt{x}$, f) $y=\dfrac{x^2-1}{1+x^4}$, g) $y=2^x+2^{-x}$, h) $y=\sqrt[3]{x^3+1}$ J: d),f),g).

8. Quyidagi funksiyalardan qaysi biri toq funksiya. a) $y=\sin^3 x$, b) $y=tgx-x$, d) $y=x^3-x+1$, e) $y=\sqrt{x^3+1}$, f) $y=\dfrac{x^3-1}{1+x^5}$, g) $y=3^x-3^{-x}$, h) $y=\dfrac{|x|}{\sqrt{x^2+1}}$ .J: a)

9. Quyidagi funksiyalardan davriy bo‘lmaganini toping.

a) $y=\sin x\cdot\cos x$, b) $y=|\cos x|$, d) $y=tg^3 x$, e) $y=\sin x+4$. J: a)

10. Quyidagi funksiyalardan qaysi birlari $\left(0;\ \dfrac{\pi}{2}\right)$ oraliqda o‘sadi.

a) $y = tgx$, b) $y = ctgx$, d) $y = \sin x$, e) $y = \cos x$, f) $y = x^2$, g) $y = \frac{1}{x}$.

J:a), d), f).

11. Quyidagi funksiyalardan qaysi birlari $(-3;\ 0)$ oraliqda kamayadi.

a) $y = x^2$, b) $y = \frac{1}{x^2}$, d) $y = 3^{-x}$, e) $y = \ell g\frac{1}{x}$. J: a),d).

13. Quyidagi funksiyalardan qaysi birlari $(\ 0;\ 1)$ intervalda chegaralangan.

a) $y = x^3$, b) $y = \sin x$, c) $y = 3^{-x}$, e) $y = ctgx$, f) $y = \frac{1}{1-x}$, g) $y = \frac{1}{\sin x}$,

h) $y = \ell g(1-x)$. J:b), f), h).

Funksiyalarning aniqlanish sohasini toping.

15. $f(x) = \frac{2x+1}{x^2+x-2}$ J: $x \in (-\infty;-2) \cup (-2;1) \cup (1;\infty)$

16. $g(x) = \frac{\sqrt[3]{x}}{x^2+1}$ J: $x \in (-\infty;+\infty)$

17. $f(x) = \sqrt{4-x} + \sqrt{x^2-1}$ J: $x \in (-\infty;-1] \cup [1;4]$

18. $y = x^3$ J: $x \in (-\infty;+\infty)$

19. $y = (x-2)^3 + 3$ J: $x \in (-\infty;+\infty)$; 20. $y = 4 - x^2$ J: $x \in (-\infty;+\infty)$

21. $y = \sqrt{x}$ J: $x \in [0;+\infty)$ 22. $y = 2\sqrt{x}$ J: $x \in [0;+\infty)$

23. $y = -2^x$ J: $x \in (-\infty;+\infty)$ 24. $f(x) = \begin{cases} 1-x^2, x \le 0 \\ 2x+1, x > 0 \end{cases}$ J: $x \in (-\infty;+\infty)$

25. Limitning ta'rifidan foydalanib quyidagilar isbotlansin:

a) $\lim_{n\to\infty}\left(1+\frac{(-1)^n}{2n+1}\right) = 1$; d) $\lim_{n\to\infty}\frac{n+1}{n+4} = 1$.

26. $\{x_n\}=\{(-1)^n\}$ ketma-ketlikning limitga ega emasligini ko‘rsating.

27. $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}=6$ ekanligini isbotlang .

28. $\lim\limits_{x\to 2}(3x-2)=4$ ekanligini isbotlang.

29. $f(x)=(x-3)^2$ funksiya x → 3 da cheksiz kichik funksiya ekanligini ko‘rsating.

30. $f(x)=\dfrac{1}{4-x}$ funksiya $x\to 4$ da cheksiz katta funksiya ekanligini ko‘rsating.

31. $f(x)=\dfrac{1}{(x+2)^2}$ funksiya $x\to -2$ da cheksiz katta funksiya ekanligini ko‘rsating.

32. $f(x)=3x+2$ funksiya $x\to\infty$ da cheksiz katta funksiya ekanligini ko‘rsating.

33. $f(x)=\begin{cases}0, & agar\ \ x\le 0\ \ bo'lsa,\\ x^2+1, & agar\ \ 0<x<1\ \ bo'lsa,\\ 3, & agar\ \ x\ge 1\ \ bo'lsa.\end{cases}$ funksiyaning x=0 va x=1 nuqtalardagi bir tomonlama limitlari topilsin

Quyidagi limitlarni hisoblang.

34. $\ell im_{x\to 1}\dfrac{x^2+2x+5}{x+1}.$ J:4

35. $\ell im_{x\to\frac{\pi}{2}}(3\sin x-6\cos x+4ctgx).$ J:3

36. $\ell im_{x\to 1}\dfrac{x^2-1}{x-1}.$ J:2

37. $\ell im_{x\to 3}\dfrac{x-3}{\sqrt{x+3}}.$ J:0

38. $\ell im_{x\to+\infty}\left(3-\dfrac{1}{x}+\dfrac{2}{\sqrt{x}}-\dfrac{5}{x^3}\right).$ J:3

39. $\ell im_{x\to\infty}\dfrac{x-6}{x}.$ J:1

40. $\ell im_{x\to\infty}\dfrac{4x^3+2x^2+x-3}{2x^3+x-2}.$ J:2

41. $\ell im_{x\to\infty}\dfrac{2x^2+x-3}{2x+3}.$ J: ∞

42. $\ell im_{x\to\infty}\dfrac{x+3}{2x^2+5}.$ J:0

43. $\ell im_{x\to 2}\dfrac{x^3-8}{x-2}.$ J:12

44. $\ell im_{x\to 3}\dfrac{x^2-7x+12}{x^2-5x+6}$. J:-1

45. $\ell im_{x\to 1}\dfrac{x^n-1}{x-1}$. J:1

46. $\ell im_{x\to 0}\dfrac{\sqrt[3]{1+x}-1}{x}$. J: $\dfrac{1}{3}$

47. $\ell im_{x\to 1}\left[\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right]$. J: 1

48. $\ell im_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$. J: $\dfrac{2}{3}$

49. $\ell im_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$. J: ∞

50. $\ell im_{x\to 0}\dfrac{\sin 3x}{tg\,2x}$. J: $\dfrac{3}{2}$

51. $\ell im_{x\to 0}\dfrac{\sin 10x}{x}$.. J: 10

52. $\ell im_{x\to\frac{\pi}{2}}\dfrac{\sin x}{x}$. J: $\dfrac{2}{\pi}$

53. $\ell im_{x\to 0}\dfrac{\sqrt{1-\cos x}}{x}$. J: 0

54. $\ell im_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x$. J: e^2

55. $\ell im_{x\to\infty}\left(1-\dfrac{1}{x}\right)^x$. J: e^{-1}

56. $\ell im_{x\to\infty}\left(\dfrac{x}{1+x}\right)^x$. J: e^{-1}

57. $\ell im_{n\to\infty}\left(1+\dfrac{1}{n}\right)^{n+9}$. J: e

58. $\ell im_{x\to 2}\dfrac{x^2-2x}{x^2-x-2}$. J: $\dfrac{2}{3}$

59. $\ell im_{x\to -1}\dfrac{7x^2+4x-3}{5x^2+4x-1}$. J: $\dfrac{5}{3}$

60. $\ell im_{x\to 2}\dfrac{\sin x}{x+tgx}$. J: $\dfrac{\sin 2}{2+tg\,2}$

61. $\ell im_{h\to\infty}\dfrac{(2+h)^5-32}{h}$. J: ∞

62. $\ell im_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$. J: $\dfrac{1}{4}$

63. $\ell im_{x\to 0}\dfrac{9^x-5^x}{x}$. J: $\ln\dfrac{9}{5}$

64. $\ell im_{x\to 4}\dfrac{x^2-4x}{x^2-3x-4}$. J: $\dfrac{4}{5}$

65. $\ell im_{x\to 16}\dfrac{4-\sqrt{x}}{x-16}$. J: - $\dfrac{1}{8}$

66. $\ell im_{x\to 0}\dfrac{1-\sqrt{1-x^2}}{x}$. J: 0

67. $\ell im_{x\to 2}\dfrac{x^2+x-6}{x-2}$. J: 5

68. $\ell im_{x\to 2}\dfrac{2x^2-7x+4}{x^3-8}$. J: ∞

69. $\ell im_{x\to 3}\dfrac{x^2-x-6}{x-3}$. J:5

MUSTAQIL YECHISH UCHUN MASHQLAR.

Funksiyaning juft va toqligi, davriyligini tekshiring.

1.1. $y=\dfrac{2x^3}{(x-2)^2}$ 1.2. $y=\dfrac{2x^3}{x^2-2}$ 1.3. $y=\dfrac{2x^2+1}{x-2}$

1.4. $y=\dfrac{2x^5}{x^4-16}$ 1.5. $y=\dfrac{x^2-1}{x-3}$. 1.6. $y=\dfrac{2x^3}{x^2-x+1}$

1.7. $y=x+\dfrac{x^3}{x^2-x-1}$ 1.8. $y=\dfrac{x^3+4}{x^2}$ 1.9. $y=\dfrac{(x-2)^2}{x+1}$

1.10. $y=\dfrac{x^3}{x^2-9}$ 1.11. $y=\dfrac{x^5-8}{x^4}$ 1.12. $y=\dfrac{x^4}{x^3-1}$

1.13. $y=\dfrac{x^2+4x+4}{x-2}$ 1.14. $y=x+\dfrac{2x^2}{x-2}$ 1.15. $y=x+\dfrac{x^2}{x-4}$

Limitlarni hisoblang.

2.1. $\lim\limits_{x\to\infty}\dfrac{3x^2+4x-6}{2x^3-7x+2}$ 2.2. $\lim\limits_{x\to\infty}\dfrac{3x^4-4x-6}{2x^4-3x^2+x}$

2.3. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{x^2+7x+2}$ 2.4. $\lim\limits_{x\to\infty}\dfrac{3x^2+2x-5}{x^3+5x+2}$

2.5. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+x}{4x^3+3x-5}$ 2.6. $\lim\limits_{x\to\infty}\dfrac{4x^2+3x+5}{2x^2-x+4}$

2.7. $\lim\limits_{x\to\infty}\dfrac{6x^2-2x+1}{2x^2-7x+2}$ 2.8. $\lim\limits_{x\to\infty}\dfrac{4x^2-2x+4}{2x^2-5x+2}$

2.9. $\lim\limits_{x\to\infty}\dfrac{5x^4+4x-6}{2x^4+7x+2}$ 2.10. $\lim\limits_{x\to\infty}\dfrac{x^4+3x-x}{2x^4-4x+2}$

2.11. $\lim\limits_{x\to\infty}\dfrac{14x^2+4x}{7x^2-7x+2}$ 2.12. $\lim\limits_{x\to\infty}\dfrac{6x^4+4x^2-6x}{2x^4-7x^3+2x}$

2.13. $\lim\limits_{x\to\infty}\dfrac{x^6+x^4}{2x^6-7x+2}$ 2.14. $\lim\limits_{x\to\infty}\dfrac{3x^2-2x+6}{5x^3-7x+2}$

2.15. $\lim\limits_{x\to\infty}\frac{7x^4-2x+2}{2x^4-7x+2}$

2.16. $\lim\limits_{x\to\infty}\frac{3x^3-4x+5}{6x^3+7x+2}$

2.17. $\lim\limits_{x\to\infty}\frac{x^4+4x^2-6}{2x^4-7x^3+2}$

2.18. $\lim\limits_{x\to\infty}\frac{8x^2-7x+6}{2x^2+7x+2}$

2.19. $\lim\limits_{x\to\infty}\frac{3x^7+4x^5-6}{2x^7-7x^5+2}$

2.20. $\lim\limits_{x\to\infty}\frac{4x^2-3x-1}{2x^2+7x+2}$

2.21. $\lim\limits_{x\to\infty}\frac{8x^6-2}{2x^6-7x+2}$

2.22. $\lim\limits_{x\to\infty}\frac{3x^2+4x-6}{2x^3-7x+2}$

2.23. $\lim\limits_{x\to\infty}\frac{2x^3+4x-6}{2x^3-7x+2}$

2.24. $\lim\limits_{x\to\infty}\frac{4x^4+4x^2-6}{x^4-7x^3+2}$

2.25. $\lim\limits_{x\to\infty}\frac{9x^3+4x-6}{3x^3-7x+2}$

2.26. $\lim\limits_{x\to\infty}\frac{3x^2-4x-6}{5x^3-7x-8}$

2.27. $\lim\limits_{x\to\infty}\frac{8x^4-4x+5}{2x^4+2}$

2.28. $\lim\limits_{x\to\infty}\frac{4x^3+4x+2}{5x^3+7x^2+2}$

2.29. $\lim\limits_{x\to\infty}\frac{10x^5+4x+7}{5x^5-7x+2}$

2.30. $\lim\limits_{x\to\infty}\frac{x^2+4x-2}{3x^3-x+2}$

2.31. $\lim\limits_{x\to-1}\frac{7x^2+4x-3}{5x^2+4x-1}$

2.32. $\lim\limits_{x\to5}\frac{3x^2-6x-45}{x^2-2x-15}$

2.33. $\lim\limits_{x\to2}\frac{x^2-12x+20}{x^2+x-6}$

2.34. $\lim\limits_{x\to3}\frac{3x^2-7x-6}{6+x-x^2}$

2.35. $\lim\limits_{x\to-1}\frac{2x^2+3x+1}{3x^2+x-2}$

2.36. $\lim\limits_{x\to-1}\frac{3x^2+2x-1}{4x^2+3x-1}$

2.37. $\lim\limits_{x\to2}\frac{x^2-5x+6}{2x^2-9x+10}$

2.38. $\lim\limits_{x\to4}\frac{x^2-5x+4}{x^2-16}$

2.39. $\lim\limits_{x\to4}\frac{x^2+x-20}{x^2-3x-4}$

2.40. $\lim\limits_{x\to-1}\frac{2+x-x^2}{x^3-3x^2-2}$

2.41. $\lim\limits_{x\to3}\frac{\sqrt{x-2}-\sqrt{4-x}}{x^2+2x-15}$

2.42. $\lim\limits_{x\to2}\frac{\sqrt{5-x}-\sqrt{x+1}}{x^2+5x-14}$

2.43. $\lim\limits_{x\to 1}\frac{3x^2+4x+1}{\sqrt{8-x}-\sqrt{4-5x}}$

2.44. $\lim\limits_{x\to 5}\frac{2x^2-7x-15}{\sqrt{x+4}-\sqrt{2x-1}}$

2.45. $\lim\limits_{x\to -5}\frac{\sqrt{3x+17}-\sqrt{7+x}}{x^2+4x-5}$

2.46. $\lim\limits_{x\to -3}\frac{2x^2-x-21}{\sqrt{7+x}-\sqrt{1-x}}$

2.47. $\lim\limits_{x\to 1}\frac{x^2-x-6}{\sqrt{2-x}-\sqrt{x+6}}$

2.48. $\lim\limits_{x\to 1}\frac{\sqrt{x+2}-\sqrt{6-x}}{x^2-3x+2}$

2.49. $\lim\limits_{x\to 1}\frac{4x^2-3x-1}{\sqrt{3+2x}-\sqrt{x+4}}$

2.50. $\lim\limits_{x\to 3}\frac{x^2+x-12}{\sqrt{5x+1}-4}$

2.51. $\lim\limits_{x\to 0}\frac{\cos 5x-\cos x}{2x^2}$

2.52. $\lim\limits_{x\to 0}\frac{\sin 3x-\sin x}{4x}$

2.53. $\lim\limits_{x\to 0}\frac{1-\cos^2 x}{x\cdot tgx}$

2.55. $\lim\limits_{x\to 0}\frac{tg2x-\sin 2x}{x^2}$

2.56. $\lim\limits_{x\to 0}\frac{5x}{\sin x+\sin 7x}$

2.57. $\lim\limits_{x\to 0}\frac{\cos^2 x-\cos^2 2x}{x^2}$

2.58. $\lim\limits_{x\to 0}\frac{tg4x}{3\sin 3x}$

2.59. $\lim\limits_{x\to 0}\frac{\arcsin 3xx}{\sin 5x}$

2.60. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}\right)$

261. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-5x+6}-x\right)$

2.62. $\lim\limits_{x\to\infty}\left(x^2-\sqrt{x^4+x^2+1}\right)$

2.63. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-2x+3}-\sqrt{x^2-x+4}\right)$

2.64. $\lim\limits_{x\to\infty}\left(\sqrt{x^4+3x^2+1}-x^2\right)$

2.65. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-x+2}\right)$

2.66. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2+x}\right)$

2.67. $\lim\limits_{x\to\infty}\left(\sqrt{x^4+3}-\sqrt{x^4-2}\right)$

2.68. $\lim\limits_{x\to+\infty}\sqrt{x+2}\left(\sqrt{x+3}-\sqrt{x-4}\right)$

2.69. $\lim\limits_{x\to\infty}\left(x-\sqrt{x(x-1)}\right)$

2.70. $\lim\limits_{x\to\infty}\left(\sqrt{(x^3+1)(x^2-3)}-\sqrt{x(x^4+2)}\right)$

2.71. $\lim\limits_{x\to\infty}\sqrt{x^3+8}\left(\sqrt{x^3-2}-\sqrt{x^3-1}\right)$

2.72. $\lim\limits_{x\to\infty}\left(\sqrt{x(x+5)}-x\right)$

2.73. $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+2}-\sqrt{x-3}\right)$

2.74. $\lim\limits_{x\to\infty}\left(\sqrt[3]{(x+2)^2}-\sqrt[3]{(x-3)^2}\right)$

2.75. $\lim\limits_{x\to\infty}\left(\sqrt[3]{5+8x^3}-2x\right)$

2.76. $\lim\limits_{x\to\infty}\left(\sqrt{x(x+2)}-\sqrt{x^2-2x+3}\right)$

2.77. $\lim\limits_{x\to\infty}\left(\sqrt{x(x+2)}-\sqrt{x^2-2x+3}\right)$

2.78. $\lim\limits_{x\to\infty}\left(x+\sqrt[3]{4-x^3}\right)$

2.79. $\lim\limits_{x\to\infty}\left(\sqrt{x^5-8}-x\sqrt{x(x^2+5)}\right)$

2.80. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-3x+2}-x\right)$

2.81. $\lim\limits_{x\to\infty}\left(\sqrt{(x^2+1)(x^2-4)}-\sqrt{x^4-9}\right)$

2.82. $\lim\limits_{x\to\infty}\left(x-\sqrt[3]{x^3-5}\right)$

2.83. $\lim\limits_{x\to\infty}x\left(\sqrt{x(x-2)}-\sqrt{x^2-3}\right)$

2.84. $\lim\limits_{x\to 4}\dfrac{\sqrt[3]{x^2-16}}{\sqrt{x+12}-\sqrt{3x+4}}$

2.85. $\lim\limits_{x\to 3}\dfrac{\sqrt{4x-3}-\sqrt{5x-6}}{\sqrt[3]{x^2-9}}$

2.86. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{2-3x}-\sqrt[3]{6-x}}{\sqrt[3]{8+x^3}}$

2.87. $\lim\limits_{x\to 5}\dfrac{\sqrt{4x+5}-\sqrt{6x-5}}{\sqrt{4+x}-\sqrt{2x-1}}$

2.88. $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-\sqrt{3x+1}}$

2.89. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{\sqrt[3]{8+x}-\sqrt[3]{8-x}}$

2.90. $\lim\limits_{x\to -0}\dfrac{\sqrt[3]{5+x}-\sqrt[3]{5-x}}{\sqrt[3]{x^2+x^4}}$

2.91. $\lim\limits_{x\to \frac{2}{3}}\dfrac{\sqrt[3]{\frac{1}{3}+x}-\sqrt[3]{2x-\frac{1}{3}}}{\sqrt[3]{3x-2}}$

2.92. $\lim\limits_{x\to 4}\dfrac{\sqrt[3]{x-3}-\sqrt[3]{2x-7}}{\sqrt{1+2x}-3}$

2.93. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{6+x}-\sqrt[3]{10+3x}}{\sqrt{2-x}-2}$

2.95. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+x}-\sqrt[3]{8-x}}{x^2+2\sqrt[3]{x}}$

2.96. $\lim\limits_{x\to -0}\dfrac{\sqrt[3]{1-x}-\sqrt[3]{x+1}}{\sqrt{1-x}-\sqrt{x+1}}$

2.97. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{2-3x}-\sqrt[3]{6-x}}{\sqrt[3]{8+x^3}}$

2.98. $\lim\limits_{x\to -8}\dfrac{\sqrt{1-x}-\sqrt{2x+25}}{\sqrt[3]{x^3}+2}$

2.99. $\lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$

2.100. $\lim\limits_{x\to -4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{\sqrt{5-x}-\sqrt{1-2x}}$

5.3. MAVZU: BIR O‘ZGARUVCHILI FUNKSIYANING DIFFERENSIAL HISOBI VA UNING TADBIQLARI . UNING GEOMETRIC VA FIZIK MA’NOSI. DIFFERENSIAL QOIDALARI VA FORMULALARI.

MAVZUGA OID NAZARIY MATERIALLAR

1. Hosila tushunchasi

Amaliyotda har xil jarayonlarni tekshirishda birinchi navbatda, shu jarayonning kechishi tezligini aniqlash kerak bo’ladi. Tezlikni aniqlash haqidagi masala fan va texnikaning eng asosiy masalalaridan biridir.

Ma’lumki, tekis kechadigan jarayonlarda uning kechishi tezligi o’zgarmasdir. **Masalan**, tekis harakatda o’tilgan yo’lning shu yo’lni o’tishga ketgan vaqtga nisbati uning tezligini bildirib u o’zgarmasdir.

Lekin tabiatdagi yoki jamiyatdagi ko’pchilik hodisalar notekis kechadigan jarayonlardir. Masalan, og’ir moddiy nuqtaning bo’shliqda og’irlik kuchi ta’sirida erkin tushushi masalasini qaraylik. Fizikadan ma’lumki, bo’shliqda moddiy nuqtaning erkin tushishi qonuni

$$S = \frac{g}{2}t^2 \qquad (1)$$

munosabat bilan ifodalanib, bu erda t erkin tushish boshlanishidan hisoblangan vaqt, S t vaqtda o’tgan yo’l, g erkin tushish tezlanishi, $g \approx 9{,}81 м/сек^2$. Bu harakat notekisdir. Notekis harakatning tezligi faqat vaqtning aniq momentiga tegishli bo’ladi. Ya’ni vaqtning har bir momentidagi oniy tezlik haqida gapirish kerak bo’ladi. Oniy tezlikni hisoblash uchun quyidagi ko’rinishdagi limitni hisoblash kerak bo’ladi.

$$v = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} \qquad (2)$$

Umuman, o’zgaruvchi miqdor ning o’zgarish tezligini topish masalasi, matematika fanining eng ahamiyatli tushunchalaridan biri - hosila tushunchasiga olib keladi. $y = f(x)$ funksiyaning orttirmasi

$$\Delta y = f(x+\Delta x) - f(x)$$

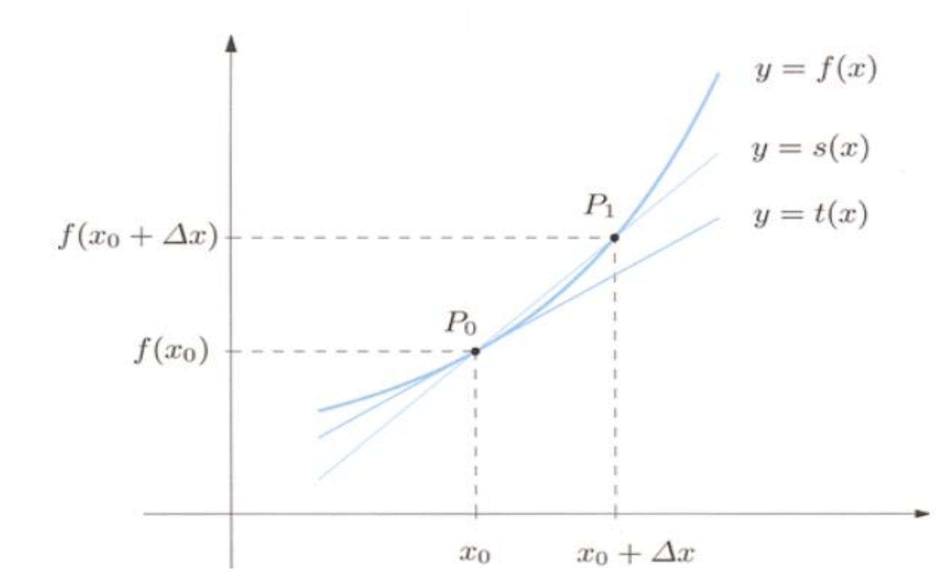
ko‘rinishda ifodalanishini eslatib o‘tamiz, bunda Δx argument *x* ni orttirmasi.

Ta’rif. $y = f(x)$ funksiyaning *x* nuqtadagi *hosilasi* deb, shu nuqtadagi funksiya orttirmasining uni shu orttirmaga erishtiradigan argument orttirmasiga nisbatining ixtiyoriy Δx nolga intilgandagi limitiga aytiladi va quyidagi belgilashlarning biri bilan belgilanadi:

$$y',\ f'(x),\ \frac{dy}{dx}.$$

Ta’rifga ko‘ra:

$$y' = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \qquad (3)$$



1-chizma

Masalan.1) $y=x^2$ funksiya hosilasi hisoblansin. *x* ga Δx orttirma berib Δy ni topamiz:

$$\Delta y = f(x+\Delta x) - f(x) = (x+\Delta x)^2 - x^2 = 2x\cdot\Delta x + \Delta x^2$$
$$\frac{\Delta y}{\Delta x} = \frac{2x\cdot\Delta x + \Delta x^2}{\Delta x} = \frac{\Delta x(2x+\Delta x)}{\Delta x} = 2x + \Delta x$$
$$y' = (x^2)' = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}(2x+\Delta x) = 2x$$

Shunday qilib, $(x^2)' = 2x$ ekan.

2). Hosila ta'rifidan foydalanib $y = \dfrac{2x}{3x+1}$ funksiyaning hosilasini toping.

Yechish. Ixtiyoriy Δx orttirma uchun funksiya orttirmasini topamiz:

$$\Delta y = \frac{2(x+\Delta x)}{3(x+\Delta x)+1} - \frac{2x}{3x+1} = \frac{6x^2+6x\Delta x+2x+2\Delta x-6x^2-6x\Delta x-2x}{(3x+3\Delta x+1)(3x+1)} = \frac{2\Delta x}{(3x+3\Delta x+1)(3x+1)}$$

Tenglikning ikkala tomonini Δx ga bo'lamiz:

$$\frac{\Delta y}{\Delta x} = \frac{2}{(3x+3\Delta x+1)(3x+1)}$$

Bu nisbatning $\Delta x \to 0$ da limitini hisoblaymiz:

$$y' = f'(x) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{2}{(3x+3\Delta x+1)(3x+1)} = \frac{2}{(3x+1)^2}.$$

Shunday qilib,

$$y' = \left(\frac{2x}{3x+1}\right)' = \frac{2}{(3x+1)^2}.$$

Funksiya hosilasining geometrik ma'nosi:

$$\lim_{\Delta x\to 0} tg\varphi = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = f'(x_0)$$

$\lim\limits_{\Delta x\to 0} tg\varphi = tg\alpha$ bo'ladi, bunda α-urinmaning OX o'qi bilan hosil qilgan burchagi.

Demak $y=f(x)$ funksiyaning $x=x_0$ nuqtadagi hosilasi $f'(x_0)$ bu funksiya grafigining $M_0(x_0;f(x_0))$ nuqtasida o'tkazilgan urinmaning burchak koeffitsientidan iborat bo'lar ekan. Bu tushunchadan quyidagi natijalar chiqadi.

$M(x_0;y_o)$ nuqtasiga o‘tkazilgan urinma tenglamasi:

$$y - y_0 = f'(x_0)(x - x_0)$$

M_0 nuqtada o‘tkazilgan normal tenglamasi:

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

Masalan. $f(x)=x^2$ funksiyaning $x=1$ nuqtasiga o‘tkazilgan urinma tenglamasi tuzilsin.

Yechish. $x_0=1,\ y_0=f(x_0)=1^2=1,\ M_0(1;1)$. Ma’lumki $f'(x) = (x^2)' = 2x$

$$f'(x_0)= f'(1)=2\bullet 1=2$$

Urinma tenglamasi:

$$y - y_0 = f'(x_0)(x - x_0)$$

$$y-1=2\cdot(x-1)\Rightarrow y=2x-1$$

Javob: $y=2x-1$

$S'(t)=v$ tezlik, S yo’ldan t vaqt bo’yicha olingan hosiladir. Shu xulosani hosilaning mexanik ma’nosi deyiladi.

Masalan. Moddiy nuqta $S = f(t) = \frac{1}{(1+t)}$ qonun bo‘yicha harakatlanadi. Nuqta harakatining $t=2c$ dagi tezligini toping.

Yechish**.**

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{\frac{1}{1+(2+h)}-\frac{1}{1+2}}{h} = \lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim_{h\to 0}\frac{\frac{3-(3+h)}{3(3+h)}}{h} =$$

$$= \lim_{h\to 0}\frac{-h}{3(3+h)h} = \lim_{h\to 0}\frac{-1}{3(3+h)} = -\frac{1}{9}$$

$$|f'(2)| = \left|-\frac{1}{9}\right| = \frac{1}{9};$$

1. Funksiya hosilasini hisoblashning asosiy qoidalari

1) O‘zgarmas funksiya hosilasi nolga teng,

ya’ni y=*f(x)=C-const* bo‘lsa , $y' = C' = 0$ bo‘ladi.

2) Agar *u=u(x), v=v(x)* bo‘lib , $u'(x)$ va $v'(x)_1$ lar mavjud bo‘lsa:

1. $(u \pm v)' = u' \pm v'$

2. $(u \cdot v)' = u'v + uv'$

3. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \quad v \neq 0$

Hosila jadvali.

1. $(x^n)' = nx^{n-1}$
2. $(c)' = 0$
3. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
4. $(\ln x)' = \frac{1}{x}$
5. $(\log_a x)' = \frac{1}{x \ln a}$
6. $(\sin x)' = \cos x$
7. $(\cos x)' = -\sin x$
8. $(\arcsin x)' = \frac{/}{\sqrt{1-x^2}}$
9. $(arc\cos x)' = -\frac{/}{\sqrt{1-x^2}}$
11. $(a^x)' = a^x \cdot \ln a$
12. $(u + v)' = u' + v'$
13. $(u - v)' = u' - v'$
14. $(u \cdot v)' = u'v + uv'$
15. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
16. $\left(\frac{c}{v}\right)' = -\frac{c \cdot v'}{v^2}$
17. $(tgx)' = \frac{1}{\cos^2 x}$
18. $(ctgx)' - \frac{1}{\sin^2 x}$
19. $(\ell^x)' = \ell^x$

10. $(arctgx)^{/} = \frac{/}{1+x^2}$ 20. $(arcctgx)^{/} = -\frac{/}{1+x^2}$

Masalan.1) $y=(x^3+3x-1)^4$ funksiyaning hosilasini toping.

Yechish. $y'=((x^3+3x-1)^4)'=4(x^3+3x-1)^{4-1}(x^3+3x-1)'=4(x^3+3x-1)^3(3x^2+3).$

2) $y=\frac{\sin^2 x}{x^3+1}$ funksiyaning hosilasini toping.

Yechish.

$$y'=\frac{(\sin^2 x)'(x^3+1)-\sin^2 x(x^3+1)'}{(x^3+1)^2}=\frac{2\sin^{2-1}x\cdot(\sin x)'(x^3+1)-\sin^2 x\cdot 3x^2}{(x^3+1)^2}=$$
$$=\frac{2\sin x\cdot\cos x(x^3+1)-3x^2\cdot\sin^2 x}{(x^3+1)^2}=\frac{\sin 2x\cdot(x^3+1)-3x^2\cdot\sin^2 x}{(x^3+1)^2}.$$

2. Funksiyaning differensiallanuvchiligi.

Agar (3) hosila formulasidagi limit mavjud bo‘lsa, *f(x)* funksiya *x* nuqtada *differensiallanuvchi* deyiladi.

Hosilani topish amali funksiyani *differensiallash* deyiladi.

1) $f(x)=x$ funksiya $x_0\in R$ da differensialga ega, va

$$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to x_0}\frac{x-x_0}{x-x_0}=1$$

2) $f(x)=C$ (C-const) funksiya uchun $f'(x)=0$ bo‘ladi.

3) $f(x)=\ell^x$ ko‘rsatkichli funksiya $x\in R$ uchun differensialga ega.

4) barcha $x\in R$ uchun $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$

5) barcha darajali funksiyalar differensiallanuvchidir.

6) $f(x)=\sqrt[3]{x}$ funksiya $x=0$ nuqtada differensialga ega emas,chunki uning limiti mavjud emas

$$\lim_{x\to 0}\frac{f(x)-f(x)}{x-0}=\lim_{x\to 0}\frac{\sqrt[3]{x}}{x}$$

7) $f(x)=|x|$ funksiya $x_0=0$ nuqtaning chap va o‘ng tomonida differensialga ega. Biroq $f_+(0)=+1, f_-(0)=-1$ bo‘lganligi uchun $x_0=0$ nuqtada hosilaga ega emas.

8) $f(x)=x^x$, $D(x)=\{x|\ x>0\}$ ko‘rinishdagi funksiya hosilasini topishdan avval

logarifmik funksiyaning hosilasini topamiz $(\ln f(x))'=\frac{d(x\ln x)}{dx}=\ln x+1$.

Bundan quyidagini hosil qilamiz $f'(x)=x^x(\ln x+1)$

3. Murakkab funksiya hosilasi

Agar $y=f(u)$ va $u=\varphi(x)$ bo‘lsa, u holda y funksiya x ning murakkab $y=f[\varphi(x)]$ funksiyasi deyiladi. U holda y dan x bo’yicha olingan hosila

$$y'_x=y'_u\cdot u'_x \quad yoki\ \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}.$$

ko'rinishga bo’lib, yuqoridagi hosila jadvali quyidagi ko’rinishga ega bo’ladi:

1) $(u^n)'=nu^{n-1}\cdot u'\ \ n\in R,\ \ u>0$;

2) $(a^u)'=a^u\cdot 1na\cdot u'$;

3) $(e^u)'=e^u\cdot u'$;

4) $(\log_a u)'=\frac{1}{u\cdot 1na}\cdot u'$;

5) $(\ln u)'=\frac{1}{u}\cdot u'$;

6) $(\sin u)'=\cos u\cdot u'$;

7) $(\cos u)'=-\sin u\cdot u'$;

8) $(tg\ u)'=\frac{1}{\cos^2 u}\cdot u'$;

9) $(ctg\, u)'=-\frac{1}{\sin^2 u}\cdot u'$;

10) $(\arcsin\ u)'=\frac{1}{\sqrt{1-u^2}}\cdot u'$;

11) $(\arccos\ u)'=-\frac{1}{\sqrt{1-u^2}}\cdot u'$;

12) $(arctg\ u)'=\frac{1}{1+u^2}\cdot u'$;

13) $(arcctg\ u)' = -\frac{1}{1+u^2}\cdot u'$;   14) $(u^v)' = vu^{м-1}\cdot u' + u^v\cdot 1nu\cdot v'$.

Masalan. $y=(10+3x^2)^7$;

Yechish: $y' = 7(10+3x^2)^6(10+3x^2)' = 42x(10+3x^2)^6$

Asosiy elementar funksiyalarning hosilalari

$$a)\ y = \ln x$$

$$x, x+\Delta x, \Delta y = \ln(x+\Delta x) - \ln x = \ln\frac{x+\Delta x}{x}$$

1) $$\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x}\ln\left(1+\frac{\Delta x}{x}\right) = \frac{1}{x}\cdot\frac{x}{\Delta x}\ln\left(1+\frac{\Delta x}{x}\right) = \frac{1}{x}\ln\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}};$$

$$y' = (\ln x)' = \lim_{\Delta x\to 0}\frac{1}{x}\ln\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}} = \frac{1}{x}\ln\left(\lim_{\Delta x\to 0}\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}}\right) = \frac{1}{x}\ln e = \frac{1}{x}\cdot 1 = \frac{1}{x}$$

Demak, $(\ln x)' = \frac{1}{x}$

Agar $y=ln\ u$, $u=u(x)$ bo‘lsa, $y'_x = \frac{u'}{u}$ bo‘ladi.

$$б)\ y = \log_a x = \frac{\ln x}{\ln a}$$

demak, $y' = (\log_a x) = \left(\frac{\ln x}{\ln a}\right)' = \frac{1}{x\ln a}$,

agar $y = \log_a u$ bo‘lib, $u=u(x)$ bo‘lsa, $y'_x = (\log_a u)'_x = \frac{u'}{u\ln a}$ bo‘ladi.

2) $y=x^{\alpha}$, α-ixtiyoriy haqiqiy son. Tenglikni hadma-had logarifmlaymiz:

$$\ln y = \alpha\ln x$$

$$(\ln y)'_x = \alpha(\ln x)'$$

$$\frac{y'_x}{y} = \alpha\cdot\frac{1}{x} \Rightarrow y'_x = \alpha\cdot\frac{1}{x}\cdot y = \alpha\cdot\frac{1}{x}\cdot x^{\alpha} = \alpha x^{\alpha-x}$$

demak, $(x^{\alpha})' = \alpha\cdot x^{\alpha-1}$

Agar $y=u^{\alpha}$, $u=u(x)$ bo‘lsa,

$$y'_x=(u^{\alpha})'_x=\alpha\cdot u^{\alpha-1}\cdot u'_x$$

Masalan. $y=(3x^2-1)^{10}$; $y'=10(3x^2-1)^9(3x^2-1)'=10(3x^2-1)^9 6x=60x(3x^2-1)^9$

3) Ko‘rsatkichli funksiyalarning hosilalari

$$а)\ y=a^x,\ \ \ln y=x\ln a,\ \ \frac{y'_x}{y}=\ln a,\ \ y'_x=y\ln a=a^x\cdot\ln a.$$

$$б)\ y=e^x,\ \ y'=e^x.$$

$$в)\ y=a^u,\ \ u=u(x),\ \ y'=a^u\cdot\ln a\cdot u'_x$$

4) Trigonometrik funksiyalarning hosilalari

$$а)\ y=\sin x,\ \ y'=\cos x,\ \ (\sin u)'=\cos u\cdot u'$$

$$б)\ y=\cos x,\ \ y'=-\sin x,\ \ (\cos u)'=-\sin u\cdot u'$$

$$в)\ y=tgx,\ \ y'=\frac{1}{\cos^2 x},\ \ (tgu)'=\frac{u'}{\cos^2 x}$$

$$г)\ y=ctgx,\ \ y'=-\frac{1}{\sin^2 x},\ \ (ctgu)'=-\frac{u'}{\sin^2 x}$$

Teskari funksiyaning hosilasi

Faraz qilaylik, $y=f(x)$ funksiya uzluksiz va monoton , $x=x_0$ nuqtada $f'(x_0)$ mavjud bo‘lib, noldan farqli bo‘lsin. U holda teskari funksiya $x=\varphi(y)$ ning $x=x_0$ nuqtadagi hosilasi quyidagicha hisoblanadi:

$$\varphi'(x_0)=\frac{1}{f'(x_0)}$$

Masalan.

$$y=\arcsin x,\ y'=?$$

$$x=\sin y\Rightarrow y'_x=(\arcsin x)'=\frac{1}{(\sin y)'}=\frac{1}{\cos y}=\frac{1}{\pm\sqrt{1-\sin^2 y}}=\frac{1}{\sqrt{1-x^2}}$$

$y\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$da $\cos y\geq 0$ bo‘lganligi uchun «+» ishora olindi.

Shunday qilib $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$.

demak, $(\arcsin x)' = \frac{u'}{\sqrt{1-u^2}}, \quad u = u(x)$. Shunga o‘xshash:

$$(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$$

$$(arctgu)' = \frac{u'}{1+u^2}$$

$$(arcctgu)' = -\frac{u'}{1+u^2}$$

ekanligini ko‘rsatish mumkin.

Murakkab ko‘rsatkichli funksiyalarning hosilasi.

Asosi ham, daraja ko‘rsatkichi ham x ning funksiyasidan iborat bo‘lgan, ya’ni

$$y = [u(x)]^{v(x)} \equiv u^v$$

ko‘rinishdagi funksiya murakkab ko‘rsatkichli funksiya deyiladi.

Misol uchun $y = (\cos x)^{x^2}, y = x^{\cos x}, y = x^x, y = (\log_a x)^x$ va shunga o‘xshash funksiyalar murakkab ko‘rsatkichli funksiyalardir. Bunday funksiyalarning hosilasini topishda berilgan funksiya logarifmining hosilasini topishdan iborat bo‘lgan usulni qo‘llash ko‘pincha hosilani birmuncha soddalashtiradi.

Masalan.1) $y = u^v$ funksiyani logarifmlab hosilasini topishdan quyidagi formulaga ega bo‘lamiz:

$$y' = u^v \ln u \cdot v' + vu^{v-1} \cdot u',$$

bunda $u = u(x)$ va $v = v(x)$.

2) $y = (\sin 4x)^{x^3}$ funksiyaning hosilasini toping.

Yechish. Tenglamani ikkala tomonini logarifmlaymiz:

$$\ln y = x^3 \ln \sin 4x.$$

Bu tenglikning ikkala tomonini *x* bo‘yicha differensiallaymiz:

$$(\ln y)' = (x^3) \cdot \ln \sin 4x + x^3 (\ln \sin 4x)'$$

Bundan

$$\frac{y'}{y} = 3x^2 \ln \sin 4x + 4x^3 \cdot \frac{1}{\sin 4x} \cos 4x.$$

Soddalashtiramiz:

$$y' = (\sin 4x)^{x^3} (3x^2 \ln \sin 4x + 4x^3 ctg\, 4x).$$

Parametrik ko‘rinishda berilgan funksiyalarni differensiallash

$$y'_x = \psi'_t \cdot t'_x = \psi'_t \cdot \frac{1}{\varphi'_t(t)} = \frac{\psi'(t)}{\varphi'(t)}$$

Ya’ni $y'_x = \dfrac{\psi'(t)}{\varphi'(t)}$

Masalan. Ellips tenglamasi.

$$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}.$$

$$y'_x = \frac{(b\sin t)'}{(a\cos t)'} = \frac{b\cos t}{-a\sin t} = -\frac{b}{a} ctgt$$

Oshkormas funksiyaning hosilasi

Agar y ga nisbatan yechilgan *F(x,y)=0* tenglama *y* ni *x* ning bir qiymatli funksiyasi sifatida aniqlasa, u holda *y* *x* ning oshkormas funksiyasi deyiladi. Bu oshkormas funksiyadan y' hosilani toppish uchun *y* ni *x* ning funksiyasi deb, *F(x,y)=0* tenglamaning ikki tomonini x bo’yicha differensiallash kerak. Hosil bo’lgan tenglamadan izlangan y' ni topamiz.

Masalan.

1) $x^2+y^2=R^2$ | 2) $y^3-3xy+x^3=0$

$2x+2y\cdot y'=0$ | $3y^2\cdot y'-3y-3x\,y'+3x^2=0$

$2y\cdot y_x'=-2x,$ | $y'=(y-x^2)/(y^2-x)$

$y'=-\frac{x}{y}$ | $y'=\frac{y-x^2}{y^2-x}$

Yuqori tartibli hosilalar.

Birinchi tartibli hosiladan olingan hosila,ya'ni

$$(y')'=(f'(x))' \ yoki \ \ y''=f''(x)$$

hosil**a** *y=f(x)* funksiyaning ikkinchi tartibli hosilasi deyiladi va $y'', f''(x), \frac{d^2y}{dx^2}$ belgilarning biri bilan belgilanadi.

Umuman, *y=f(x)* funksiyaning *n*- tartibli hosilasi deb,uning *(n-1)* – tartibli hosilasining hosilasiga aytiladi va $y^{(n)}, f^{(n)}(x), \frac{d^ny}{dx^n}$ belgilarning biri bilan belgilanadi.

$$y''=f''(x)=(y')'=\left(f'(x)\right)'_x$$

$$y^{(n)}=f^{(n)}(x)=\left(y^{(n-1)}\right)'=\left(f^{(n-1)}(x)\right)'_x$$

Odatda 2, 3, 4, 5...-tartibli hosilalar rim raqamlarida belgilanadi:

Masalan.1) $y=\ln(x+\sqrt{x^2+a^2})$ funksiyaning ikkinchi tartibli hosilasini toping.

Yechish**.** Dastlab hosilalar jadvalidan foydalanib birinchi tartibli hosilasini topamiz:

$$y' = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left(x+\sqrt{x^2+a^2}\right)' = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left(1+\frac{x}{\sqrt{x^2+a^2}}\right) =$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}.$$

Hosil bo‘lgan natijadan yana hosila olamiz:

$$y'' = (y')' = \left(\frac{1}{\sqrt{x^2+a^2}}\right)' = \left(\left(x^2+a^2\right)^{-\frac{1}{2}}\right)' =$$

$$= -\frac{1}{2}(x^2+a^2)^{-\frac{3}{2}} \cdot 2x = -\frac{x}{\sqrt{\left(x^2+a^2\right)^3}}.$$

2) $S(t)=2t^3-3t^2+5\ (m)$ qonun bilan to‘g‘ri chiziqli harakat qilayotgan jismning $t=2$ sek momentdagi tezlanishini toping.

Yechish. $S'(t)=6t^2-6t$, $S''(t)=12t-6$

$S''(2)=12 \cdot 2-6=18$ Javob: 18 m/sek^2

3) a) $y=0{,}5x^4$, $y'''=?$ b) $y=e^{kx}$, $y^{(n)}=?$ k-const

$y'=2x^3$, $y'=ke^{kx}$

$y''=6x$ $y''=k^2e^{kx}$

$y'''=12x$ $y^{(n)}=k^ne^{kx}$

4) $y=\sin x$ funksiyaning n- tartibli hosilasini toping.

Yechish.

$$y' = \cos x = \sin\left(x+\frac{\pi}{2}\right),$$

$$y'' = \cos\left(x+\frac{\pi}{2}\right) = \sin\left(x+2\cdot\frac{\pi}{2}\right),$$

$$y''' = \cos\left(x+2\cdot\frac{\pi}{2}\right) = \sin\left(x+3\cdot\frac{\pi}{2}\right),$$

....

$$y^{(n)} = \cos\left(x+(n-1)\frac{\pi}{2}\right) = \sin\left(x+n\cdot\frac{\pi}{2}\right).$$

Oshkormas funksiyaning yuqori tartibli hosilasi

Oshkormas funksiyaning yuqori tartibli hosilasini topish uchun, *F(x,y)=0* tenglikni *y* va uning barcha hosilalari *x* ning funksiyasi ekanligini e`tiborga olgan holda tegishli marta differensiallanadi . Masalan, y'' ni topish uchun *F(x,y)=0* tenglamani *x* bo'yicha ikki marta differensiallash kerak va hokazo.

Masalan. Oshkormas holda

$$x^2 + y^2 = 64$$

tenglama bilan berilgan *y* funksiyaning y' *va* y'' hosilalarini toping.

Yechish. *y* o'zgaruvchi *x* ning funksiyasi deb hisoblab, berilgan tenglamani ikkala qismini *x* bo'yicha differensiallaymiz:

$$2x + 2yy' = 0.$$

Bundan $y' = -\frac{x}{y}$. Topilgan birinchi y' hosilani yana *x* bo'yicha differensiallaymiz:

$$y'' = (y')' = -\frac{y - xy'}{y^2}.$$

Endi $y' = -\frac{x}{y}$. ekanini hisobga olib,

$$y'' = -\frac{y - x(-\frac{x}{y})}{y^2}$$

ni hosil qilamiz.

Shunday qilib, $y'' = -\frac{y^2 + x^2}{y^3}$ *yoki* $y'' = -\frac{64}{y^3}$, chunki shartga ko'ra $x^2 + y^2 = 64$.

Parametrik funksiyalarning yuqori tartibli hosilasi

Agar *x* va *y* o'zgaruvchilar *t* parametr orqali funksional bo'lsa:

$$\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases}$$

$$y'_x = \frac{\psi'(t)}{\varphi'(t)}$$

tenglikni x bo‘yicha differensiallab, bunda t ni x ning funksiyasi deb olinadi, ikkinchi tartibli y''_{xx} hosilani topish mumkin:

$$y''_{xx} = \left(\frac{\psi'_t}{\varphi'_t}\right)'_x = \left(\frac{\psi'_t}{\varphi'_t}\right)'_t \cdot t'_x = \frac{\psi''_{tt} \cdot \varphi'_t - \psi'_t \cdot \varphi''_{tt}}{(\varphi'_t)^2} \cdot \frac{1}{\varphi'_t};$$

demak, $y''_{xx} = \dfrac{\psi''_{tt} \cdot \varphi'_t - \psi'_t \cdot \varphi''_{tt}}{(\varphi'_t)^3};$

Uchinchi va undan yuqori tartibli hosilalar ham shu tartibda hisoblanadi.

4. Funksiyaning differensiali.

y=f(x) funksiyaning differensiali deb,uning orttirmasining erkli o‘zgaruvchi

x ning orttimasiga nisbatan chiziqli bo‘lgan bosh qismiga aytiladi.

y=f(x) funksiyaning differensiali uning hosilasi bilan erkli o‘zgaruvchi orttirmasining ko‘paytmasiga teng.

$$dy = f'(x) \cdot \Delta x \ yoki \ dy = y' \cdot \Delta x$$

Ravshanki $dx = \Delta x$. Shu sababli

$$dy = f'(x) \cdot dx \ yoki \ dy = y' \cdot dx$$

Differensial geometrik jihatdan *y=f(x)* funksiya grafigiga *M(x;y)* nuqtadan o‘tkazilgan urinma ordinatasining orttirmasiga teng.

Funksiyaning differensiali *dy* uning Δy orttirmasidan Δx ga nisbatan yuqori tartibli cheksiz kichik miqdorga farq qiladi.

Agar $u = u(x)\ va v = v(x)$ funksiyalar differensiallanuvchi bo‘lsa, u holda differensialning ta’rifi va differensiallash qoidalaridan bevosita differensialning asosiy xossalariga ega bo‘lamiz

Agar *u(x)* va *v(x)* funksiyalar differensiallanuvchi bo‘lsa, ushbu formulalar ham o‘rinli bo‘ladi:

1. $d(C) = 0, bunda\ C - o`zgarmas.$
2. $d(Cu) = Cdu.$
3. $d(u \pm v) = du \pm dv.$
4. $d(u \cdot v) = udv + vdu.$
5. $d(\frac{u}{v}) = \frac{vdu - udv}{v^2},\ bunda\ v \neq 0.$
6. $df(yu) = f'_u(u) \cdot u'dx = f'(u)du.$

Masalan.1) $y = tg^4 2x$ funksiya differensialini toping.

Yechish. Oldin berilgan funksiyaning hosilasini topamiz:

$$y' = 8tg^3 2x \cdot \frac{1}{\cos^2 2x} = 8tg^3 2x \sec^2 2x.$$

U holda

$$dy = 8tg^3 2x \cdot \sec^2 2xdx.$$

2) *y=sin3x,* *dy=?*

$$dy = (sin3x)'\, dx=3cos3xdx$$

Oxirgi tenglikdan $y' = f'(x)$ =*dy/dx* ekanligi kelib chiqadi.

Yuqori tartibli differensiallar

y=f(x) funksiyaning ikkinchi tartibli differensiali deb birinchi tartibli differensialdan olingan differensialga aytiladi va

$$d^2 y = d(dy)$$

kabi belgilanadi.

y=f(x) funksiyaning *n-* tartibli differensiali deb *(n-1)* – tartibli differensialdan olingan differensialga aytiladi, ya'ni:

$$d^{(n)}y = d(d^{n-1}y).$$

y=f(x) funksiya berilgan bo'lib, bunda *x*-erkli o'zgaruvchi bo'lsa, u holda uning yuqori tartibli differensiallari ushbu formulalar bo'yicha hisoblanadi:

$$d^2y = y''dx^2, d^3y = y'''dx^3, ... d^n y = y^{(n)}dx^n.$$

Masalan.1) $y = x(\ln x - 1)$ funksiyaning ikkinchi tartibli differensialini toping.

Yechish.Berilgan funksiyaning birinchi va ikkinchi tartibli hosilalarini topamiz:

$$y' = \ln x - 1 + x \cdot \frac{1}{x} = \ln x,\ y'' = \frac{1}{x}.$$

Demak,

$$dy = \ln x dx, d^2x = \frac{dx^2}{x}.$$

2) $y=cosx$, $d^2y=?$

$$dy=-sinxdx, \qquad d^2y=-cosxdx^2$$

Agar $y=f(x)$, $x=\varphi(t)$ bo'lsa d^2y qanday hisoblanishini ko'rib chiqamiz:

$dy= y' dx$, bunda $dx= \varphi' (t)dt$

$d^2y=d(dy)=d(y' dx)=d (y') dx+ y' d(dx)= y'' dxdx+ y' d^2x= y'' dx^2+ y' d^2x$

3) $y=cosx$, $x=lnt$ bo'lsa, $d^2y=?$

Yechish. $dy= y' dx=-\frac{\sin xdt}{t}t$=- $sinxdx$

$$d^2y = -\cos x\left(\frac{dt}{t}\right)^2 + \sin x \cdot \frac{dt^2}{t^2} = -\cos x dx^2 - \sin x d^2x, \text{ chunki } \frac{dt^2}{t^2} = -d^2x.$$

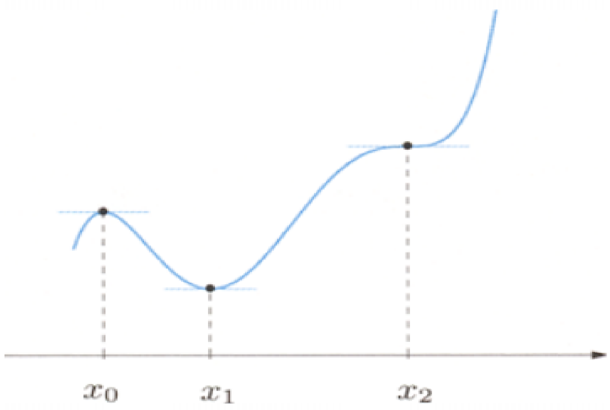
6. Differensiallanuvchi funksiyalar haqida teoremalar.

1. Ferma teoremasi.

Agar *(a, b)* oraliqda aniqlangan $y=f(x)$ funksiya $x_0 \in (a; b)$ nuqtada o'zining eng katta yoki eng kichik qiymatiga erishsa va $f'(x_0)$ mavjud bo'lsa, u holda $f'(x_0)=0$ bo'ladi.

Bu teoremaning geometrik ma'nosi shundan iboratki, agar funksiya

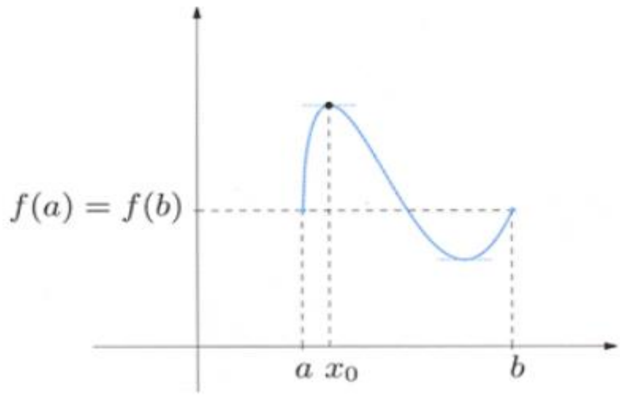
differensiallanuvchi bo'lib, uning biror nuqtasida eng katta (kichik) qiymatga erishsa, funksiya grafigining shu nuqtasiga o'tkazilgan urinma *OX* o'qiga parallel bo'ladi.



2- chizma

2. Roll teoremasi.

(a, b) oraliqda differensiallanuvchi va *[a, b]* kesmada uzluksiz bo'lgan $y=f(x)$ funksiya berilgan bo'lib, $f(b)=f(a)$ o'rinli bo'lsa, bu *(a, b)* oraliqda shunday $c \in (a, b)$ nuqta mavjudki, $f'(c)=0$ bo'ladi.



3- chizma

Masalan. $[1;5]$ kesmada $f(x) = x^2 - 6x + 100$ funksiya uchun Roll teoremasi bajariladimi.

Yechish. *f(x)* funksiya *x* ning barcha qiymatlarida uzluksiz va (1;5) oraliqda differensiallanuvchi hamda $[1;5]$ kesmaning chetki qiymatlarida $f(1) = f(5) = 95$ teng, shuning uchun Roll teoremasi shu kesmada bajariladi.

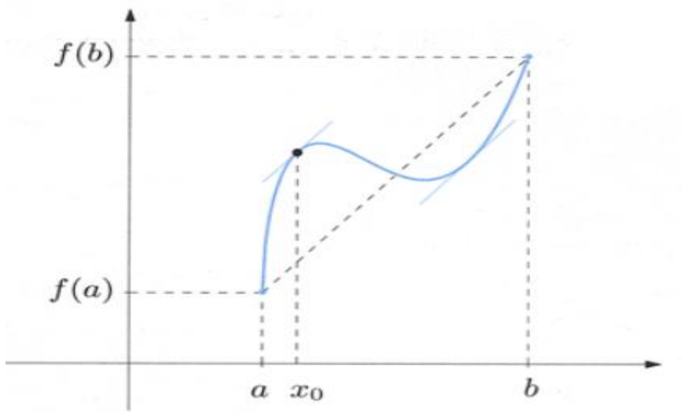
3. Lagranj teoremasi.

[a, b] kesmada uzluksiz *y=f(x)* funksiya *(a, b)* oraliqda hosilasi mavjud bo‘lsa, u holda shunday $c \in (a, b)$ nuqta mavjudki, bu nuqta uchun quyidagi tenglik o‘rinli bo‘ladi:

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Lagranj teoremasining geometrik ma’nosi quyidagicha:

MN-kesuvchi; bunda (vatar) *M(a;f(a)), N(b;f(b))* $tg\alpha$-koordinatalari *(c;f(c))* bo‘lgan nuqtaga o‘tkazilgan urinmaning burchak koeffitsienti bo‘lsin. U holda *y=f(x)* ning grafigida kamida bitta shunday nuqta topish mumkinki, bu nuqtaga o‘tkazilgan urinma *MN* kesuvchi (vatarga) parallel bo‘ladi.



4- chizma

4. Koshi teoremasi.

Agar *f(x)* va *φ(x)* funksiyalar *[a,b]* da uzluksiz va uning ichki nuqtalarida

differensiallanuvchi bo‘lib *φ’(x)≠0* bo‘lsa, u holda *(a,b)* intervalda shunday *c*∈*(a,b)* nuqta mavjudki, bu nuqta uchun ushbu tenglik o‘rinli bo‘ladi:

$$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)}$$

Ravshanki *φ(x)=x* bo‘lganda bu teorema Lagranj teoremasidan iborat bo‘ladi.

Masalan. $f(x)=x^3$ va $g(x)=x^2$ funksiyalar uchun Koshi formulasi yozilsin va c topilsin.

Yechish. $f'(x)=3x^2$, $g'(x)=2x$, $f(b)=b^3$, $f(a)=a^3$, $g(b)=b^2$, $g(a)=a^2$, $f'(c)=3c^2$, $g'(c)=2c$ bo‘lgani uchun $\dfrac{f(b)-f(a)}{g(b)-g(a)}=\dfrac{f'(c)}{g'(c)}$

Koshi formulasi quyidagi ko‘rinishga ega bo‘ladi.

$$\frac{b^3-a^3}{b^2-a^2}=\frac{3c^2}{2c};\quad \frac{(b-a)(b^2+ba+a^2)}{(b-a)(b+a)}=\frac{3}{2}c.$$

Bundan, $c=\dfrac{2(b^2+ba+a^2)}{3(b+a)}.$

5. Lopital qoidasi.

Lopitalning birinchi qoidasi. Agar $\lim_{x\to a} f(x) = \lim_{x\to a} \varphi(x) = 0$ va $\lim\limits_{x\to a}\frac{f'(a)}{\varphi'(a)} = A$ limit mavjud bo‘lsa, u holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)} = \lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)} = A$ tenglik o‘rinli bo‘ladi.

Lopitalning ikkinchi qoidasi. Agar $\lim_{x\to a} f(x) = \lim_{x\to a} \varphi(x) = \infty$ va $\lim\limits_{x\to a}\frac{f'(a)}{\varphi'(a)} = A$ limit mavjud bo‘lsa, u holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)} = \lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)} = A$ tenglik o‘rinli bo‘ladi.

Isboti. *f(a)=0, φ(a)=0*$\Rightarrow \frac{f(x)}{\varphi(x)} = \frac{f(x)-f(a)}{\varphi(x)-\varphi(a)}$

a nuqtaning atrofidagi *x* lar va *C* nuqta uchun Koshi teoremasiga ko‘ra:

$\frac{f(x)}{\varphi(x)} = \frac{f'(c)}{\varphi'(c)}$ o‘rinli bo‘ladi.

Bu tenglikda $x\to a$ deb limitga o‘tsak, $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)} = \lim\limits_{x\to a}\frac{f'(c)}{\varphi'(c)} = \lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)} = A$ isbotlandi.

Masalan. $\lim\limits_{x\to 0}\frac{(\sin 3x)'}{(4x)'} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0}\frac{3\cos 3x}{4} = \frac{3}{4}.$

1-Eslatma. Agar $f'(x)$ va $\varphi'(x)$ funksiyalar ham *f(x)* va *φ(x)* funksiyalar uchun qo‘yilgan shartlarni bajarsa $\frac{f'(x)}{\varphi'(x)}$ ning limitini topish uchun Lopital qoidasini qo‘llash mumkin.

Masalan.

1) $\lim\limits_{x\to\infty}\frac{x+1}{e^x} = \lim\limits_{x\to\infty}\frac{(x+1)'}{(e^x)'} = \lim\limits_{x\to\infty}\frac{1}{e^x} = 0$

2) $\lim\limits_{x\to 0}\frac{\ln x}{ctgx} = \lim\limits_{x\to 0}\frac{(\ln x)'}{(ctgx)'} = \lim\limits_{x\to 0}\frac{-\sin^2 x}{x} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0}\frac{-2\sin x\cos x}{1} = 0$

Masalan.

$$1)\lim_{x\to 0+} x\ln x = (0\cdot\infty) = \lim_{x\to 0+}\frac{\ln x}{1/x} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to 0+}\frac{(\ln x)'}{(1/x)'} = \lim_{x\to 0+}\frac{1/x}{-1/x^2} =$$

$$= \lim_{x\to 0+} x = 0$$

$$2)\lim_{x\to\frac{\pi}{2}} (tgx)^{2\cos x} = \lim_{x\to\frac{\pi}{2}} e^{2\cos x\ln tgx} = \lim_{x\to\frac{\pi}{2}} e^{\frac{2\ln tgx}{1/\cos x}} = e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{2\ln tgx}{1/\cos x}} = e^{2\lim\limits_{x\to\frac{\pi}{2}}\frac{\sec x}{tg^2 x}} =$$

$$e^{2\lim\limits_{x\to\frac{\pi}{2}}\frac{\sec x\cdot tgx}{2tgx\sec^2 x}} = e^{2\lim\limits_{x\to\frac{\pi}{2}}\cos x} = e^0 = 1$$

2-Eslatma. $0\bullet\infty, \infty-\infty$, 0^0 va $\infty^{\,0}$ ko‘rinishdagi limitlarni hisoblash uchun ularni avval $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko‘rinishdagi aniqmasliklarga keltirib, so‘ngra Lopital qoidasi qo‘llaniladi.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1 – 8. Hosila ta’rifidan foydalanib funksiyani hosilasini toping.

1. $f(x) = 5x+3$ J: 5 ;

2. $f(x) = 5-4x+3x^2$ J: $6x-4$;

3. $f = x^3 - x^2 + 2x$ J: $3x^2-2x+2$;

4. $f(x) = x+\sqrt{x}$ J: $1+\frac{1}{2\sqrt{x}}$;

5. $g(x) = \sqrt{1+2x}$ J: $\frac{1}{\sqrt{1+2x}}$

6. $f(x) = \frac{x+1}{x-1}$ J: $-\frac{2}{(x-1)^2}$

7. $g(x) = \frac{4-3x}{2+x}$ J: $-\frac{10}{(2+x)^2}$;

8. $g(x) = \frac{1}{x^2}$ J: $-\frac{2}{x^3}$;

Quyidagi funksiyalar uchun: a) Hosila ta’rifidan foydalanib funksiyani hosilasini toping. b) f va f' funksiyalarning grafigi yasalsin.

9. $f'(x) = 3x^2 - 1$ J: $6x$;

10. $f(x) = \sqrt{x-1}$ J: $\frac{1}{2\sqrt{x-1}}$

11. $y = 3x^3 - 2x^2 + 3x - 1$ hosila ta’rifidan foydalanib funksiyani hosilasini toping.

J: $27x^2 - 4x + 3$

Jadval asosida funksiya hosilasi olinsin.

12. $f(x)=x^4$ J: $4x^3$; 13. $f(x)=x^6$ J: $6x^5$;

14. $y=x^4-6x^2+4$ J: $4x^3-12x$; 15. $f(r)=r^{\frac{1}{4}}$ J: $\frac{1}{4}r$;

16. $f(x)=3x^4$ J: $12x^3$; 17. $f(x)=x^{1000}$ J: $1000x^{999}$

18. $F(x)=(6x^3)(7x^4)$ J: $294x^6$; 19. $y=\frac{x^2+x-2}{x^3}$ J: $\frac{6-2x-x^2}{x^4}$

20. $y=1/x$ J: $-\frac{1}{x^2}$ 21. $y=\frac{\sqrt{x}}{(1+x^2)}$ J: $\frac{(1-3x^2)\sqrt{x}}{2x(1+x^2)^2}$

22. $g(s)=(s^2+s+1)(s^2+2)$ J: $4s^3+3s^2+6s+2$

23. $y=2\sqrt{x^3}-\frac{7}{x}+3x^2-\frac{2}{x^5}$ J: $\frac{1}{3\sqrt{x}}+\frac{7}{x^2}+6x+\frac{10}{x^6}$

24. $y=5x^2-\sqrt[3]{x^8}+\frac{4}{x^3}-\frac{5}{x}$ J: $10x-\frac{8}{3}x\sqrt[3]{x^2}-\frac{12}{x^4}-5\ln|x|$ $+5/x^2$

25. $y=7x+\frac{5}{x^2}-\sqrt[5]{x^4}+\frac{6}{x}$ J: $7-\frac{10}{x^4}-\frac{4}{5\sqrt[5]{x}}+6\ln|x|$ $-6/x^2$

26. $y=3x^5-\frac{3}{x}\sqrt{x^3}+\frac{10}{x^2}$ J: $15x^4-\frac{3}{2\sqrt{x}}-\frac{20}{x^3}$

27. $y=x^2-8x+9$ parabolaga $(3;-6)$ nuqtadan o‘tkazilgan urinma tenglamasi tuzilsin. J: $y=-2x$

28. $y=\frac{\sqrt{x}}{(1+x^2)}$ funksiyaga $\left(1;\frac{1}{2}\right)$ nuqtada o‘tkazilgan urinma tenglamasi tuzilsin.

J: $y=-\frac{1}{4}x+\frac{3}{4}$

29. $y = x^3 + 2x - 2$ egri chiziqqa abssissasi $x_o = 1$ bo‘lgan nuqtadan o‘tkazilgan urinma va normalning tenglamasi tuzilsin. J: $y = 5x - 4; y = -\frac{1}{5}x + \frac{6}{5}$

30. $y = 3tg2x + 1$ egri chiziqqa abssissasi $x = \frac{\pi}{2}$ bo‘lgan nuqtadan o‘tkazilgan normalning tenglamasini yozing. J: $y = -\frac{1}{6}x + \frac{\pi}{12} + 1$

31. Moddiy nuqtaning t vaqt ichida bosib o‘tgan masofasi $S = \frac{1}{4}t^4 - \frac{1}{3}t^3 + 2t + 1$ ga teng. Berilgan nuqtaning tezligini toping. J: $V = t^3 - t^2 + 2$

32. *OY* o‘qi bo‘yicha ikkita moddiy nuqta $x_1 = \frac{t^2}{3} - 4$ va $x_2 = \frac{7}{2}t^2 - 12t + 3$ qonun bo‘yicha harakatlanadi. Qanday vaqtdan keyin ularning tezligi teng bo‘ladi. J: $t = 2$

33. Moddiy nuqta $S = t^4 - 3t^2 + 2t - 4$ qonun bo‘yicha harakatlanadi. Nuqta harakatining *t=2sek.* dagi tezligini toping. J: 22

34. Moddiy nuqta $S = 4t^3 - 2t + 11$ qonun bo‘yicha harakatlanadi. Necha sekunddan keyin uning tezligi *190 m/c* ga teng bo‘ladi. J: $t = 16\,s$

Murakkab funksiyaning hosilasi topilsin.

35. $y = \frac{2\lg(4x+5)}{(x+6)}$

J: $\frac{8(x+6) - 2(4x+5)\ln(4x+5)}{(4x+5)(x+6)^2}$

36. $y = \frac{\log_5(3x-7)}{ctg7x^3}$

J: $\frac{3\cos 7x^3 + 42x^2\ln(4x+5)}{(3x-7)\ln 5\cos^2 7x^3 ctg^2 7x^3}$

37. $f(x) = \sin x$ funksiyaning hosilasi $f'(x) = \cos x$ ekanligi isbotlansin.

Parametrik ko‘rinishdagi funksiyaning birinchi tartibli hosilasi topilsin.

38. $\begin{cases} y = t^3 + t^2 - 1 \\ x = t^2 + t + 1 \end{cases}$ J: $\dfrac{3t^2 + 2t}{2t + 1}$ 39. $\begin{cases} y = 2\sin^3 t \\ x = 2\cos^3 t \end{cases}$ J: $-tgt$

40. $\begin{cases} y = t^3 + t^2 + 1 \\ x = 1/t \end{cases}$ J: $-t^3(3t+2)$ 41. $\begin{cases} y = t^3 + t \\ x = t^2 - 2t \end{cases}$ J: $\dfrac{3t^2 + 1}{2t - 2}$

42. $\begin{cases} y = (2t+1)\cos t \\ x = \ln t \end{cases}$ J: $2\cos t - (2t+1)\sin t$ 43. $\begin{cases} y = 1 - \cos t \\ x = t - \sin x \end{cases}$ J: $\dfrac{\sin t}{1 - \cos t}$

44. $\begin{cases} x = \dfrac{2t}{1+t^2} \\ y = \dfrac{t^2}{1+t^2} \end{cases}$ J: $\dfrac{t}{1-t^2}$ 45. $\begin{cases} x = \sqrt{t} \\ y = \sqrt[3]{t} \end{cases}$ J: $\dfrac{2}{3}\dfrac{1}{\sqrt[6]{t}}$

Oshkormas funksiyaning birinchi tartibli hosilasini toping.

46. $y^2 - x = \cos x$; 47. $3x + \sin y = 5y$; 48. $tgy = 3x + 5y$;

49. $xy = ctgy$; 50. $y = \ell^y + 4x$; 51. $\ln y - \dfrac{y}{x} = 7$;

Berilgan funksiyalarning n- tartibli hosilasini toping

52. $y = \dfrac{1}{x+5}$; 53. $y = \ell^{-2x}$; 54. $y = x\ \ell^{3x}$; 55. $y = \ln(x-3)$;

56. $y = \ln(3 = x)$; 57. $y = \sqrt{x}$; 58. $y = \ln(5+x)^2$;

Oshkormas va parametric ko‘rinishdagi funksiyalardan ikkinchi tartibli hosila olinsin.

59. $x^3 + 3xy + 3y = 7$; 60. $\ell^{y-1} = \cos x$; 61. $\cos^2 x + \cos^2 y = \dfrac{2}{3}$;

62. $x^6 + y^5 = 6x$; 63. $xy = y^5$; 64. $\begin{cases} x = \sin^2 t \\ y = \cos^4 t \end{cases}$;

65. $\begin{cases} x = \ell^{\sin t} \\ y = \ell^{3t} \end{cases}$; 66. $\begin{cases} \sqrt[3]{(t+1)} \\ y = \sqrt{t+1} \end{cases}$; 67. $\begin{cases} x = \ln^3 t \\ y = t - \ln t \end{cases}$;

68. $\begin{cases} x = \ell^t \\ y = \dfrac{1}{\ell^t} \end{cases}$; 69. $x^3 + y^5x^4 + 6x - 7y = 0$; 70. $\sqrt{x^2 + y^2} = x^3$;

Funksiyalarning birinchi tartibli differensialini toping

71. $f(x) = 5x - 1$

72. $f(x) = -4x^{10}$

73. $f(x) = x^2 + 3x - 4$

74. $g(x) = 5x^8 - 2x^5 + 6$

75. $v(t) = 6t^{-9}$

76. $v(r) = \dfrac{4}{3}\pi r^3$

77. $r(t) = 5t^{-\frac{3}{5}}$

78. $y = 4\pi^2$

79. $r(x) = \dfrac{\sqrt{10}}{x^7}$

Ikkinchi tartibli differensiali topilsin.

80. $y = (x^2 + 4x + 6)^2$

81. $y = \cos(tgx)$

82. $y = \sqrt{\sin x}$

83. $y = tg3x$

84. $y = \sqrt[4]{1 + x^2}$

85. $y = \sin\sqrt{x}$

86. $y = (3x-2)^{10}(5x^2 - x + 1)^{12}$

87. $(6t^2 + 5)^3(t^3 - 7)^4$

88. $y = (2x-5)^4(8x^2 - 5)^{-3}$

89. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

90. $y = x^3 \cos nx$

91. $[0;8]$ kesmada $f(x) = \sqrt[3]{8x - x^2}$ funksiya uchun Roll teoremasi bajariladimi.

J: Bajariladi.

92. Roll teoremasini $f(x) = \sqrt[3]{x^2}$ funksiyaga $[-1,1]$ kesmada tadbiq qilish mumkinmi?

J: Tatbiq qilib bo‘lmaydi, chunki $x=o$ bo‘lganda hosila mavjud emas.

93. $f(x)=x^2-6x+100$ funksiya uchun $[1,5]$ oraliqda Roll teoremasi o‘rinlimi?

J: O‘rinli

94. $[-1;0]$ va $[0;1]$ kesmada $f(x)=x-x^3$ funksiya uchun Roll teoremasi o‘rinlimi?

x ning qanday qiymatida o‘rinli bo‘ladi? J: $x=\pm\frac{1}{\sqrt{3}}$.

95. $f(x)=\sqrt[3]{(x-8)^2}$ funksiya uchun $[0;10]$ kesmada Logranj teoremasi o‘rinlimi?

J: O‘rinli emas.

96. $f(x)=x^3$ va $\varphi(x)=x^2$ funksiyalar uchun Koshi formulasini yoying va c nuqtani toping.

Lopital qoidasini qo‘llab limitni toping.

97. $\lim\limits_{x\to 0}\frac{x^3-7x^2+4x+2}{x^3-5x+4}$; 98. $\lim\limits_{x\to 0}\frac{\ell^x-\ell^{-x}}{\ln(1+x)}$; 99. $\lim\limits_{x\to\infty}\frac{\ell^x}{x^3}$;

100. $\lim\limits_{x\to 1}\frac{tg\frac{\pi x}{2}}{\ln(1-x)}$; 101. $\lim\limits_{x\to\infty}\frac{xe^{\frac{x}{2}}}{x+e^x}$; 102. $\lim\limits_{x\to 0}\frac{1-\cos x^2}{x^2-\sin x^2}$;

103. $\lim\limits_{x\to o}\frac{x-arctgx}{x^3}$; 104. $\lim\limits_{x\to 0}\frac{tgx-\sin x}{x-\sin x}$; 105. $\lim\limits_{x\to 0}\frac{\ell^{5x}-1`}{\sin 3x}$;

MUSTAQIL YECHISH UCHUN MASHQLAR.

Hosila ta'rifidan foydalanib funksiyani hosilasini toping

3.1. $y=2x^3+5x^2-7x-4$

3.2. $y=x^4$

3.3. $y=\sqrt{x}$

3.4. $y=\sqrt{x}$

3.5. $y=\frac{1}{x^2}$

3.6. $y=x^2+4x$

3.7. $y=x^3$

3.8. $y=x^2-2x+5$

Funksiyalarning urinma tenglamasi va normal tenglamasini tuzing :

3.9. $y=x^3-5x^2+7x-2$ egri chiziqning $(1;1)$ nuqtasida o'tkazilgan normalning tenglamasini yozing.

3.10. $y=x^2-6x+2$ egri chiziqqa abssissasi *x=2* nuqtada o'tkazilgan urinma tenglamasi tuzilsin.

3.11. $y=\frac{x^2}{4}-x+5$ egri chiziqqa abssissasi x=4 nuqtadan o'tkazilgan urinma tenglamasini yozing.

3.12. $y=\frac{x^4}{4}-27x+60$ egri chiziqqa abssissasi *x=2* nuqtada o'tkazilgan urinma tenglamasini yozing.

3.13. $y=\frac{2}{3}x^5-\frac{1}{9}x^3$ egri chiziqqa abssissasi *x=1* nuqtada o'tkazilgan urinma tenglamasi tuzilsin.

3.14. $\frac{x^2}{9}-\frac{y^2}{8}=1$ egri chiziqqa *M(-9;-8)* nuqtada o'tkazilgan urinmaning tenglamasi tuzilsin.

3.15. $y=\frac{8}{4+x^2}$ lokonga (zulfga) *x=2* nuqtada o‘tkazilgan urinma tenglamasi tuzilsin.($y=-\frac{x}{2}+2$)

3.16. $y=\frac{x^3}{3}$ egri chiziqqa abssissasi *x*=-2 nuqtada o‘tkazilgan urinma tenglamasi tuzilsin.

3.17. $y^2=x^3$ egri chiziqqa abssissasi *x=0 va x=1* nuqtalarda o‘tkazilgan urinma tenglamasi tuzilsin.

3.18. $y=\sin x$ sinusoidaga $x=\pi$ nuqtada o‘tkazilgan urinma tenglamasi tuzilsin.

3.19. $y=\frac{4}{x}$ giperbolaga *x=1 va x=-4* nuqtalarda o‘tkazilgan urinma tenglamasi tuzilsin.

3.20. $y=4x-x^2$ funksiyaga *OX* o‘qi bilan kesishgan nuqtalarida o‘tkazilgan urinmalarning tenglamasi tuzilsin.

3.21. $y^2=4-x$ funksiyaga *OY* o‘qi bilan kesishgan nuqtalarida o‘tkazilgan urinma tenglamasi tuzilsin.

3.22. $y=x^2-4x+5$ parabolaga *OY* o‘qi bilan kesishgan nuqtalarida o‘tkazilgan urinma tenglamasi tuzilsin.

3.23. $y=\frac{x^2}{4}$ parabolaga abssissasi *x=2* nuqtada o‘tkazilgan urinma tenglamasi tuzilsin.

3.24. $y=x^3-5x^2+7x-2$ egri chiziqning $(1;1)$ nuqtasida o‘tkazilgan urinma tenglamasini yozing.

3.25. $y=x^2-6x+2$ egri chiziqqa abssissasi *x=2* nuqtada o‘tkazilgan normal tenglamasi tuzilsin.

3.26. $y=\frac{x^2}{4}-x+5$ egri chiziqqa abssissasi $x=4$ nuqtada o‘tkazilgan normal tenglamasini yozing.

3.27. $y=\frac{x^4}{4}-27x+60$ egri chiziqqa abssissasi $x=2$ nuqtada o‘tkazilgan normal tenglamasini yozing.

3.28. $y=\frac{2}{3}x^5-\frac{1}{9}x^3$ egri chiziqqa abssissasi $x=1$ nuqtada o‘tkazilgan normal tenglamasi tuzilsin.

Funksiya hosilasini jadval asosida toping.

3.29. $y=2x^3-5x^2+7x+4$; 3.30. $y=x^3arctgx$; 3.31. $y=\frac{\sqrt{4x+1}}{x^2}$;

3.32. $y=x^2\sqrt{1-x^2}$; 3.33. $y=\frac{\arcsin x}{x}$; 3.34. $y=\frac{1}{2}x^3\ell^x$;

3.35. $y=tg^6x$9. $y=\cos^2 x$; 3.36. $y=\sin(2x+3)$; 3.37. $y=\lg\ln x$;

3.38. $y=\sin^3\frac{x}{3}$; 3.39. $y=\ln(x^2+5)$; 3.40. $y=\frac{\sin x}{\cos^2 x}$;

3.41. $y=\frac{7}{x^3}$; 3.42. $y=\ln(2x^3+3x^2)$; 3.43. $y=\sqrt{1-3x^2}$;

3.44. $y=2x^3-1$ egri chiziqning qaysi nuqtasidan o‘tkazilgan urinma o‘qi bilan $\frac{\pi}{4}$ burchak tashkil etishini aniqlang.

3.45. Moddiy nuqta $f(t)=t^2-6t-5$ qonun bo‘yicha harakatlanadi. Nuqta harakatining $t=2cek$. dagi tezligini toping.

3.46. Moddiy nuqta $f(t)=2t^2-t+1$ qonun bo‘yicha harakatlanadi. Nuqta harakatining $t=2cek$. dagi tezligini toping.

Teskari funksiyaning hosilasi toping.

3.47. $y=(x-4)^5 arcctg3x^2$

3.63. $y=\frac{3\arcsin(2x-7)}{(x+2)^4}$

3.49. $y=\frac{\arcsin^2 4x}{arctg(5x-3)}$

3.65. $y=4(x-7)^6 \arcsin 3x^5$

3.51. $y=arctg^2 5x(\ln(x-4))$

3.67. $y=(x+5)^2 \arccos^3 5x$

3.53. $y=(x-3)^4 \arccos 5x^3$

3.69. $y=\arccos\frac{9-x^2}{9+x^2}$

3.55. $y=\frac{\ell^{\arccos x}}{\sqrt{x+5}}$

3.71. $y=\sqrt{x}\arcsin\sqrt{x}$

3.48. $y=(x-2)^4 \arcsin 5x^4$

3.58. $y=arktg\frac{x}{\sqrt{a^2-x^2}}$

3.50. $y=\frac{8arctg(2x+3)}{(x+1)^3}$

3.60. $y=arctg\frac{1-\sqrt{x}}{x}$

3.52. $y=\frac{7\arccos(4x-1)}{(x+2)^4}$

3.62. $y=\arcsin\frac{2x^3 1+x^6}{}$

3.54. $y=\sqrt{\arccos 2x * 3^{-x}}$

3.64. $y=xarcccos\frac{x}{2}-\sqrt{4-x^2}$

3.56. $y=arctg^4 x*\log_2(x-3)$

3.57. $y=arctg^2 4x*3^{\arcsin x}$

3.66. $y=arctg\sqrt{\frac{1-x}{1+x}}$

3.59. $y=\frac{7arctg(4x+1)}{(x-4)^2}$

3.68. $y=\arcsin\sqrt{1-0{,}2x^2}$

3.61. $y=5^{-x^2}\arccos 5x$

Murakkab funksiya hosilasini toping.

3.73. $y=3^{x^2}$

3.79. $y=x^{\frac{2}{x}}$

3.75. $y=x^3 tg^3 x$

3.81. $y=x^{\ell^x}$

3.77. $y=x^2\sin 2x$

3.83. $y == x^{\arcsin x}$

3.78. $y = (\ln(x+7))^{ctg\,2x}$

3.85. $y = (\cos x)^{\cos x}$

3.80. $y = (ctg(7x+4))^{\sqrt{x+3}}$

3.87. $y = (\ln x)^{x}$

3.82. $y = (\ln(5x+4))^{arctgx}$

3.89. $y = 2x^{\sqrt{x}}$

3.84. $y = (\cos 7x)^{x^2}$

3.91. $y = (\cos x)^{x^2}$

3.86. $y = x^{x^2}$

3.93. $y = (\sin x)^{\cos x}$

3.88. $y = (\ln x)^{x}$

3.95. $y = x^{\arccos x}$

3.90. $y = (\cos x\ )^{x^2}$

3.97. $y = x^{tgx}$

3.92. $y = \left(\sqrt{5x+3}\right)^{4x}$

3.99. $y = \left(\sqrt{3x+2}\right)^{x}$

3.94. $y = (\sin 5x)^{arctgx}$

3.101. $y = (\cos 5x)^{arctgx}$

3.96. $y = (tg5x)^{\sqrt{x+1}}$

3.74. $y = (tg3x)^{x^2}$

3.98. $y = (ctgx^2)^{x}$

3.76. $y = (ctg5x)^{x^3-1}$

3.100. $y = (tgx)^{\sqrt{x+2}}$

Parametrik ko‘rinishda berilgan funksiyaning hosilasi olinsin.

3.102. $\begin{cases} x = \ln\cos 2t \\ y = \sin 2t \end{cases}$

3.107. $\begin{cases} x = \sin^3 4t \\ y = \dfrac{1}{2}\cos^3 4t \end{cases}$

3.103. $\begin{cases} x = x - \ell^{3t} \\ y = \dfrac{1}{3}\left(\ell^{-3t}\right) \end{cases}$

3.104. $\begin{cases} x = \dfrac{1}{3}t^3 + t \\ y = \ln\left(t^2+1\right) \end{cases}$

3.105. $\begin{cases} x = \dfrac{1-t}{t^2} \\ y = \dfrac{1+t}{t^2} \end{cases}$

3.106. $\begin{cases} x = tgt \\ y = \dfrac{1}{\sin^2 t} \end{cases}$

3.108. $\begin{cases} x = \ln(1+t^2) \\ y = t - arctgt \end{cases}$

3.109. $\begin{cases} x = \dfrac{\sin t}{1+\sin t} \\ y = \dfrac{\cos t}{1+\sin t} \end{cases}$

3.111. $\begin{cases} x = 4 - \ell^{-2t} \\ y = \dfrac{3}{\ell^{2t}+1} \end{cases}$

3.113. $\begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases}$

3.115. $\begin{cases} x = t\sin t \\ y = t\cos t \end{cases}$

3.117. $\begin{cases} x = \cos\dfrac{t}{2} \\ y = t - \sin t \end{cases}$

3.119. $\begin{cases} x = t + \sin t \\ y = 1 - \cos t \end{cases}$

3.121. $\begin{cases} x = t^2 \\ y = \dfrac{1}{3}t^3 - t \end{cases}$

3.123. $\begin{cases} x = \cos 3t \\ y = \sin 3t \end{cases}$

3.125. $\begin{cases} x = \sin\dfrac{t}{2} \\ y = \cos t \end{cases}$

3.127. $\begin{cases} x = \ell^{2t} \\ y = \cos t \end{cases}$

3.129. $\begin{cases} x = t^2 + 1 \\ y = \ell^{t^3} \end{cases} 1$

3.110. $\begin{cases} x = 3\cos^2 t \\ y = 2\sin^3 t \end{cases}$

3.112. $\begin{cases} x = t + \ln\cos t \\ y = t - \ln\sin t \end{cases}$

3.114. $\begin{cases} x = 2t - \sin 2t \\ y = \sin^2 t \end{cases}$

3.116. $\begin{cases} x = t + \frac{1}{2}\sin 2t \\ y = \cos^3 t \end{cases}$

3.118. $\begin{cases} x = t^5 + 2t \\ y = t^3 8t + 1 \end{cases}$

3.120. $\begin{cases} x = \dfrac{1}{3}t^3 + \dfrac{1}{2}t^2 + t \\ y = \dfrac{1}{2}t^2 + \dfrac{1}{t} \end{cases}$

3.122. $\begin{cases} x = \arcsin(t^2 - 1) \\ y = \arccos 2t \end{cases}$

3.124. $\begin{cases} x = t^2 + t + 1 \\ y = t^3 + t \end{cases}$

3.126. $\begin{cases} x = ctgt \\ y = \dfrac{1}{\cos^2 t} \end{cases}$

3.128. $\begin{cases} x = \dfrac{2-t}{2+t^2} \end{cases}$

3.130. $\begin{cases} x = 2\cos^3 2t \\ y = \sin^3 2t \end{cases}$

Oshkormas holda quyidagi tenglamalar bilan berilgan funksiyalarning birinchi tartibli hosilasi topilsin.

3.131. $x^3+y^3-3xy=0$

3.132. $arctgy=4x+5y$

3.133. $y^2-x=\cos x$

3.134. $3x+\sin y=5y$

3.135. $tgy=3x+5y$

3.136. $xy=ctgy$

3.137. $y=\ell^y+4x$

3.138. $\ln y-\frac{y}{x}=7$

3.139. $\ell^y=4x-7y$

3.140. $4\sin(x+y)=x$

3.141. $\sin y=7x+3y$

3.142. $y^2+x^2=\sin y$

3.143. $tgy=4y-5x$

3.144. $y=7x-ctgy$

3.145. $xy=6+\cos y$

3.146. $3y=7+xy^3$

3.147. $y^2=x+\ln\frac{y}{x}$

3.148. $xy^2-y^3=4x-5$

3.149. $x^2y^2+x=5y$

3.150. $x^4+x^3y^2+y=4$

Ikkinchi tartibli hosilani toping.

3.151. $y=\ln\left(x+x^2\right)$;

3.152. $y=\frac{x}{x^2-1}$

3.153. $y=x^3\ln x$;

3.154. $y=arctgx$

3.155. $y=x\ell^{x^2}$;

3.156. $y=x-arctgx$

3.157. $y=arctgx^2$;

3.158. $y=x^2\ln x$

3.159. $y=\ln tg4x$;

3.160. $y=\cos^2 x$

3.161. $y=x\ell^{\frac{1}{x}}$;

3.162. $y==x\ell^{-x}$

3.163. $y = \ln(\ln x)$;

3.164. $y = (1 + x^2)arctgx$

3.165. $y = \ell^{\sqrt{x}}$;

3.166. $y = \frac{1}{1+x^2}$

3.167. $y = \sqrt{4 - x^2}$;

3.168. $y = \frac{1}{4+\sqrt{x}}$

3.169. $y = \sqrt{1 - x^2 \arcsin x}$;

3.170. $y = \sin^4 x$

3.171. $y = \cos^4 x$;

3.172. $y = \ln(x + \sqrt{x})$

Berilgan funksiyalarning *n*- tartibli hosilasini toping.

3.173. $y = \ell^{-5x}$

3.174. $y = \ln(4 + x)$

3.175. $y = \frac{1}{x-6}$

3.176. $y = 10^x$

3.177. $y = 7^x$

3.178. $y = \cos 3x$

3.179. $y = \ln(3x - 5)$

3.180. $y = \frac{x}{x+5}$

3.181. $y = \ln\frac{1}{4-x}$

3.182. $y = \sqrt{x+7}$

3.183. $y = x\ell^{5x}$

3.184. $y = \frac{4}{x+3}$

3.185. $y = \frac{1+x}{\sqrt{x}}$

3.186. $y = \frac{1}{x+1}$

Birinchi va ikkinchi tartibli differensial topilsin.

3.187. $y = arctgx$; 3.188. $s = \ell^{t^3}$

3.189. $y = (2x-3)^3$; 3.190. $y = 2x^3 + 5x^2$

3.191. $y = x(\ln x - 1)$; 3.192. $y = \ell^{-x^3}$

3.193. $y = x^3 - 9x^2 24x - 15$; 3.194. $y = x^5 - \frac{5}{3}x^3$

3.195. $y = (x-3)^2(x-2)$; 3.196. $y = x^4 - 8x^3 + 24x^2$

3.197. $y = x^5 - x^3 - 2x$; 3.198. $y = \frac{1}{10}(x^4 - 12x)$.

3.199. $y = (x = 2)(x-1)^2$; 3.200. $y = x^2 \ell^{2x}$

3.201. $y = \ell^{-x} \sin 3x$; 3.202. $y = x^3 \sin(4x+1)$

3.203. $y = \ln(4x+1)$y; 3.204. $y = \frac{1}{x-6}$

3.205. $y = 10^x$; 3.206. $y = \cos 3x$

3.207. $y = \ln \frac{1`}{4-x}$; 3.208. $y = \sqrt{x+7}$

3.209. $y = x\ell^{5x}$; 3.210. $y = \frac{1+x}{\sqrt{x}}$

3.211. $y = \ln(5x-1)$; 3.212. $y = \frac{x^2}{\cos x}$

3.213. $g(x) = x^2 + \frac{1}{x^2}$ 3.214. $f(x) = \frac{x}{\sqrt{7-3x}}$

3.215. $y = \frac{1}{2}x^2 \ell^x$ 3.216. $y = \arcsin x$

3.217. $y = (5x-4)^5$ 3.218. $y = x \sin 2x$

3.219. $y = x^2 \ln x$ 3.220. $y = x \sin 2x$

3.221. $y = x\cos 2x$

3.222. $y = x^4 \ln x$

3.223. $y = x + arctgx$

3.224. $y = \cos^2 x$

3.225. $[-1;1]$ kesmada $f(x) = 1 - \sqrt[3]{x}$ funksiya uchun Roll teoremasi bajariladimi.

3.226. $[-1;2]$ kesmada $y = \dfrac{4}{x}$ va $y = 1 - \sqrt[3]{x^2}$ funksiyalarga Logranj teoremasini qo‘llab bo‘ladimi.

3.227. Ushbu

a) $f(x) = arctgx$ funksiya uchun $[0;1]$ kesmada;

b) $f(x) = \arcsin x$ funksiya uchun $[0;1]$ kesmada;

v) $f(x) = \ln x$ funksiya uchun $[1;2]$ kesmada Logranj formulasini yozing va $x = c$ ni toping.

Lopital qoidasini qo‘llab limitni toping.

3.228. $\lim\limits_{x\to 0} \dfrac{\cos\left(\ell^{x^2} - 1\right)}{\cos x - 1}$

3.229. $\lim\limits_{x\to 0} \dfrac{tgx - x}{2\sin x + x}$

3.230. $\lim\limits_{x\to 0} \dfrac{\ell^{\alpha x} - \cos\alpha x}{\ell^{\beta x} - \cos\beta x}$

3.231. $\lim\limits_{x\to \infty} \dfrac{\ell^{\frac{1}{x^2}} - 1}{2arctgx^2 - \pi}$

3.232. $\lim\limits_{x\to a} \dfrac{x^m - a^m}{x_n - a^n}$

3.233. $\lim\limits_{x\to \frac{\pi}{6}} \dfrac{1 - 2\sin x}{\cos 3x}$

6-BOB. FUNKSIYANI TEKSHIRISH

6.1-MAVZU: FUNKSIYANING O‘SISH VA KAMAYISHI. FUNKSIYANING EKSTREMUMI

MAVZUGA OID NAZARIY MATERIALLAR

1. Funksiyaning o‘sishi va kamayishi.

y=f(x) funksiya X to‘plamda aniqlangan va $X_1 \subset X$ bo‘lsin.

1-**Ta’rif**. Agar $x_1, x_2 \in X$ uchun (X_1 to’plamdan olingan istalgan x_1, x_2 uchun) $x_1 < x_2$ bo’lganda, $f(x_1) < f(x_2)$($f(x_1) > f(x_2)$)tengsizlir bajarilsa, *y=f(x)* funksiya X_1 to’plamda ***o‘suvchi*** (*kamayuvchi*) deyiladi; $f(x_1) \le f(x_2)$ ($f(x_1) \ge f(x_2)$)tengsizlik bajarilsa, *y=f(x)* funksiyaga X_1 to’plamda *kamaymaydigan (o‘smaydigan)* deyiladi.

1. Funksiyaning kamaymaydigan yoki o‘smaydigan intervallari uning *monotonlik intervallari* deyiladi.

2. Agar berilgan kesmada *y=f(x)* funksiya faqat o‘suvchi yoki faqat kamayuvchi bo‘lsa, shu kesmada *y=f(x)* funksiya *monoton* deyiladi.

3. Funksiyaning birinchi tartibli hosilasini nolga aylantiradigan yoki uzilishga ega bo‘ladigan nuqtalari *y=f(x)* funksiyaning kritik nuqtalari deyiladi.

Masalan. $y = 2x^2 - \ln x$ funksiyaning monotonlik intervallari va kritik nuqtalarini toping.

Yechish. Berilgan funksiya *x>0* da aniqlangan. Uning hosilasini topamiz:

$$y' = 4x - \frac{1}{x} = \frac{4x^2 - 1}{x}$$

$$y' = 0, 4x^2 - 1 = 0, \ bundan\ x_1 = \frac{1}{2},\ x_2 = -\frac{1}{2}$$

$x_2 = -\frac{1}{2}$ kritik nuqta funksiyaning aniqlanish sohasiga kirmagani uchun uni tashlab yuboramiz. Topilgan $x_1 = \frac{1}{2}$ kritik nuqta funksiyaning aniqlanish sohasini $\left(0;\frac{1}{2}\right)$ va $\left(\frac{1}{2};+\infty\right)$ intervallarga bo‘ladi. Bu intervallarda y' hosilaning ishorasini aniqlaymiz.

a) $\left(o;\frac{1}{2}\right)$ $da\ y'\left(\frac{1}{3}\right) = -\frac{5}{3} < 0$ $\quad$ $b)\left(\frac{1}{2};+\infty\right)$ $da\ \ y'(1) = 3 > 0$

Bu esa birinchi intervalda funksiya kamayuvchi, ikkinchi intervalda o‘suvchi ekanini bildiradi.

Funksiya ekstremumi mavjud bo‘lishining zaruriy sharti. Agar $y=f(x)$ funksiya $x = x_0$ nuqtada ekstremumga ega bo‘lsa, u holda $f'(x_0) = 0$ bo‘ladi yoki $f'(x_0)$ mavjud bo‘lmaydi.

Ekstremum nuqtasidan differensiallanuvchi funksiya grafigiga o‘tkazilgan urinma OX o‘qiga parallel bo‘ladi.

Masalan. $y = (x+2)^3$ funksiyaning ekstremumini toping.

Yechish. Berilgan funksiyaning hosilasini topamiz.

$$y' = 3(x+2)^3, \quad y' = 0, \ x_1 = -2$$

$x_1 = -2$ nuqtada berilgan funksiya ekstremumga ega emas, chunki $x>-2$ da

$$y = (x+2)^3 > 0, \ x < -2 \ da \ y = (x+2)^3 < o, \ x = -2 \ da \ y = (x+2)^3 = 0$$

Demak, funksiyaning hosilasini nolga aylantiradigan nuqtaning mavjud bo‘lishi funksiyaning ekstremumi mavjud bo‘ladi, deyish noto‘g‘ri ekan.

Ekstremum mavjudligini yetarli sharti. $y = f(x)$ funksiya $x=x_0$ kritik nuqta bo‘lgan biror intervalda uzluksiz va bu intervalning hamma nuqtalarida differensiallanuvchi bo‘lsin. Agar $x<x_0$ da $f'(x) > 0$ musbat, $x>x_0$ da $f'(x) < 0$

manfiy bo‘lsin, u holda $y = f(x)$ funksiya maksimumga ega bo‘ladi. Agar $x<x_0$ da $f'(x) < 0$ manfiy, $x>x_0$ da $f'(x) > 0$ musbat bo‘lsa, $y = f(x)$ funksiya minimumga ega bo‘ladi.

1-Misol. $y = x^2$ funksiyaning monotonlik intervallarini aniqlang.

Yechish. y' hosilani topamiz: $y' = 2x$. $x < 0$ da $y' < 0$ va funksiya $(-\infty; 0)$ intervalda kamayadi; $x > 0$ da $y' > 0$ va funksiya $(0; +\infty)$ intervalda o‘sadi;

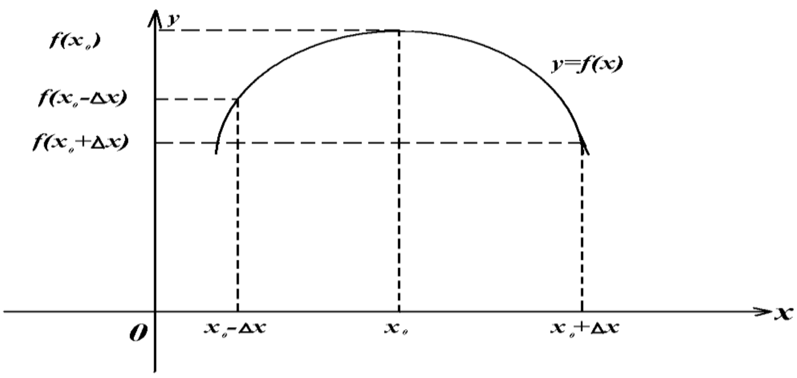
2-Misol. $y = 4x + \sin x$ funksiyaning monotonlik intervallarini aniqlang.

Yechish**.** y' hosilani topamiz: $y' = 4 + \cos x$. Barcha $x \in (-\infty; +\infty)$ uchun $y' > 0$ bo‘lganligi sababli berilgan funksiya $(-\infty; +\infty)$ intervalda o‘sadi.

2. Funksiyaning maksimum va minimum qiymatlari

x_0 nuqtada va uning atrofida aniqlangan $y = f(x)$ funksiyani qaraymiz.

3-Ta’rif. Agar $y = f(x_0)$ qiymat x_0 nuqtaning qandaydir ikki tomonlama atrofida $y = f(x)$ funksiyaning eng katta qiymati bo‘lsa, $y = f(x)$ funksiya x_0 nuqtada ***maksimum*** (maximum)ga ega deyiladi.



1-chizma

4-Ta’rif. Agar $y = f(x_0)$ qiymat x_0 nuqtaning qandaydir ikki tomonlama atrofida $y = f(x)$ funksiyaning eng kichik qiymati bo‘lsa, $y = f(x)$ funksiya x_0 nuqtada ***minimum*** ga ega deyiladi.

Funksiyaning maksimumlari va minimumlari funksiyaning ***ekstremumlari*** yoki ***ekstremal*** qiymatlari deyiladi.

Agar x_0 nuqtada funksiyaning ekstremumga ega bo‘lsa u holda bu nuqta funksiyaning ***ekstrimum*** nuqtasi deyiladi.

Masalan. $y = 1 - \sqrt[3]{x^2}$ funksiyaning hosilasi $y' = -\frac{2}{3\sqrt[3]{x}}$ x=0 nuqtada cheksizlikka aylanadi. Funksiya x=0 nuqtada maksimumga ega (1-chizma).

Shunday qilib, funksiya ekstremumga ega bo‘lgan nuqtalarda funksiyaning hosilasi nolga teng, yoki cheksizlikka teng yoki mavjud bo‘lmas ekan. Bunday nuqtalar funksiyaning ***kritik (statsionar)*** nuqtalari deyiladi. Demak funksiya ekstremal qiymatlarini faqatgina o‘zining kritik nuqtalarida qabul qilishi mumkin.

Teskari tasdiq o‘rinli emas, ya’ni nuqtaning kritik nuqta ekanligidan shu nuqtada funksiyaning ekstremumga ega ekanligi kelib chiqmaydi. Masalan, $y = x^3$ funksiyaning hosilasi $y' = 3x^2$ x=0 nuqtada nolga aylanadi. Ammo bu nuqtada funksiya ekstremumga ega emas, chunki u o‘suvchi ($y' \geq 0$).

Masalan. $y = 2x^3 - 9x^2 + 12x + 1$ funksiyaning monotonlik intervallarini va ekstremumini toping.

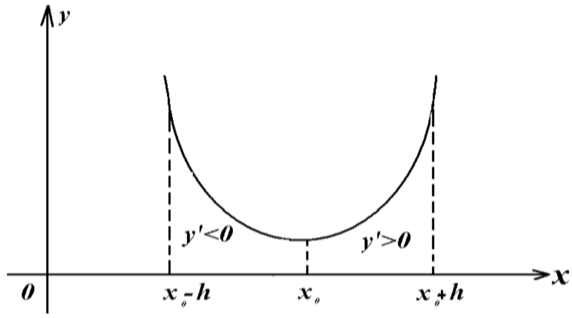
Yechish**.**1) Berilgan funksiya $(-\infty; +\infty)$ intervalda aniqlangan va differensiallanuvchi.

2) Funksiyaning hosilasini topamiz: $y' = 6x^2 - 18x + 12$.

3) Kritik nuqtalarini topamiz: $6x^2 - 18x + 12 = 0$; $x^2 - 3x + 2 = 0$;

$x_{1,2} = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = \frac{3}{2} \pm \frac{1}{2}$; $x_1 = 1,\ x_2 = 2$-kritik nuqtalar.

$y' = 6(x-1)(x-2)$ hosilaning ishorasini intervallar usulidan foydalanib tekshiramiz.



2-chizma

Demak, $(-\infty;1)$ va $(2;+\infty)$ intervallarda $y'>0$ bo‘lgani uchun bu intervallarda funksiya o‘sadi, $(1;\,2)$ intervalda $y'<0$ bo‘lgani uchun bu intervalda funksiya kamayadi. x=1 kritik nuqtaning chap tomonidan o’ng tomoniga o’tishda hosila ishorasini musbatdan manfiyga o‘zgartirganligi uchun x=1 kritik nuqtada funksiya maksimumga ega. x=2 kritik nuqtaning chap tomonidan o‘ng tomoniga o‘tishda hosila ishorasini manfiydan musbatga o‘zgartirganligi uchun bu kritik nuqtada funksiya minimumga ega (2-chizma).

$$y_{max} = y(1) = 6, \; y_{min} = y(2) = 5.$$

Masalan. $f(x) = 2x^3 - 21x^2 + 72x + 6$ funksiyaning $[2;\,5]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechish.1. Funksiyaning $[2;\,5]$ kesmadagi kritik nuqtalarini topamiz va bu nuqtalarda $f'(x)$ hosilani hisoblaymiz. $f'(x) = 6x^2 - 42x + 72.$ $f'(x) = 0$ tenglamani yechamiz:

$6x^2 - 42x + 72 = 0;\;\; x^2 - 7x + 12 = 0;\; x_1 = 3,\; x_2 = 4$ -kritik nuqtalar. Kritik nuqtalarning har ikkalasi berilgan kesmaga tegishli. Funksiyaning $x_1 = 3$ va $x_2 = 4$ nuqtalardagi qiymatlarini hisoblaymiz:

$$f(3) = 2\cdot 3^3 - 21\cdot 3^2 + 72\cdot 3 + 6 = 87,\;\; f(4) = 2\cdot 4^3 - 21\cdot 4^2 + 72\cdot 4 + 6 = 80.$$

2. Funksiyaning $[2;\,5]$ kesmaning oxirlari $x = 2$ va $x = 5$ nuqtalardagi qiymatlarini hisoblaymiz:

$$f(2)=2\cdot 2^3-21\cdot 2^2+72\cdot 2+6=82\,,\ f(5)=2\cdot 5^3-21\cdot 5^2+72\cdot 5+6=91\,.$$

Topilgan qiymatlar 82, 87, 80, 91 dan eng kichigi 80 berilgan funksiyaning $[2;\,5]$ kesmadagi eng kichik qiymati, ulardan eng kattasi 91 uning shu kesmadagi eng katta qiymati bo‘ladi.

3. Ekstremumlarni ikkinchi hosila yordamida tekshirish

Ba’zi hollarda funksiyaning ekstremumlarini uning ikkinchi hosilasi yordamida tekshirish qulay bo‘ladi.

Faraz qilaylik $y=f(x)$ funksiyaning hosilasi $x=x_0$ nuqtada nolga aylansin, ya’ni $f'(x_0)=0$ va funksiya shu nuqtada hamda uning biror atrofida ikkinchi tartibli uzluksiz hosilaga ega bo‘lib, $f''(x_0)\neq 0$ bo‘lsin.

3-Teorema (ekstremum mavjudligining ikkinchi yetarlilik sharti).

Agar $f''(x_0)<0$ bo‘lsa $f(x)$ funksiya $x=x_0$ kritik nuqtada maksimumga ega bo‘ladi, $f''(x_0)>0$ bo‘lganda y $x=x_0$ kritik nuqtada minimumga ega bo‘ladi.

Masalan. 1) $y=x+2\cos x$ funksiyaning $[0;2\pi]$ kesmadagi ekstremumini toping.

Yechish.1. Birinchi tartibli hosilani topamiz: $y'=1-2\sin x$.

2. $(0;2\pi)$ intervalga tegishli kritik nuqtalarni topamiz: $1-2\sin x=0$; $\sin x=\frac{1}{2}$, $x_1=\frac{\pi}{6}$, $x_2=\frac{5\pi}{6}$.

3. Ikkinchi tartibli hosilani topamiz: $y''=-2\cos x$.

4. Ikkinchi tartibli hosilaning $x_1=\frac{\pi}{6}$ ва $x_2=\frac{5\pi}{6}$ kritik nuqtalardagi ishoralarini aniqlaymiz.

$$f''\left(\frac{\pi}{6}\right)=-2\cos\frac{\pi}{6}=-2\cdot\frac{\sqrt{3}}{2}=-\sqrt{3}<0\,,\ f''\left(\frac{5\pi}{6}\right)=-2\cos\frac{5\pi}{6}=2\cdot\frac{\sqrt{3}}{2}=\sqrt{3}>0$$

1- teoremaga binoan berilgan funksiya $x_1 = \frac{\pi}{6}$ nuqtada maksimumga va $x_2 = \frac{5\pi}{6}$ nuqtada minimumga ega bo‘ladi.

$$y_{\max} = y\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\frac{\pi}{6} = \frac{\pi}{6} + 2\cdot\frac{\sqrt{3}}{2} = \frac{3{,}14}{6} + \sqrt{3} \approx 2{,}23,$$

$$y_{\min} = y\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\cos\frac{5\pi}{6} = \frac{5\pi}{6} - 2\cdot\frac{\sqrt{3}}{2} \approx 1{,}78.$$

2) $f(x) = x^4$ funksiyaning ekstremumini toping.

Yechish. Bu funksiya butun sonlar o‘qida aniqlangan va differensiallanuvchi.

1. Birinchi tartibli hosilani topamiz: $f'(x) = 4x^3$.

2. Hosilani nolga tenglashtirib uning ildizlarini topamiz: $f'(x) = 0$; $4x^3 = 0$; $x = 0$- kritik nuqta.

3. Ikkinchi tartibli hosilani topamiz: $f''(x) = 12x^2$.

Kritik nuqtada ikkinchi tartibli hosila nolga teng, ya’ni $f''(0) = 12\cdot 0^2 = 0$. Demak, qaralayotgan hol uchun ikkinchi yetarlilik sharti bajarilmaydi. Birinchi yetarlilik shartiga murojaat etib topamiz: $x < 0$ da $f'(x) < 0$ va $x > 0$ da $f'(x) > 0$. Shunday qilib, $x = 0$ kritik nuqtaning chap tomonidan o‘ng tomoniga o‘tishda hosila ishorasini minusdan plyusga o‘zgartirganligi sababli $x = 0$ nuqtada funksiya minimumga ega.

Demak, kritik nuqtada ikkinchi tartibli hosila mavjud bo‘lib u noldan farqli bo‘lgandagina ikkinchi yetarlilik shartidan foydalanish mumkin ekan. Agar kritik nuqtada ikkinchi tartibli hosila nolga teng bo‘lsa yoki mavjud bo‘lmasa, u holda ikkinchi yetarlilik shartidan foydalanib bo‘lmaydi.

Shunday qilib, differensiallanuvchi $y = f(x)$ funksiyaning ekstremumini quyidagi sxema asosida izlash maqsadga muvofiqdir.

1. Funksiyaning hosilasi $f'(x)$ topiladi.

2. Kritik nuqtalar topiladi; buning uchun: a) $f'(x)=0$ tenglamaning haqiqiy ildizlari topiladi.

b) x ning $f'(x)$ hosila mavjud bo‘lmagan yoki cheksizlikka aylanadigan qiymatlari topiladi.

3. Yetarlilik shartlarining birortasidan foydalanib topilgan kritik nuqtalarning har birida funksiyaning maksimum yoki minimumga ega ekanligi yoki ekstremumning mavjud emasligi aniqlanadi.

4. $f(x)$ funksiyaning ekstremumi mavjud kritik nuqtalaridagi qiymatlarini hisoblab uning ekstremumi topiladi.

4. Ekstremumlar nazariyasining masalalar yechishga tadbiqi.

Ekstremumlar nazariyasi yordamida geometriya, mexanika va hokazolarga doir ko‘pgina masalalar yechiladi. Shunday masalalarning ba’zilarini yechish usuli bilan tanishamiz.

1-Masala. Uzunligi 120 metrlik panjara bilan bir tomondan uy bilan chegaralangan eng katta yuzga ega to‘g‘ri to‘rtburchak shaklidagi maydon o‘rab olinishi kerak. To‘g‘ri to‘rtburchakli maydonning o‘lchovlari (bo‘yi va eni) aniqlansin.

Yechish. Maydonning uzunligini x, enini y, yuzini S orqali belgilaymiz. U holda to‘g‘ri to‘rtburchakning yuzini topish formulasiga ko‘ra maydonning yuzi $S=xy$ bo‘ladi.

S yuz hozircha ikkita erkli o‘zgaruvchilar x va y ga bog‘liq. Ulardan birortasini ikkinchisi orqali ifodalash uchun masalaning shartidan foydalanamiz. Shartga ko‘ra maydonning bir tomoni tayyor uy (devor) bilan, qolgan uch tomoni uzunligi $120m$ panjara bilan chegaralanishi lozim, ya’ni $x+2y=120$. Bundan $x=120-2y$ kelib chiqadi. x ning ushbu qiymatini S yuzni topish formulasiga qo‘yamiz. U holda $S=(120-2y)y=120y-2y^2$ bitta erkli o‘zgaruvchining funksiyasi hosil bo‘ladi. Endi shu $S(y)$ funksiyaning eng katta qiymatini topamiz.

$$S'(y)=120-4y,\ S''(y)=(120-4y)'=-4,$$

$S'(y)=0$ yoki $120-4y=0$ dan $4y=120;\ y=30$ yagona kritik nuqta kelib chiqadi.

$S''(30)=-4<0$ bo‘lgani uchun ikkinchi yetarlilik shartga ko‘ra *x*=30 qiymatda funksiya maksimumga ega. Bu yagona maksimum uning eng katta qiymati ham bo‘ladi. Shunday qilib bir tomoni uy bilan qolgan uch tomoni 120 *m* uzunlikdagi panjara bilan chegaralangan to‘rtburchak shaklidagi maydonlar orasida eni *y*=30*м,* bo‘yi (uzunligi) $x=120-2\cdot 30=60$ *m* bo‘lgan maydon eng katta $S=60\cdot 30m^2=1800m^2$ yuzga ega bo‘lar ekan.

2-Masala. 180 soni ko‘paytmasi eng katta va ulardan ikkitasi $1:2$ nisbatda bo‘lgan uchta qo‘shiluvchiga ajratilsin.

Yechish. Faraz qilaylik $180=x+y+z$ ko‘rinishda tasvirlansin. Shartga ko‘ra *x,y,z* sonlardan ikkitasi, masalan *x,y* $1:2$ nisbatda, ya’ni $\frac{x}{y}=\frac{1}{2}$, $y=2x$ bo‘lishi lozim. U holda $180=x+2x+z$ yoki $180=3x+z,\ z=180-3x$ hosil bo‘ladi. Demak $180=x+2x+(180-3x)$ ko‘rinishdagi uchta , $2x$, $180-3x$ qo‘shiluvchilarga ajratildi. Shularning ko‘paytmasi $y=x\cdot 2x\cdot(180-3x)=360x^2-6x^3$ ifodaning eng katta qiymatini topishimiz kerak. $y'(x)=720x-18x^2;\ y''(x)=720-36x;\ y'(x)=0$ yoki $720x-18x^2=0;$ dan $x\cdot(720-18x)=0;\ x\neq 0$ bo‘lgani uchun $720-18x=0;\ x=\frac{720}{18}=40$ kritik qiymat kelib chiqadi. $y''(40)=720-36\cdot 40<0$ bo‘lgani uchun ikkinchi yetarlilik shartiga binoan *x*=40 qiymatda $y=x\cdot 2x\cdot(180-3x)$ funksiya eng katta qiymatga ega bo‘ladi.

Demak, $y = 2x = 2 \cdot 40 = 80$, $z = 180 - 3x = 180 - 3 \cdot 40 = 60$. Shunday qilib, 180 soni 40, 80, 60 sonlarning yig‘indisi ko‘rinishida tasvirlanganda qo‘shiluvchilardan ikkitasi $1:2$ nisbatda bo‘lib, qo‘shiluvchilarning ko‘paytmasi eng katta bo‘lar ekan, ya’ni $y_{\max} = 40 \cdot 80 \cdot 60 = 192000$.

3-Masala. Marvaridni bahosi uning massasi kvadratiga proporsional. Ishlov berish vaqtida marvarid ikki bo‘lakka ajralib ketdi va natijada eng ko‘p qiymatini (bahosini) yo‘qotdi. Bo‘laklarning massalari topilsin.

Yechish. Marvaridning massasini *m*, bahosini *z*, bo‘laklarning massalarini $m_1, m_2 (m_1 + m_2 = m)$ va ularning baholarini mos ravishda z_1, z_2 orqali belgilaymiz. U holda butun marvaridning bahosi $z = \alpha \cdot m^2$ bo‘laklarning baholari esa $z_1 = \alpha \cdot m_1^2, z_2 = \alpha \cdot m_2^2$ bo‘ladi, bunda $\alpha > 0$ -proporsionallik koeffitsienti. Shartga ko‘ra, butun marvaridning bahosi $z = \alpha \cdot m^2$ bilan siniq ikki bo‘lak marvaridning bahosi $\alpha \cdot m_1^2 + \alpha \cdot m_2^2$ orasidagi farq $y = \alpha \cdot m^2 - \alpha \cdot (m_1^2 + m_2^2)$ eng katta ekanligi bizga ma’lum, $m = m_1 + m_2$ yoki $m_2 = m - m_1$ ekanini hisobga olsak

$$y = \alpha \cdot m^2 - \alpha \cdot (m_1^2 + m_2^2) = \alpha \cdot m^2 - \alpha \cdot m_1^2 - \alpha \cdot (m - m_1)^2 =$$
$$= \alpha \cdot m^2 - \alpha \cdot m_1^2 - \alpha \cdot m^2 + 2\alpha \cdot m \cdot m_1 - \alpha \cdot m_1^2 = 2\alpha \cdot (m \cdot m_1 - m_1^2)$$

kelib chiqadi. Bu yerda α, *m* o‘zgarmas miqdorlar, m_1 эса o‘zgaruvchi miqdordir. Endi $y(m_1)$ funksiyaning eng katta qiymatini topamiz. $y' = 2\alpha(m - 2m_1); y'' = -4\alpha.$ $y'(m_1) = 0$ yoki $m - 2m_1 = 0$ dan $m_1 = \frac{m}{2}$ kritik qiymat kelib chiqadi. $y''\left(\frac{m}{2}\right) = -4\alpha < 0$ bo‘lgani uchun ikkinchi yetarlilik shartiga ko‘ra $y = 2\alpha(mm_1 - 2m_1^2)$ funksiya $m_1 = \frac{m}{2}$ qiymatda maksimumga ega bo‘ladi. Demak, marvarid teng $\left(m_1 = m_2 = \frac{m}{2}\right)$ ikki bo‘lakka bo‘linganda o‘zining eng ko‘p bahosini yo‘qotar ekan.

4-Masala. Jism $v_0 = 60\,m/sek$ tezlik bilan tik yo‘nalishda yuqoriga otilgan. Jismning eng yuqori ko‘tarilish balandligi topilsin.

Yechish. Fizika kursidan ma’lumki tik yo‘nalishda yuqoriga v_0 boshlang‘ich tezlik bilan otilgan jismning harakat tenglamasi $H = v_0 t - \frac{gt^2}{2}$ bo‘ladi. Bunda *H*-otilgan jismning yerdan balandligi, $g \approx 10\,m/sek^2$ erkin tushish tezlanishi, t esa sarflangan vaqt. Masalaning shartiga asosan $v_0 = 60\,m/sek$ va binobarin, $H = 60t - 5t^2$. Endi shu $H(t)$ funksiyaning eng katta qiymatini topamiz. $H'(t) = 60 - 10t;\ H''(t) = -10$.

$H'(t) = 0$ yoki $60 - 10t = 0$ dan $10t = 60,$ $t = 6$ kritik nuqta kelib chiqadi. $H''(6) = -10 < 0$ bo‘lgani uchun ikkinchi yetarlilik shartiga asosan $t = 6$ qiymatda $H = 60t - 5t^2$ funksiya maksimumga ega bo‘ladi. Demak, $H_{\max} = H(6) = 60 \cdot 6 - 5 \cdot 6^2 = 180\ (m)$.

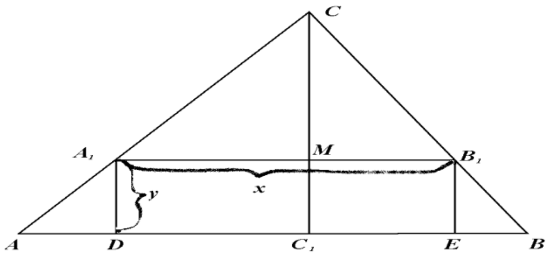
Shunday qilib $v_0 = 60\,м/сек$ tezlik bilan yuqoriga tik otilgan jism taqriban

6 sek.dan so‘ng eng yuqori *H*=180*m* balandlikka ko‘tarilar ekan.

5-Masala. Asosi *a* va balandligi *h* bo‘lgan uchburchakka eng katta yuzli to‘g‘ri to‘rtburchak ichki chizilgan. To‘g‘ri to‘rtburchakning yuzi aniqlansin.

Yechish. *ABC*(4-chizma) uchburchakka ichki chizilgan to‘g‘ri to’rtburchakning tomonlarini x va *y* orqali belgilaymiz. U holda to‘g‘ri to‘rtburchakning yuzi $S = xy$ bo‘ladi. *ABC* va A_1B_1C uchburchaklarning o‘xshashligidan

$$\frac{A_1B_1}{AB} = \frac{CM}{CC_1} \qquad (1)$$



4-chizma.

proporsiya kelib chiqadi. Masalaning shartiga ko‘ra $AB = a,\ CC_1 = h$. Belgilashimizga asosan $A_1B_1 = x,\ B_1E = MC_1 = y,\ CM = CC_1 - CM = h - y$ bo‘lgani uchun (1) munosabat quyidagi ko‘rinishga ega bo‘ladi.

$$\frac{x}{a} = \frac{h-y}{h},$$

bundan $x = \frac{a}{h}(h-y)$ kelib chiqadi. x ning ushbu qiymatini $S = xy$ ga qo‘yib

$S = \frac{a}{h}(h-y)y = \frac{a}{h}\left(hy - y^2\right)$ bir o‘zgaruvchining funksiyasiga ega bo‘lamiz.

Endi shu $S(y)$ funksiyaning eng katta qiymatini topamiz.

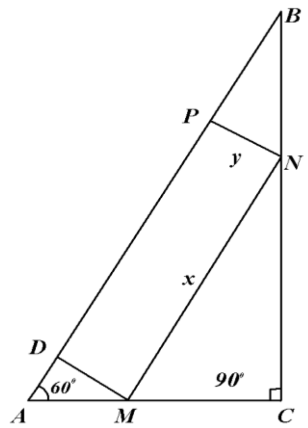
$S'(y) = \frac{a}{h}(h-2y),\ S''(y) = -\frac{2a}{h}$.

$S'(y) = 0$ yoki $\frac{a}{h}(h-2y) = 0$ dan $h - 2y = 0,\ y = \frac{h}{2}$ kritik qiymat kelib chiqadi.

$S''\left(\frac{h}{2}\right) = -\frac{2a}{h} < 0$ bo‘lgani uchun ikkinchi yetarlilik shartiga ko‘ra $S(y)$ funksiya $y = \frac{h}{2}$ da maksimumga ega bo‘ladi. Bu yagona maksimum uning eng katta qiymati ham bo‘ladi. Shunday qilib uchburchakka ichki chizilgan to‘g‘ri to‘rtburchaklardan asosi $x = \frac{a}{h}(h-y) = \frac{a}{h}\left(h - \frac{h}{2}\right) = \frac{a}{2}$ va balandligi $y = \frac{h}{2}$ bo‘lgan to‘rtburchak eng katta yuzga ega bo‘lar ekan. Bu to‘rtburchakning yuzi esa $S = \frac{ah}{4}$ bo‘ladi.

6-Masala. Gipotenuzasi 24 sm, burchagi 60^0 to‘g‘ri burchakli uchburchakka asosi gipotenuzada bo‘lgan to‘g‘ri to‘rtburchak ichki chizilgan. Shu to‘g‘ri to‘rtburchak eng katta yuzga ega bo‘lishi uchun uning tomonlari qanday bo‘lishi kerak?

Yechish. To‘g‘ri to‘rtburchakning tomonlarini x va y orqali belgilaymiz. U holda uning yuzi $S = xy$ bo‘ladi. Endi y ni x orqali ifodalaymiz(5-chizma).



5-chizma.

Shartga ko‘ra $\angle A = 60^0,\ \angle C = 90^0$. Demak $\angle B = 30^0$. Ma’lumki to‘g‘ri burchakli uchburchakning 30^0 li burchagi qarshisidagi tomoni gipotenuzaning yarmiga teng. Shuning uchun $AC = \frac{AB}{2} = \frac{24}{2} = 12(sm)$. To‘g‘ri burchakli uchburchak MNC ning 30^0 li burchagi qarshisidagi MC tomoni uning gipotenuzasi x ning yarmiga teng, ya’ni $MC = \frac{x}{2}$. Demak $AM = AC - MC = 12 - \frac{x}{2}$. ΔADM dan $AD = \frac{AM}{2}$. Pifagor teoremasiga ko‘ra $y^2 = DM^2 = AM^2 - AD^2 = AM^2 - \left(\frac{AM}{2}\right)^2 = \frac{3}{4}AM^2$ yoki

$$y = \frac{\sqrt{3}}{2}AM = \frac{\sqrt{3}}{2}\left(12 - \frac{x}{2}\right) = \sqrt{3}\left(6 - \frac{x}{4}\right)$$ bo‘ladi.

Demak, to‘g‘ri to‘rtburchakning yuzi $S = xy = x\sqrt{3}\left(6 - \frac{x}{4}\right) = \sqrt{3}\left(6x - \frac{x^2}{4}\right)$ bo‘ladi.

Endi shu $S(x)$ funksiyaning eng katta qiymatini topamiz.

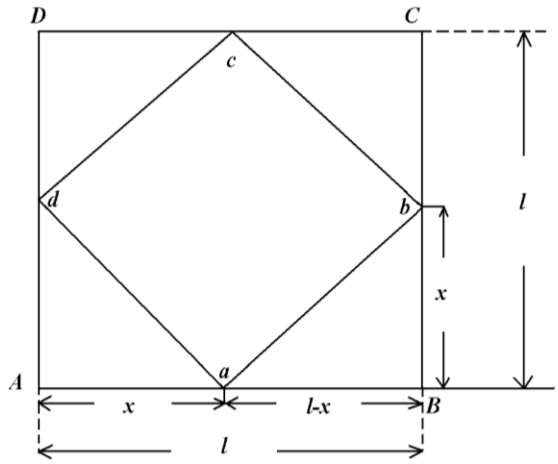
$S'(x)=\sqrt{3}\left(6-\frac{2x}{4}\right)$, $S''(x)=-\frac{\sqrt{3}}{2}$; $S'(x)=0$ yoki $6-\frac{x}{2}=0$ dan x=12 kritik qiymat kelib chiqadi. $S''(12)=-\frac{\sqrt{3}}{2}<0$ bo‘lgani uchun ikkinchi yetarlilik shartiga ko‘ra

$S(x)=\sqrt{3}\left(6x-\frac{x^2}{4}\right)=\sqrt{3}\left(6-\frac{x}{4}\right)x$ funksiya x=12 qiymatda maksimumga ega bo‘ladi.

Shunday qilib, uchburchakka ichki chizilgan va bir tomoni uning 24 sm li gipotenuzasida bo‘lgan to‘g‘ri to‘rtburchaklardan tomonlari x=12, $y=\sqrt{3}\left(6-\frac{12}{4}\right)=3\sqrt{3}$ ga teng bo‘lgani eng katta yuzga ega bo‘lib, $S_{\max}=12\cdot 3\sqrt{3}=36\sqrt{3}\ \left(sm^2\right)$ ga teng ekan.

7-Masala. *ABCD* kvadrat berilgan. Uning uchlaridan bir xil *Aa, Bb, Cc, Dd* kesmalar ajratilgan va *a, b, c, d* nuqtalarni birlashtirib kvadrat hosil qilingan. *Aa* ning qanday qiymatida *abcd* kvadratning yuzi eng kichik bo‘ladi.(6-chizma).

Yechish. $Aa=x,\ AB=\ell$ deb belgilasak, $aB=\ell-x$ va Pifagor teoremasiga ko‘ra $ab^2=x^2+\left(\ell-x\right)^2=x^2+\ell^2-2\ell x+x^2=2x^2-2\ell x+\ell^2$ bo‘ladi. Tomoni *ab* ga teng *abcd* kvadratning yuzi $S=ab^2$ ga teng. Demak, $S=2x^2-2\ell x+\ell^2$. Endi shu $S(x)$ funksiyaning eng kichik qiymatini topamiz. $S'(x)=4x-2\ell,\ S''(x)=4.\ S'(x)=0$ yoki $4x-2\ell=0$ dan $x=\frac{\ell}{2}=\frac{AB}{2}$ kritik qiymat kelib chiqadi. $S'\left(\frac{\ell}{2}\right)=4>0$ bo‘lgani uchun ikkinchi yetarlilik shartiga binoan $S=2x^2-2\ell x+\ell^2$ funksiya $x=\frac{\ell}{2}$ qiymatda eng kichik qiymatga ega bo‘ladi. Shunday qilib, *ABCD* kvadratga masalaning shartida ko‘rsatilgandek qilib ichki chizilgan kvadratlardan *ABCD* kvadrat tomonlarini o‘rtasini birlashtirib hosil qilingan kvadrat eng kichik yuzga ega bo‘lar ekan. (6-chizma).



6-chizma.

8-Masala. Tagi kvadrat shaklidagi, hajmi 108 m^3 ga teng ochiq hovuzning o'lchovlari shunday aniqlansinki, uning devorlari bilan tagini qoplash uchun mumkin qadar oz material sarf etilsin. Hovuzning o'lchovlari deganda uning tagini tomonlari va balandligi (chuqurligi) tushuniladi.

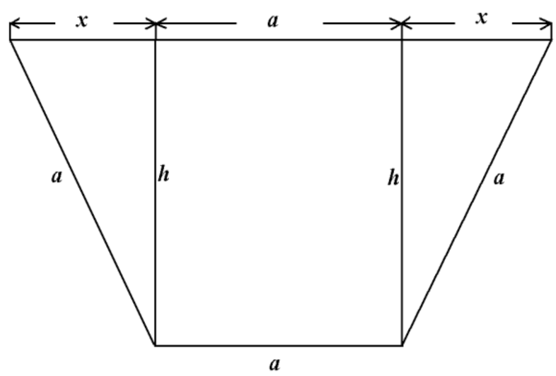
Yechish. Hovuz tagini tomoni x orqali va hovuz balandligini h orqali belgilaymiz. U holda hovuz parallelepiped shaklida bo'lgani uchun uning hajmi $v = x^2 h$ bo'ladi. Shartga ko'ra $x^2 h = 108$. Hovuz tagi x^2, devori $4xh$ yuzga ega bo'lgani uchun jami $S = x^2 + 4xh$ yuzni material bilan qoplash lozim. S yuzni birgina erkli o'zgaruvchining funksiyasi sifatida ifodalash uchun $x^2 h = 108$ tenglikdan topilgan $h = \frac{108}{x^2}$ qiymatni unga qo'yamiz. U holda $S = x^2 + 4x\frac{108}{x^2} = x^2 + \frac{432}{x}$ kelib chiqadi. Endi shu $S(x)$ funksiyaning eng kichik qiymatini topamiz.

$$S'(x) = 2x - \frac{432}{x^2};\ S''(x) = 2 + \frac{864}{x^3}\ (x > 0).$$

$S'(x) = 2x - \frac{432}{x^2} = 0$ dan $2x^3 - 432 = 0;\ x^3 = 216,\ x = 6$ kritik nuqta kelib chiqadi. Ikkinchi hosila $S''(6) = 2 + \frac{864}{216} > 0$ bo'lgani uchun ikkinchi yetarlilik shartiga asosan $x = 6$ qiymatda $S(x) = x^2 + \frac{432}{x}$ funksiya eng kichik qiymatga ega bo'ladi.

Demak, hajmi 108 $м^3$ ga teng ochiq hovuzning tagi 6*m* kvadratdan iborat, balandligi $h=\frac{108}{36}=3\ m$ bo‘lgandagina uning devorlariga ishlov berish uchun eng kam material sarflanar ekan. Ya’ni hovuzning o‘lchovlari $6m\times 6m\times 3m$ bo‘lishi lozim ekan.

9-Masala. Trapetsiyaning kichik asosi va yon tomonlarining har biri *a* ga teng. Uning katta asosi shunday aniqlansinki, trapetsiyaning yuzi eng katta bo‘lsin(7-chizma).



7-chizma

Yechish. Chizmaga binoan trapetsiyaning katta asosi 2*x*+*a* ga teng. Trapetsiyaning balandligini *h* orqali belgilaymiz. Ma’lumki trapetsiyaning yuzi asoslari yig‘indisining yarmi bilan balandligi ko‘paytmasiga teng, ya’ni

$$\frac{2x+a+a}{2}h=(x+a)h$$

Pifagor teoremasiga ko‘ra chizmadan $h=\sqrt{a^2-x^2}$ bo‘lgani uchun trapetsiyaning yuzi $S=(x+a)\sqrt{a^2-x^2}$ bo‘ladi. Endi shu $S(x)$ funksiyaning eng katta qiymatini topamiz.

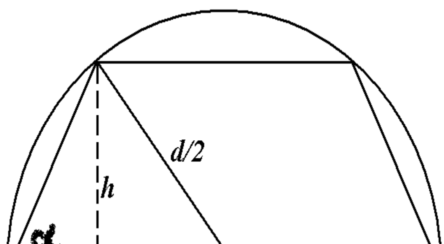
$$S'(x)=\sqrt{a^2-x^2}+(x+a)\frac{-x}{\sqrt{a^2-x^2}}=\frac{a^2-x^2-x(x+a)}{\sqrt{a^2-x^2}}=\frac{-2x^2-ax+a^2}{\sqrt{a^2-x^2}}.$$

$S'(x)=0$ yoki $-2x^2-ax+a^2=0$ dan $x_{1,2}=\frac{a\pm\sqrt{a^2+4\cdot 2\cdot a^2}}{-4}=\frac{a\pm 3a}{-4}$;

$x_1 = \frac{a}{2}$; $x_2 = -a$ kelib chiqadi. Masalaning shartiga ko‘ra $x > 0$ bo‘lgani uchun $x = \frac{a}{2}$ kritik qiymatga ega bo‘lamiz. Hosilani

$S'(x) = \frac{-2\left(x - \frac{a}{2}\right)(x+a)}{\sqrt{a^2 - x^2}}$ ko‘rinishda tasvirlasak $x < \frac{a}{2}$ bo‘lganda $x - \frac{a}{2} < 0$ va $S'(x) > 0$ ekani kelib chiqadi. Xuddi shuningdek $x > \frac{a}{2}$ bo‘lganda $S'(x)$ *<0* ekani kelib chiqadi. $S'(x)$ hosila $x = \frac{a}{2}$ kritik qiymatning chap tomonidan o‘ng tomoniga o‘tganda o‘z ishorasini «+» dan «-» ga o‘zgartiradi. Shuning uchun birinchi yetarlilik shartiga ko‘ra $S = (x+a)\sqrt{a^2 - x^2}$ funksiya $x = \frac{a}{2}$ qiymatda maksimumga ega bo‘ladi. Bu yagona maksimum uning eng katta qiymati ham bo‘ladi. Shunday qilib trapetsiyaning katta asosi $2x + a = 2\frac{a}{2} + a = 2a$ bo‘lganda u eng katta yuzga ega bo‘lar ekan.

10-Masala. Yarim doiraga asosi yarim doira diametridan iborat bo‘lgan trapetsiya ichki chizilgan. Trapetsiyaning asosiga yopishgan burchagi qanday bo‘lganda trapetsiyaning yuzi eng katta bo‘ladi(8-chizma).



8-chizma

Yechish. Doiraning diametrini *d*, trapetsiyaning balandligini *h*, trapetsiya yon tomonining katta asosidagi proeksiyasini *x*, shu tomon bilan asos orasidagi burchakni α deb olamiz. U holda trapetsiyaning kichik asosi *d*-2*x*, balandligi

$h = xtg\alpha$ va yuzi $S = \dfrac{d + d - 2x}{2}h = (d - x)xtg\alpha$ bo‘ladi. Ikkinchi tomondan chizmadan Pifagor teoremasiga ko‘ra

$h^2 = \left(\dfrac{d}{2}\right)^2 - \left(\dfrac{d}{2} - x\right)^2 = dx - x^2$ ga ega bo‘lamiz. Bunga $h = xtg\alpha$ qiymatni qo‘ysak

$$x^2tg^2\alpha = d \cdot x - x^2;\ x^2tg^2\alpha + x^2 = d \cdot x;\ x^2\left(1 + tg^2\alpha\right) = d \cdot x;\ x \cdot \frac{1}{\cos^2\alpha} = d;\ x = d\cos^2\alpha$$

kelib chiqadi. Shunday qilib,

$$S = (d - x)xtg\alpha = \left(d - d\cos^2\alpha\right)d\cos^2\alpha\frac{\sin\alpha}{\cos\alpha} = d\left(1 - \cos^2\alpha\right)d\cos\alpha \cdot \sin\alpha = d^2\sin^3\alpha\cos\alpha$$

bo‘ladi.

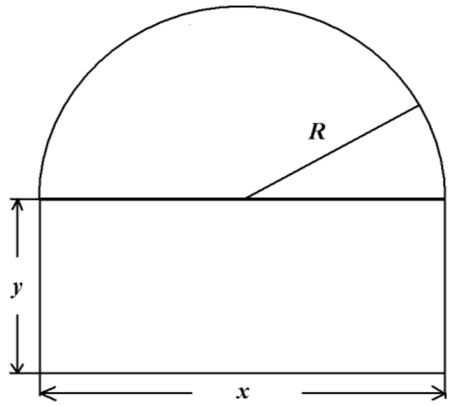
Endi $S(\alpha) = d^2\sin^3\alpha\cos\alpha$ funksiyaning eng katta qiymatini topamiz.

$$S'(\alpha) = d^2\left(\sin^3\alpha\cos\alpha\right)' = d^2\left(3\sin^2\alpha\cos^2\alpha - \sin^4\alpha\right) = d^2\sin^2\alpha\cos^2\alpha\left(3 - tg^2\alpha\right).$$

$S'(\alpha) = 0$ yoki $d^2\sin^2\alpha\cos^2\alpha\left(3 - tg^2\alpha\right) = 0$ dan $\sin^2\alpha\cos^2\alpha \neq 0$ bo‘lgani uchun $3 - tg^2\alpha = 0;\ tg^2\alpha = 3,\ tg\alpha = \pm\sqrt{3}$ kelib chiqadi. Shartga ko‘ra $0 < \alpha < \dfrac{\pi}{2}$ bo‘lgani sababli $tg\alpha = \sqrt{3}$, $\alpha = 60^0$ kritik qiymatga ega bo‘lamiz. $0 < \alpha < 60^0$ bo`lsa $tg\alpha < tg60^0 = \sqrt{3},\ tg^2\alpha < 3,\ 3 - tg^2\alpha > 0$ va $S'(\alpha) = d^2\sin^2\alpha\cos^2\alpha\left(3 - tg^2\alpha\right) > 0$ bo‘ladi.

$60^0 < \alpha < 90^0$ bo‘lganda $tg60^0 < tg\alpha;\ \sqrt{3} < tg\alpha;\ 3 < tg^2\alpha;\ 3 - tg^2\alpha < 0$ bo‘lib, $S'(\alpha) < 0$ bo‘ladi, chunki $y = tgx$ funksiya o‘suvchi. $S(\alpha)$ funksiyaning hosilasi $\alpha = 60^0$ kritik qiymatning chapidan o‘ngiga o‘tganda ishorasini “+” dan “-” ga o‘zgartirganligi uchun birinchi yetarlilik shartiga binoan funksiya $\alpha = 60^0$ bo‘lganda uning yuzi eng katta bo‘lar ekan.

11-Masala. Tunnelning ko‘ndalang kesimi bir tomoni yarim doiradan iborat to‘g‘ri to‘rtburchak shakliga ega. Kesim perimetri 25*m*. Yarim doira radiusi qanday bo‘lsa, kesim yuzi eng katta bo‘ladi(9-chizma).



9-chizma

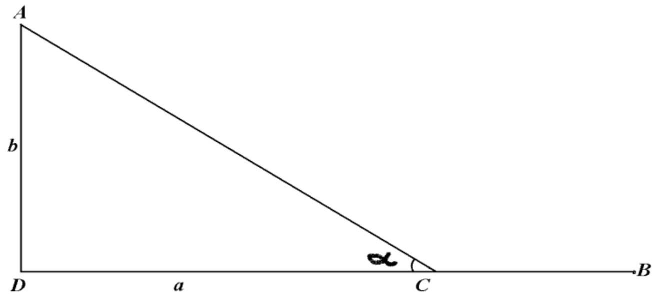
Yechish. Aylana uzunligini topish formulasi ($\ell = 2\pi R$) ga binoan yarim doiraning uzunligi πR (R-yarim doiraning radiusi). To‘g‘ri to‘rtburchakning asosini x, balandligini y orqali belgilasak kesimning perimetri shartga ko‘ra (α) bo‘ladi. Kesimning yuzi to‘g‘ri to‘rtburchak yuzi bilan yarim doira yuzining yig‘indisidan iborat, ya’ni $S = xy + \frac{1}{2}\pi R^2$ (β) bo‘ladi. $x = 2R$ bo‘lgani uchun (α) dan $2R + 2y + \pi R = 25;\ y = 12{,}5 - \frac{\pi + 2}{2}R$ kelib chiqadi. y ning topilgan qiymatini (β) ga qo‘yamiz. U holda $S = 2R\left(12{,}5 - \frac{\pi + 2}{2}R\right) + \frac{1}{2}\pi R^2 =$

$= 25R - (\pi + 2)R^2 + \frac{1}{2}\pi R^2$ bir o‘zgaruvchi R ning funksiyasi kelib chiqadi. Endi shu $S(R)$ funksiyaning eng katta qiymatini topamiz.

$S'(R) = 25 - 2(\pi + 2)R + \pi R;\ S''(R) = -2(\pi + 2) + \pi = -\pi - 4 = -(\pi + 4);$ $S'(R) = 0$ yoki $25 - 2(\pi + 2)R + \pi R = 0$ dan $25 - \pi R - 4R = 0$; $R = \frac{25}{\pi + 4} \approx 3{,}5$ kelib chiqadi.

$S''(R) = -(\pi + 4) < 0$ bo‘lgani uchun ikkinchi yetarlilik shartiga asosan funksiya *R=3,5m* bo‘lganda tunnel kesimining yuzi eng katta bo‘lar ekan.

12-Masala. *A* zavodga yaqin bo‘lgan joydan berilgan to‘g‘ri chiziq bo‘yicha *B* shaharga qarab temir yo‘l o‘tkazilgan. Agar bir tonna yukni bir km ga tosh yo‘l bo‘yicha tashish temir yo‘l bo‘yicha tashishga qaraganda m marta qimmatroq bo‘lsa, A dan B ga yuk tashish eng arzon bo‘lishi uchun, A zavoddan temir yo‘lgacha tosh yo‘lni temir yo‘lga nisbatan qanday α burchak ostida o‘tkazish kerak?(10-chizma).



10-chizma

Yechish. A zavoddan temir yo‘lgacha masofani b ($AD=b$), D dan B gacha masofani a, tosh yo‘l bilan temir yo‘l orasidagi burchakni α orqali belgilaymiz.

1 tonna yukni tosh yo‘lda 1 *km* ga tashish uchun d so‘m sarf bo‘lsin. U holda 1 tonna yukni temir yo‘lda 1 km ga tashish uchun $\frac{d}{m}$ so‘m sarflanadi. Yuk A dan B gacha AC *km* tosh yo‘lda, CB *km* temir yo‘lda tashiladi. ΔACD dan trigonometrik funksiyalarning ta’rifiga ko‘ra quyidagilarga ega bo‘lamiz. $\frac{AD}{AC}=\sin\alpha,\ AC=\frac{AD}{\sin\alpha}=\frac{b}{\sin\alpha},\ \frac{DC}{AD}=ctg\alpha;\ DC=ADctg\alpha=bctg\alpha$.

Demak $CB=DB-DC=a-bctg\alpha$. Shunday qilib tashilgan yuk A dan B gacha $AC=\frac{b}{\sin\alpha}$ km.ni tosh yo‘lda o‘tib uni tashishga $\frac{bd}{\sin\alpha}$ so’mni $CB=(a-bctg\,\alpha)$ km.ni temir yo‘lda o‘tib uni tashishga $(a-bctg\alpha)\frac{d}{m}$ so‘m sarflanadi. U holda yukni tashish uchun hammasi bo‘lib $f(\alpha)=\frac{bd}{\sin\alpha}+(a-bctg\alpha)\frac{d}{m}$ so‘m pul sarflanadi. Endi a, b, d, m larni o‘zgarmas hisoblab $f(\alpha)$ funksiyaning eng kichik qiymatini topamiz.

$f'(\alpha)=\dfrac{bd\cos\alpha}{\sin^2\alpha}+\dfrac{bd}{m\sin^2\alpha}=bd\dfrac{1-m\cos\alpha}{\sin^2\alpha}=mbd\dfrac{\frac{1}{m}-\cos\alpha}{\sin^2\alpha}$. $f'(\alpha)=0$ yoki $\frac{1}{m}-\cos\alpha=0$ dan $\cos\alpha=\frac{1}{m}$; $\alpha=\arccos\frac{1}{m}$ kritik qiymat kelib chiqadi. $\cos\alpha$ funksiya $\left[0;\ \frac{\pi}{2}\right]$ da kamayuvchi ekanini hisobga olsak $\alpha<\arccos\frac{1}{m}$ bo‘lganda $\cos\alpha>\frac{1}{m}$;

$\frac{1}{m}-\cos\alpha<0$, $f'(\alpha)=mbd\dfrac{\frac{1}{m}-\cos\alpha}{\sin^2\alpha}<0$ va $\alpha>\arccos\frac{1}{m}$ bo‘lganda $\cos\alpha<\frac{1}{m}$;

$\frac{1}{m}-\cos\alpha>0$, $f'(\alpha)>0$ kelib chiqadi. Hosila $\alpha=\arccos\frac{1}{m}$ kritik nuqtaning chapidan o‘ngiga o‘tganda ishorasini “-” dan “+”ga o‘zgartirganligi uchun birinchi yetarlilik shartiga ko‘ra funksiya shu qiymatda minimumga ega bo‘ladi.

Shunday qilib yukni A zavoddan B shaharga tashish eng arzon bo‘lishi uchun tosh yo‘lni temir yo‘lga $\alpha=\arccos\frac{1}{m}$ burchak ostida qurish lozim ekan.

Hususiy holda yukni tosh yo‘lda tashish temir yo‘ldagiga qaraganda 5 marta qimmat bo‘lganda eng kam xarajat qilish uchun tosh yo‘lni temir yo‘lga $\alpha=\arccos\frac{1}{5}\approx 78^0$ burchak ostida o‘tkazish kerak ekan.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. Berilgan S yuzga ega barcha to‘g‘ri to‘rtburchaklardan kvadrat eng kichik perimetrga ega ekanligi isbotlansin.

Ko‘rsatma. To‘g‘ri to’rtburchakning bir tomonini x desak, ikkinchi tomonini $y=\frac{S}{x}$, perimetri $P=2\cdot\left(x+\frac{S}{x}\right)$ bo‘ladi. J: $x=\sqrt{S}$; $y=\sqrt{S}$

2. P perimetrli barcha to‘g‘ri to’rtburchaklar orasidan kvadrat eng katta yuzga ega ekanligi isbotlansin.

Ko‘rsatma. To‘g‘ri to‘rtburchakning bir tomonini x, ikkinchi tomonini $y=\frac{P}{2}-x$, yuzi $S=x\cdot\left(\frac{p}{2}-x\right)$ bo‘ladi. J: $x=\frac{P}{4}$; $y=\frac{P}{4}$

3. Uchburchakning asosi a ga, perimetri esa P ga teng. Uchburchakning qolgan ikki tomoni shunday aniqlansinki, uning yuzi eng katta bo‘lsin.

Ko‘rsatma. Uchburchakning ikkinchi tomoni $b=x$ desak, uchinchi tomoni $c=P-a-x$ bo‘ladi. Geron formulasiga ko‘ra uchburchakning yuzi $S=\sqrt{P(P-a)(P-b)(P-c)}$ yoki, belgilashimizga binoan,

$$S=\sqrt{P(P-a)(P-x)(a+x)},\ (0<x<P)$$

bo‘ladi. S yuzning eng katta qiymatini topish kerak. Buning uchun $f(x)=(P-x)(a+x)$ funksiyaning eng katta qiymatini topish yetarli.

J: $b=c=\frac{P-a}{2}$, ya’ni uchburchak teng yonli bo‘lishi kerak.

4. Uchburchakning bir tomoni a ga va uning qarshisidagi burchagi α ga teng. Uchburchakning qolgan ikki burchagi shunday aniqlansinki, uning yuzi eng katta bo‘lsin. J: $x=\frac{1}{2}(\pi-\alpha)$; $\gamma=\frac{1}{2}(\pi-\alpha)$, uchburchak teng yonli bo‘ladi.

5. v hajmga ega yopiq silindrik idish(bak) tayyorlash talab etiladi. Idishni tayyorlashga eng kam material sarflanishi uchun uning o‘lchamlari qanday bo‘lishi kerak? J: $R=\sqrt[3]{\frac{v}{2\pi}},\ H=2R,\ H:R=2,$ bunda R silindr radiusi H esa balandligi.

6. S to‘la sirtga ega silindrning hajmi eng katta bo‘lishi uchun uning (radiusi R va balandligi H) o‘lchamlari qanday bo‘lishi kerak? J: $R=\sqrt{\frac{S}{6\pi}},\ H=2R,\ H:R=2.$

7. Berilgan hajmga ega to‘g‘ri doiraviy konusning yon sirti $r^2 : h^2 : \ell^2 = 1:2:3$ munosabat bajarilgandagina eng kichik bo‘lishi isbotlansin. Bu yerda r – konus asosining radiusi, h – konusning balandligi, ℓ – konusning yasovchisi.

J: $r = \sqrt[3]{\frac{3v}{\sqrt{2}\pi}},\ h = \sqrt[3]{\frac{6v}{\pi}},\ \ell = \sqrt{3}\cdot\sqrt[3]{\frac{3v}{\sqrt{2}\pi}}$; $r^2 : h^2 : \ell^2 = 1:2:3.$

8. O‘lchovlari $80\text{sm} \times 50\text{sm}$ bo‘lgan to‘g‘ri to‘rtburchakli tunuka berilgan. Tunukaning to‘rtta uchidan kattaligi bir xil kvadratlar kesib olinib, qolgan qismdan qopqoqsiz to‘g‘ri to’rtburchakli quti yasalgan. Qutining hajmi eng katta bo‘lishi uchun kesib tashlangan kvadratning tomoni qanday bo‘lishi kerak? J: 10 sm.

9. Berilgan doiraga eng katta yuzga ega to‘g‘ri to‘rtburchak ichki chizilsin.

J: tomoni $R\sqrt{2}$ bo‘lgan kvadrat, bunda R doiraning radiusi

10. Berilgan sharga yon sirti eng katta bo‘lgan tsilindr ichki chizilsin.

J: $r = \frac{\sqrt{2}}{2}R,\ h = \sqrt{2}R$, bunda r doiraninng radiusi h esa uning balandligi.

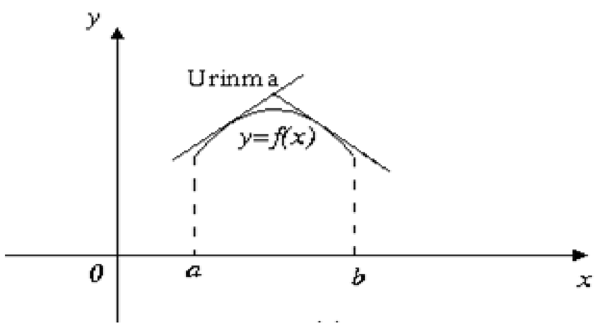
6.2.MAVZU: FUNKSIYA GRAFIGINING QAVARIQLIGI VA BOTIQLIGI. EGILISH NUQTA. EGRI CHIZIQ ASIMPTOTALARI

MAVZUGA OID NAZARIY MATERIALLAR

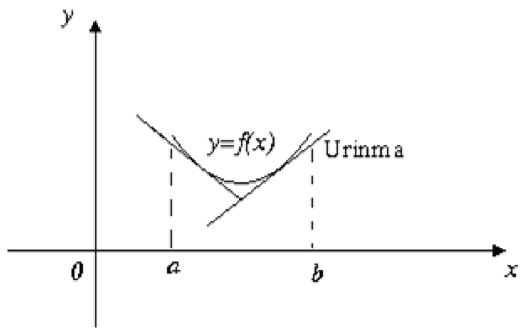
1. Funksiya grafigining qavariqligi va botiqligi. Egilish nuqta.

$(a;\ b)$ intervalda differensiallanuvchi $y=f(x)$ funksiyaning grafigini qaraymiz. $y=f(x)$ funksiya grafigining $(a;\ b)$ intervaldagi urinmasi deyilganda grafikning abssissasi $(a;\ b)$ intervalga tegishli istalgan nuqtalaridan grafikka o‘tkazilgan urinmalar tushuniladi.

1-Ta’rif. Agar funksiyaning grafigi uning ixtiyoriy nuqtasidan o’tkazilgan urinmadan pastda (yuqorida) joylashgan bo’lsa, $(a;\ b)$ intervalda differensiallanuvchi $y=f(x)$ funksiyaning grafigi shu $(a;\ b)$ intervalda ***qavariq*** (*botiq*) deyiladi (1,2-chizmalar).

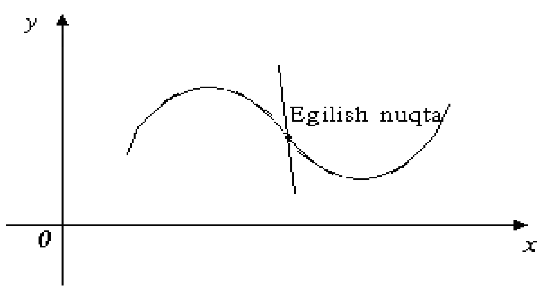


1-chizma



2-chizma

3-Ta’rif. Uzluksiz funksiya grafigining qavariq qismini botiq qismidan ajratuvchi nuqtasi grafikning ***egilish nuqtasi*** deyiladi. (3-chizma).



3-chizma

Funksiya grafigining botiqlik, qavariqlik intervallari hamda grafikning egilish nuqtalarini aniqlash funksiyaning ikkinchi tartibli hosilasidan foydalanib amalga oshiriladi.

1-Teorema. Agar $(a;\ b)$ intervalning barcha nuqtalarida $f''(x)$ mavjud va $f''(x)<0$ bo‘lsa, u holda $(a;\ b)$ intervalda $y=f(x)$ funksiyaning grafigi qavariq bo‘ladi.

2-Teorema. Agar $(a;\ b)$ intervalning barcha nuqtalarida $f''(x)$ mavjud va $f''(x)>0$ bo‘lsa, u holda $(a;\ b)$ intervalda $y=f(x)$ funksiyaning grafigi botiq bo‘ladi.

3- **Teorema. (**Burilish nuqtasi mavjud bo’lisining yetarli sharti **).** Agar funksiyaning ikkinchi tartibli hosilasi $f''(x_0)=0$ bo’lsa yoki mavjud bo’lmasa va x_0 nuqtadan o’tayotganda $f''(x)$ o’z ishorasini o’zgartirsa, $x=x_0$ absisali nuqta $y=f(x)$ egri chiziqning burilish nuqtasi bo’ladi.

Masalan. $y=x^3-9x^2+5x+43$ funksiya grafigining qavariqlik, botiqlik intervallarini hamda egilish nuqtalarini toping.

Yechish. Ikkinchi hosilani topamiz:

$y'=3x^2-18x+5$, $y''=6x-18$. Ikkinchi tartibli hosilani nolga tenglashtirib hosil bo‘lgan tenglamani yechamiz: $y''=0$, $6x-18=0$, $x=3$.

$x<3$ da $y''=6(x-3)<0$ bo‘lganligi sababli $(-\infty;\ 3)$ intervalda funksiyaning grafigi qavariq bo‘ladi.

$x>3$ da $y''=6(x-3)>0$ bo‘lganligi sababli $(3,+\infty)$ intervalda grafik botiq bo‘ladi. $y(3)=3^3-9\cdot 3^2+5\cdot 3+43=4$ ekanligini hisobga olsak $M_0(3;\ 4)$ nuqta grafikning egilish nuqtasi ekanligi kelib chiqadi.

Masalan. $y=\ell nx$ funksiya grafigining qavariqlik, botiqlik intervallarini hamda egilish nuqtalarini toping.

Yechish. $y=\ell nx$ funksiya $(0,+\infty)$ intervalda aniqlangan. Ikkinchi tartibli hosilani topamiz: $y'=(\ell nx)'=\frac{1}{x}$, $y''=\left(\frac{1}{x}\right)'=-\frac{1}{x^2}<0$.

Ikkinchi tartibli hosila $(0,+\infty)$ intervalda manfiy bo‘lganligi sababli $y=\ell nx$ funksiyaning grafigi bu intervalda qavariq bo‘ladi. Grafik egilish nuqtaga ega emas.

2. Egri chiziqning asimptotalari

4-Ta’rif. Agar $y=f(x)$ funksiya grafigining o‘zgaruvchi nuqtasi grafik bo‘ylab cheksiz uzoqlashganda undan biror to‘g‘ri chiziqqacha masofa nolga intilsa, bu to‘g‘ri chiziq $y=f(x)$ funksiya grafigining ***asimptotasi*** deb ataladi.

Boshqach aytganda , egri chiziq ***asimptotasi*** deb shunday to‘g’ri chiziqqa aytiladiki, egri chiziqda yotuvchi M nuqta egri chiziq bo‘ylab xarakat qilib koordinata boshidan cheksiz uzoqlashgani sari M nuqtaning bu to‘g’ri chiziqqacha bo‘lgan masofasi nolga intiladi.

Asimptotalar vertikal (ya’ni $0Y$ o‘qqa parallel) hamda og‘ma (ya’ni OY oqqa paralel bo‘lmagan) asimptotalarga ajratilib o‘rganiladi.

Agar $\lim_{x\to x_0+0} f(x)$ yoki $\lim_{x\to x_0-0} f(x)$ limitlardan hech bo’lmaganda bittasi cheksiz ($+\infty$ $yoki$ $-\infty$) bo’lsa, $x=x_0$ to’g’ri chiziqqa $y=f(x)$ funksiya grafigining ***vertical*** asimptotasi deyiladi.

Agar shunday k va b sonlari mavjud bo’lib, $x\to\infty$ $(x\to-\infty)$ da $f(x)$ funksiya $f(x)=kx+b+\alpha(x)$, $\lim_{x\to\infty}\alpha(x)=0$ ko’rinishda ifodalansa, y $=kx+b$ to’g’ri chiziqqa y= *f*(x) funksiya grafigining ***og‘ma*** asimptotasi deyiladi. Bu yerda $k=\ell im_{x\to\infty}\frac{f(x)}{x}$, $b=\ell im_{x\to\infty}(f(x)-kx)$.

Xususan, agar k=0 bo’lsa, y $=b$ to‘g‘ri chiziqqa *f*(x) funksiya grafigining gorizontal asimptotasi deyiladi.

Izoh. $y = f(x)$ funksiya grafigining asimptotalari $x \to +\infty$ da va $x \to -\infty$ xar xil bo'lishi mumkin. Shu sababli k va b ni aniqlashda $x \to +\infty$ va $x \to -\infty$ hollarini alohida qarash lozim.

a).Vertikal asimptotalar. Vertikal asimptotaning ta'rifidan, agar $\lim\limits_{x \to x_0 - 0} f(x) = \infty$ yoki $\lim\limits_{x \to x_0 + 0} f(x) = \infty$ yoki $\lim\limits_{x \to x_0} f(x) = \infty$ bo'lsa, u holda $x = x_0$ to'g'ri chiziq $y = f(x)$ egri chiziqning asimptotasi ekanligi kelib chiqadi; va aksincha, agar $x = x_0$ to'g'ri chiziq $y = f(x)$ egri chiziqning vertikal asimptotasi bo'lsa, u holda yozilgan tengliklardan biri albatta bajariladi.

Demak, $y = f(x)$ egri chiziqning vertikal asimptotalarini topish uchun argument x ning $f(x)$ funksiyani cheksizlikka aylantiradigan qiymatlarini topish kerak ekan

(4-chizma).

Masalan. 1) $y = x - \dfrac{2}{x+3}$ funksiya grafigining vertikal asimptotasini toping.

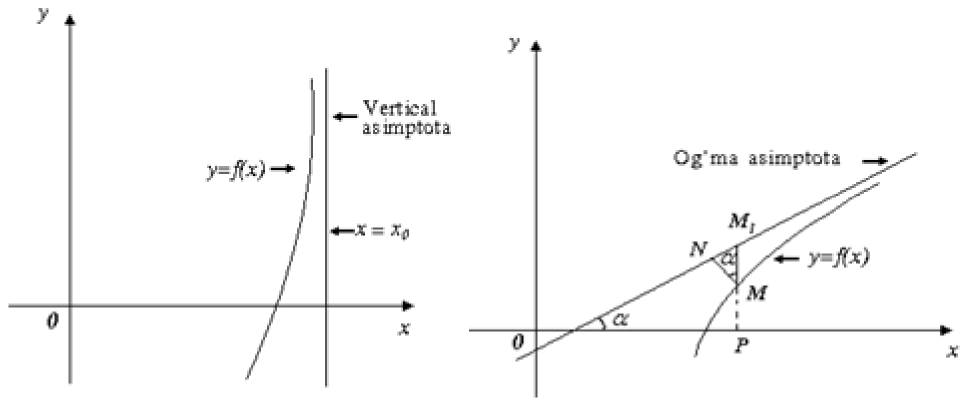
Yechish. $\lim\limits_{x \to -3}\left(x - \dfrac{2}{x+3}\right) = \infty$, shu sababli x=-3 to'g'ri chiziq grafikning vertikal asimptotasidir.

2) $y = ctgx$ funksiya grafigining vertikal asimptotasini toping.

Yechish. $\lim\limits_{x \to n\pi} ctgx = \infty$, bo'lganligi sababli funksiyaning grafigi cheksiz ko'p vertikal asimptotalarga ega:

$$x = 0,\ x = \pm\pi,\ x = \pm 2\pi,\ x = \pm 3\pi,\ \ldots.$$

b) Og'ma asimptotalar. $y = f(x)$ funksiyaning grafigi $0Y$ o'qqa parallel bo'lmagan asimptotalarga ega bo'lsin. U holda bu asimptotaning tenglamasi $y = kx + b$ ko'rinishga ega bo'lishi ravshan. Xususiy holda $k = 0$ bo'lganda $0X$ o'qqa parallel gorizontal asimptota hosil bo'ladi. k va b parametrlarni aniqlashga kirishamiz. Grafikning M nuqtasidan asimptotaga MN perpendikulyar o‘tkazamiz(4-chizma). Asimptotaning ta‘rifidan $\lim_{x\to+\infty} MN = 0$ ekani kelib chiqadi. ΔM_1MN dan $\frac{MN}{M_1M} = \cos\alpha$ yoki bundan $M_1M = \frac{MN}{\cos\alpha}$ hosil bo‘ladi. α $\left(\alpha \neq \frac{\pi}{2}\right)$ o‘zgarmasligini hisobga olsak $\lim_{x\to+\infty} M_1M = \frac{1}{\cos\alpha}\lim_{x\to+\infty} MN = 0$ bo‘ladi.



4-chizma

Shu sababli

$$M_1M = y_{asimptota} - y_{график} = (kx+b) - f(x) \text{ ва}$$

$$\lim_{x\to+\infty} M_1M = \lim_{x\to+\infty}(kx+b-f(x)) = 0$$

Bundan $\lim_{x\to+\infty} x\left(k + \frac{b}{x} - \frac{f(x)}{x}\right) = 0$.

Ma’lumki ikki ifodaning ko‘paytmasi nolga teng bo‘lishi uchun kamida ulardan biri nolga teng bo‘lishi lozim. Shuning uchun oxirgi tenglikda $x \to +\infty$, shu sababli $\lim_{x\to\infty}\left(k + \frac{b}{x} - \frac{f(x)}{x}\right) = 0$ bo‘ladi. $\lim_{x\to\infty}\frac{b}{x} = 0$ ekanini hisobga olsak $\lim_{x\to\infty}\left(k - \frac{f(x)}{x}\right) = 0$ yoki bundan $k = \lim_{x\to\infty}\frac{f(x)}{x}$ hosil bo‘ladi, tenglikdan $b = \lim_{x\to\infty}(f(x) - kx)$ ga ega bo‘lamiz. Bunga k ning topilgan qiymatini qoysak b topiladi.

Masalan. 1) $y = \frac{x^2+1}{x+1}$ funksiya grafigining asimptotalarini toping.

Yechish. $\lim\limits_{x\to -1}\frac{x^2+1}{x+1}=\infty$, bo‘lganligi sababli *x=-1* to‘g‘ri chiziq grafikning vertikal asimptotasidir. $y=kx+b$ og‘ma asimptotani izlaymiz.

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2+1}{x(x+1)}=\lim_{x\to\infty}\frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(1+\frac{1}{x}\right)}=\lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{1+\frac{1}{x}}=\frac{1+0}{1+0}=1,$$

$$b=\lim_{x\to\infty}(f(x)-kx)=\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-1\cdot x\right)=\lim_{x\to\infty}\frac{x^2+1-x^2-x}{x+1}=\lim_{x\to\infty}\frac{1-x}{x+1}=\lim_{x\to\infty}\frac{\frac{1}{x}-1}{1+\frac{1}{x}}=\frac{0-1}{1+0}=-1.$$

Demak, $y=x-1$ to‘g‘ri chiziq grafikning og‘ma asimptotasidir.

2) $y=e^{-x}+x$ funksiya grafigining asimptotalarini toping.

Yechish. Vertikal asimptota mavjud emas, chunki funksiya butun son o‘qida aniqlangan.

Og‘ma asimptotalarni izlaymiz:

a) $k=\lim\limits_{x\to+\infty}\frac{f(x)}{x}=\lim\limits_{x\to+\infty}\frac{e^{-x}+x}{x}=\lim\limits_{x\to+\infty}\left(\frac{1}{e^x x}+1\right)=1,$

$$b=\lim_{x\to+\infty}(f(x)-k\cdot x)=\lim_{x\to+\infty}\left(e^{-x}+x-1\cdot x\right)=\lim_{x\to+\infty}\frac{1}{e^x}=0.$$

Demak, $y=x$ to‘g‘ri chiziq grafikning $x\to+\infty$ dagi og‘ma asimptotasidir.

b) $k=\lim\limits_{x\to-\infty}\frac{f(x)}{x}=\lim\limits_{x\to-\infty}\left(\frac{e^{-x}}{x}+1\right)=\lim\limits_{x\to-\infty}\frac{e^{-x}}{x}+1.$

$x\to-\infty$ da $\frac{e^{-x}}{x}$ nisbat $\frac{\infty}{\infty}$ ko‘rinishdagi aniqmaslik. Unga Lopital qoidasini qo‘llasak $k=\lim\limits_{x\to-\infty}\frac{\left(e^{-x}\right)'}{x'}+1=\lim\limits_{x\to-\infty}\frac{-e^{-x}}{1}+1=\infty$ bo‘ladi. Demak, $x\to-\infty$ da grafik og‘ma asimptotaga ega emas.

6.3.-MAVZU: FUNKSIYANI HOSILA YORDAMIDA TO‘LA TEKSHIRISH VA GRAFIGINI CHIZISH.

MAVZUGA OID NAZARIY MATERIALLAR

Funksiyani hosila yordamida to‘la tekshirish deyilganda quyidagilar nazarda tutiladi.

1. Funksiyaning aniqlanish sohasi, juft yoki toqligi, davriyligi tekshiriladi.

2. Funksiyaning uzilish nuqtalari, uning grafigining koordinata o‘qlari bilan kesishish nuqtalari aniqlanadi.

3. Funksiyaning monotonligi va ekstremumlari tekshiriladi.

a) y' birinchi tartibli hosilasi topiladi;

b) birinch tartibli hosilani nolga tenglab, mavjud bo‘lmagan va kritik nuqtalar aniqlanadi;

v) har bir kritik nuqtadan chap va o‘ng tomonda $f'(x)$ ning ishorasi aniqlanadi; funksiya x_1,x_2 va x_3 kritik nuqtalarga ega bo‘lsa:

1) agar x_1 kritik nuqtaning chap tomonida hosilaning ishorasi musbat, o‘ng tomonida manfiy bo‘lsa, bu nuqtada $f(x)$ funksiya lokal maksimumga ega;

2) agar x_2 kritik nuqtaning chap tomonida hosilaning ishorasi manfiy, o‘ng tomonida musbat bo‘lsa, bu nuqtada $f(x)$ funksiya lokal minimumga ega;

3) Agar x_3 kritik nuqtaning chap va o‘ng tomonida hosilaning ishorasi bir xil bo‘lsa, bu nuqtada funksiya ekstremumga erishmaydi.

4. Qavariq va botiqlilik intervallari, burilish nuqtasi aniqlanadi;

a) birinch va ikkinchi tartibli hosila olinadi;

b) ikkinchi tartibli hosilani nolga tenglab ildizlarni topamiz. Shu nuqtalar atrofida ikkinch tartibli hosila ishorasining o‘zgarish qonunini aniqlaymiz:

1) intervallarda $y''>0$ ishora musbat bo‘lsa egri chiziq- botiq;

2)) intervallarda $y''<0$ ishora manfiy bo‘lsa egri chiziq -qavariq;

5. Funksiyaning asimptotalari topiladi;

a) funksiya ikkinchi tur uzilishga ega bo‘lsa; $\lim\limits_{x\to a} f(x)=\infty \Rightarrow x=a$

b) k= $\lim\limits_{x\to\pm\infty}\frac{f(x)}{x}$, b= $\lim\limits_{x\to\pm\infty}(f(x)-kx)$ limitlar mavjud va chekli bo‘lsa, u holda $y=kx+b$ to‘g‘ri chiziq egri chiziqning og‘ma asimptotasi bo‘ladi;

v) $y=kx+b$ og‘ma asimptota tenglamasini aniqlashda $k=0$ bo‘lsa, u holda $y=b$ to‘g‘ri chiziq gorizontal asimptota bo‘ladi;

6. Bu ma’lumotlar grafikni chizish uchun kamlik qilsa, qo‘shimcha zarur bo‘lgan hisoblashlarni bajarish kerak;

7. Yuqoridagi ma’lumotlarga ko‘ra funksiya grafigi yasaladi.

Eslatma. Grafikni yasashda uning koordinata o‘qlari bilan kesishish nuqtalarini topish foydalidir.

Eslatma. Grafikni yasash oldidan funksiyaning juft yoki toqligini aniqlash foydalidir.

Masalan. $y=e^{-x^2}$ funksiyaning grafigi chizilsin.

Yechish. 1. Funksiya $(-\infty;+\infty)$ intervalda aniqlangan.

2. Funksiya butun son o‘qida uzluksiz.

3. $y'=\left(e^{-x^2}\right)'=e^{-x^2}\cdot\left(-x^2\right)'=-2x\cdot e^{-x^2}$.

$e^{-x^2}>0$ bo‘lganligi sababli $x<0$ da $y'>0$ va $x>0$ da $y'<0$ bo‘ladi. Demak funksiya $(-\infty;0)$ intervalda o‘sadi, $(0;+\infty)$ intervalda esa kamayadi.

4. Funksiyaning hosilasini nolga tenglashtirib, hosil bo‘lgan tenglamani yechib funksiyaning kritik nuqtalarini aniqlaymiz:

$y' = -2x \cdot e^{-x^2} = 0$. Demak, $x = 0$ kritik nuqta.

Bu kritik nuqtaning chapidan o’ngiga o‘tganda hosila ishorasini plyusdan minusga o‘zgartirganligi uchun $x = 0$ nuqtada funksiya maksimumga ega. $y_{\max} = y(0) = e^0 = 1$.

5. Ikkinchi tartibli hosilani topamiz:

$$y'' = \left(-2x \cdot e^{-x^2}\right)' = -2 \cdot e^{-x^2} - 2x \cdot e^{-x^2}\left(-x^2\right)' = -2 \cdot e^{-x^2} + 4x^2 \cdot e^{-x^2} = 2\left(2x^2 - 1\right) \cdot e^{-x^2}.$$

Buni nolga tenglashtirib yechsak grafikning egilish nuqtalarining abssissalari hosil bo‘ladi.

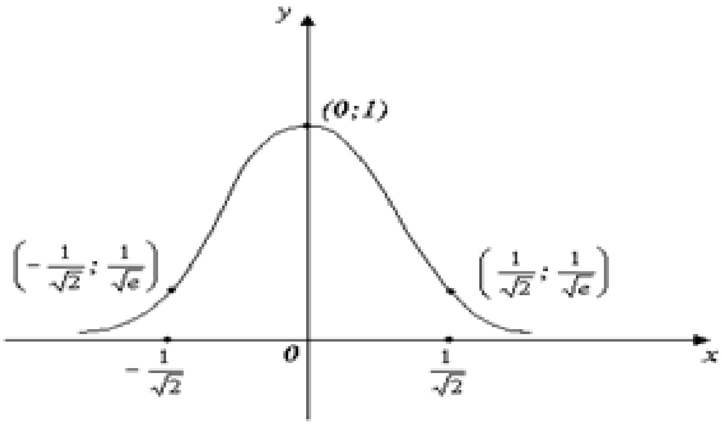
$e^{-x^2} \neq 0$ bo‘lganligi uchun $2\left(2x^2 - 1\right) \cdot e^{-x^2} = 0$ tenglamadan $2x^2 - 1 = 0$, $x^2 = \frac{1}{2}$, $x = \pm\frac{1}{\sqrt{2}}$ ga ega bo‘lamiz. Demak grafikning $x_1 = -\frac{1}{\sqrt{2}}$ va $x_2 = \frac{1}{\sqrt{2}}$ abssissali $\left(-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{e}}\right)$ va $\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{e}}\right)$ nuqtalari uning egilish nuqtalaridir. $\left(-\infty; -\frac{1}{\sqrt{2}}\right)$ va $\left(\frac{1}{\sqrt{2}}; +\infty\right)$ oraliqlarda $y'' > 0$ bo‘lgani uchun grafik bu oraliqlarda botiq, $\left(-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$ oraliqda $y'' < 0$ bo‘lgani uchun bu oraliqda grafik qavariq.

6. Funksiya x ning barcha qiymatlarida aniqlanganligi uchun uning grafigi vertikal asimptotalarga ega emas. Grafikning og‘ma asimptotalarini aniqlaymiz.

$$k = \lim_{x\to\infty} \frac{y}{x} = \lim_{x\to\infty} \frac{e^{-x^2}}{x} = \lim_{x\to\infty} \frac{1}{xe^{x^2}} = 0, \quad b = \lim_{x\to\infty}(y - k \cdot x) = \lim_{x\to\infty}\left(e^{-x^2} - 0 \cdot x\right) = \lim_{x\to\infty} \frac{1}{e^{x^2}} = 0.$$

Demak, $y = 0$, ya’ni $0x$ o‘q grafikning asimptotasidir.

$y = e^{-x^2}$ funksiyaning grafigi ***Gauss egri chizig‘i*** deb ataladi. U 5-chizmada tasvirlangan.



5-chizma

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

Quyidagi egri chiziqlarning egilish nuqtalari hamda botiqlik va qavariqlik intervallari aniqlansin.

1. $y = x^7$. J: $(-\infty;\ 0)$ intervalda grafik qavariq, $(0;\ +\infty)$ intervalda grafik botiq, $(0;\ 0)$ grafikning egilish nuqtasi.

2. $y = 4 - x^2$. J: $(-\infty;\ +\infty)$ intervalda grafik qavariq.

3. $y = x^3 - 3x^2 - 9x + 9$. Javob: $(-\infty;\ 1)$ intervalda grafik qavariq, $(1;\ +\infty)$ intervalda grafik botiq, $(1;\ -2)$ grafikning egilish nuqtasi.

4. $y = x^4$. J: Grafik hamma yerda botiq.

5. $y = tgx$. Javob: $(n\pi;\ 0)$ grafikning egilish nuqtalari.

Quyidagi egri chiziqlarning asimptotalari topilsin.

6. $y = \dfrac{3}{x-2}$. J: $x = 2,\ \ y = 0$.

7. $y = e^{\frac{1}{x}} - 1$. J: $x = 0,\ \ y = 0$.

8. $y = \ell nx$. J: $x = 0$.

9. $y^3 = 6x^2 + x^3$. J: $y = x + 2$.

10. $y = c + \frac{a^3}{(x-b)^2}$. J: $x = b,\ y = c$.

MUSTAQIL YECHISH UCHUN MASHQLAR.

Funksiyani birinchi va ikkinch tartibli hosila yordamida ekstremumlarini tekshiring.

1.1. $y = 3\left(\frac{x^4}{2} - x^2\right)$

1.2. $y = x^3 - 9x^2 + 24x - 15$

1.3 $y = x^5 - \frac{5}{3}x^3$

1.4. $y = 2x^3 + 3x^2 - 12x - 5$

1.5. $y = (x-3)^2(x-2)$

1.6. $y = x^4 - 8x^3 + 16x^2$

1.7. $y = x^2 + \frac{1}{3}x^3 - \frac{x^4}{4}$

1.8. $y = \frac{1}{10}(2x^3 - 6x^2 - 18x + 15)$

1.9. $y = x^5 - x^3 - 2x$

1.10. $y = 1 - x^2 - \frac{x^4}{8}$

1.11. $y = -4x + x^3$

1.12. $y = (x+1)(x-2)^2$

1.13. $y = x^3 - 3x^2 + 4$

1.14. $y = x^3 - 9x^2 + 24x - 7$

1.15. $y = x^4 - 8x^2 + 16$

1.16. $y = x^3 - \frac{1}{2}x^2 - 4x + 2$

1.17. $y = -4x^3 + 6x^2 - 3x - \frac{1}{2}$

1.18. $y = \frac{1}{10}(x^4 - 12x)$

1.19. $y = x^4 - 2x^2 + 3$

1.20. $y = (x+2)(x-1)^2$

1.21. $y = x^3 - 3x^2 + 2$ 1.22. $y = 8 + 2x^2 - 4x^4$

1.23. $y = \frac{1}{5}x^5 - 4x^2$ 1.24. $y = 2x^3 - 15x^2 + 36x$

Berilgan *y=f(x)* funksiyaning *[a;b]* kesmadagi eng kata va eng kichik qiymatlarini toping.

1.25. $y = \frac{x^3}{x^2 - x + 1}; \quad [-1;1]$ **1.26.** $y = \frac{(x+1)^3}{x^3}; \quad [1;2]$

1.27. $y = \sqrt{x - x^3}; \quad [-2;2]$ 1.28. $y = 4 - e^{-x^2}; \quad [0;1]$

1.29. $y = \frac{x^3 + 4}{x^2}; \quad [1;2]$ 1.30. $y = xe^x; \quad [-2;0]$

1.31. $y = (x - x^2)e^x; \quad [-2;1]$ 1.32. $y = (x-1)e^{-x}; \quad [0;3]$

1.33. $y = \frac{x}{9 - x^2}; \quad [-2;2]$ 1.34. $y = \frac{1 + \ln x}{x}; \quad \left[\frac{1}{e};e\right]$

1.35. $y = e^{4x - x^2}; \quad [1;3]$ 1.36. $y = \frac{x^5 - 8}{x^4}; \quad [-3;1]$

1.37. $y = \frac{e^{2x} + 1}{e^x}; \quad [-1;2]$ 1.38. $y = x\ln x; \quad \left[\frac{1}{e^2};1\right]$

1.39. $y = x^3 e^{x+1}; \quad [-4;0]$ 1.40. $y = x^2 - 2x + \frac{2}{x-1}; \quad [-1;3]$

1.41. $y = (x+1)\sqrt{x}; \quad \left[-\frac{4}{3};3\right]$ 1.42. $y = e^{6x - x^2}; \quad [-3;3]$

1.43. $y = \frac{\ln x}{x}; \quad [1;4]$ 1.44. $y = 3x^4 - 16x^3 + 2; \quad [-3;1]$

1.45. $y = x^5 - 5x^4 + 5x32 + 1; \quad [-1;2]$ 1.46. $y = (3 - x)e^{-x}; \quad [0;5]$

1.47. $y=\frac{\sqrt{3}}{2}+\cos x;\quad \left[0;\frac{\pi}{2}\right]$

1.48. $y=108x-x^4;\quad [-1;4]$

1.49. $y=\frac{x^4}{49}-6x^3+7;\quad [16;20]$

1.50. $y=x^3+3x^2-12x+1;\quad [-1;5]$

Quyidagi funksiyalarni qavariq, botiq va egilish nuqtalarini toping.

2.1. $y=x^4-2x+10$.

2.2. $y=\frac{x}{1+x^2}$

2.3. $y=\frac{x^2}{1+x^4}$

2.4. $y=\frac{x}{1+x}$

2.5. $y=1+3x^2-x^4$

2.6. $y=1+3x^3-x^5$

2.7. $y=\frac{1}{2+x}$

2.8. $y=(x-1)^3$

2.9. $y=2+x^2-\frac{x^4}{2}$

2.10. $y=5-\frac{2}{x}-x^2$

2.11. $y=\frac{x^4}{(1+x)^3}$

2.12. $y=\frac{3}{2+x}-\frac{3}{x-2}-1$

2.13. $y=\frac{x^2}{1-x^2}$

2.14. $y=x^3-3x+2$

2.15. $f(x)=\frac{x+4}{x^2-9}$

2.16. $f(x)=\frac{2x^2-5}{x^2+x-6}$

2.17. $f(x)=\sqrt[3]{2x-1}$

2.18. $y=\frac{(x-1)^2}{x^2+1};$

2.19. $y=\frac{x^2+1}{x^2};$

2.20. $y=\frac{x}{3-x^2};$

2.21. $y=\frac{x-1}{x^2-2x};$

2.22. $y=\frac{x}{(x-1)^2};$

2.23. $y = 2 + \frac{12}{x^2 - 4}$.

2.24. $y = \frac{x^2 + 1}{x}$;

2.25. $y = x + \frac{4}{x + 2}$.

Funksiyalarni hosila yordamida to‘liq tekshiring va grafigini yasang.

3.1 $y = \frac{x - 1}{x^2 - 2x}$;

3.2. $y = \frac{2 - 4x^2}{1 - 4x^2}$;

3.3. $y = \frac{2x^2}{4x^2 - 1}$;

3.4. $y = \frac{2x + 1}{x^2}$;

3.5. $y = \frac{1}{x^2 - 9}$;

3.6. $y = \frac{4x^2}{x^2 - 1}$;

3.7. $y = \frac{x^4}{x^3 - 1}$;

3.8. $y = \frac{x^2 - x - 1}{x^2 - 2x}$;

3.9. $y = \frac{(x - 3)^2}{(x - 1)}$;

3.10. $y = \frac{x^2 + 4x + 1}{x^2}$;

3.11. $y = \frac{x^2 + 16}{4x}$;

3.12. $y = \frac{3x}{x^2 + 1}$;

3.13. $y = \frac{3 - x^2}{x + 2}$;

3.14. $y = \frac{5x^2}{x^2 - 25}$;

3.15. $y = \frac{2x - 1}{(x - 1)^2}$;

3.16. $y = \frac{x^3}{2(x + 1)^2}$;

3.17. $y = \frac{1}{1 - x^2}$;

3.18. $y = \frac{2}{x^2 + x + 1}$;

3.19. $y = \frac{x^3 - 1}{4x^2}$;

3.20. $y = \left(\frac{x + 2}{x - 1}\right)^2$;

3.21. $y = \frac{x^3 + 16}{x^2 - 2x}$;

3.22. $y = \frac{4x}{(x + 1)^2}$;

3.23. $y=\frac{x^2-3x+3}{x-1}$;

3.24. $y=\frac{4}{x^2+2x-3}$;

3.25. $y=x-\ell n(x+1)$.

3.26. $y=\frac{6x}{1+x^2}$.

3.27 . $y=\frac{x^2-x-6}{2-x}$

3.28. $y=x^2+6x+4$

3.29. $y=x^3e^x$

3.30. $y=\frac{e^x}{1+x}$

3.31. $y=\ln(x^2-2x+2)$

3.32. $y=x+arctgx$

3.33. $y=2x-\arcsin x$

3.34. $y=\frac{x}{16+x^2}$

3.35. $f(x)=\frac{1}{\sqrt[3]{x^2+5x}}$

3.36. $f(x)=\frac{x+1}{1+\frac{1}{x+1}}$

7-BOB. KO‘P O‘ZGARUVCHILI FUNKSIYA.

7.1-Mavzu: IKKI O‘ZGARUVCHILI FUNKSIYA TUSHUNCHASI.

MAVZUGA OID NAZARIY MATERIALLAR

1. Funksiyaning aniqlanish va o‘zgarish sohasi.

1-Ta’rif. Agar bir-biriga bog‘liq bo‘lmagan ikki o‘zgaruvchi x va y ning biror D o‘zgarish sohasidagi har bir (x,y) qiymatlari juftiga E to‘plamdagi z o‘zgaruvchining aniq bir qiymati mos kelsa, u holda D sohada ***ikki o‘zgaruvchining funksiyasi*** z aniqlangan deyiladi.

x va y ***erkli o‘zgaruvchilar*** yoki ***argumentlar***, z esa ***erksiz o‘zgaruvchi*** yoki ***funksiya*** deb ataladi.

Ikki o‘zgaruvchining funksiyasi

$z=z(x,y)$, $z=f(x,y)$, $z=F(x,y)$ va hokazo ko‘rinishda belgilanadi.

D to‘plam funksiyaning ***aniqlanish sohasi*** deyiladi. z o‘zgaruvchini qiymatlari to‘plami E funksiyaning ***o‘zgarish sohasi*** (qiymatlar to‘plami) deyiladi. $z=f(x,y)$ fnuktsiyaning argumentlarning $x=x_0$, $y=y_0$ tayin (aniq) qiymatlariga mos z_0 xususiy qiymati $z_0=z\big|_{\substack{x=x_0\\ y=y_0}}$ yoki $z_0=f(x_0, y_0)$ kabi yoziladi. Masalan, $x=-1$, $y=2$ da $z=x^3+y^2$ funksiyaning qiymati $z\big|_{\substack{x=-1\\ y=2}}=(-1)^3+2^2=3$ bo‘ladi.

Bir o‘zgaruvchili funksiya kabi ikki o‘zgaruvchili funksiya ham, umuman aytganda argumentlar x va y ning barcha qiymatlarida mavjud bo‘lavermaydi.

Masalan, $z=\sqrt{1-x^2-y^2}$ funksiya faqatgina x va y ning $1-x^2-y^2\geq 0$ tengsizlikni qanoatlantiradigan qiymatlaridagina mavjud.

2-Ta’rif. $z=f(x,y)$ funksiya aniqlangan x va y qiymatlarining (x,y) juftligidan iborat to‘plami funksiyaning ***aniqlanish sohasi*** yoki ***mavjudlik sohasi*** deb ataladi.

z=F(x,y) tenglama ***geometrik*** nuqtai nazardan qandaydir sirtni aniqlaydi, *x* va *y* ning qiymatlari jufti *xOy* tekslikda *P(x,y)* nuqtani aniqlaydi, *z=F(x,y)* esa sirtdagi unga mos *M(x,y,z)* nuqtaning aplikatasini aniqlaydi. Shu sababli *z* o'zgaruvchi *P(x,y)* nuqtaning funksiyasi deyiladi va *z=F(P)* deb yoziladi.

1-Misol. $z=x^2+y^2$ funksiyaning aniqlanish va o'zgarish sohasi topilsin.

Yechish. Funksiya *x* va *y* ning barcha qiymatlarida aniqlangan, ya'ni butun *0XY* tekislik funksiyaning aniqlanish sohasi. Funksiyaning qiymatlar sohasi $E=[0;\infty)$.

2-Misol. $z=\sqrt{4-x^2-y^2}$ funksiyaning aniqlanish va o'zgarish sohalari topilsin.

Yechish. Bu funksiyaning aniqlanish sohasi $\sqrt{4-x^2-y^2}$ ifoda aniqlangan (ma'noga ega) nuqtalar to'plamidan ya'ni *x* va *y* ning $4-x^2-y^2\geq 0$ yoki $4\leq x^2+y^2$ tengsizlikni qanoatlantiruvchi qiymatlaridan iborat. $x^2+y^2=4$ markazi koordinatalar boshida bo'lgan radiusi *2* ga teng aylana tenglamasi ekaligini hisobga olsak $x^2+y^2\leq 4$ shu aylana bilan chegaralangan sohani, ya'ni doirani ifodalaydi. Aylananing niqtalari ham funksiyaning aniqlanish sohasiga tegishli. Bu yerda kvadrat ildiz ildiz ostidagi ifoda nomanfiy bo'lganda ma'noga ega ekanligi hisobga olindi. Funksiyaning qiymatlar sohasi *E=[0,2]* kesmadan iborat.

3-Misol. $z=\dfrac{1}{\sqrt{x^2+y^2-1}}$ funksiyaning aniqlanish va o'zgarish sohalari topilsin.

Yechish. Funksiyaning aniqlanish sohasi $\dfrac{1}{\sqrt{x^2+y^2-1}}$ ifoda aniqlangan (ma'noga ega) nuqtalar to'plami, ya'ni $x^2+y^2-1>0$ yoki $x^2+y^2>1$ bajariladigan nuqtalar to'plami bo'ladi. Bu to'plamga *0XY* tekislikning markazi koordinatalar boshida bo'lib, radiusi 1 ga teng aylanadan tashqarida yotgan barcha nuqtalari tegishli. Bu yerda kasrning maxraji noldan farqli bo‘lganda u ma’noga ega ekanligi hisobga olindi.

Bir o‘zgaruvchili funksiya kabi ikki o‘zgaruvchi funksiya ham bir necha usullar bilan berilishi mumkin. Biz asosan analitik usulda berilgan funksiyani qaraymiz.

Dekartning fazodagi koordinatalar sistemasi $0XYZ$ ni qaraganda fazoning ixtiyoriy P nuqtasiga shu nuqtaning koordinatalari- haqiqiy sonlarning tartiblangan uchligi (x,y,z) mos kelishini va istalgan tartiblangan haqiqiy sonlarning uchligi (x,y,z) ga $0XYZ$ fazoning $P\ (x,y,z)$ nuqtasi mos kelishini ko‘rgan edik. Shuning uchun ikki o‘zgaruvchining funksiyasi $z=f(P)$ da $0XY$ tekisligining $P(x,y)$ nuqtalari to‘plami o‘rnida $0XYZ$ fazoning biror $P(x,y,z)$ nuqtalari to‘plami qaralsa uch o‘zgaruvchining funksiyasi $u=f(P)$ yoki $u=f(x,y,z)$ hosil bo‘ladi. Uch o‘zgaruvchi funksiyasining aniqlanish sohasi butun uch o‘lchovli $0XYZ$ fazodan yoki uning biror qismidan iborat bo‘ladi.

Masalan, $u=x^2+y^2+z^2$ funksiya butun $0XYZ$ fazoda aniqlangan. $u=\ell n(4-x^2-y^2-z^2)$ funksiya esa fazoning markazi koordinatalar boshida bo‘lib radiusi 2 ga teng sfera bilan chegaralangan (shar) qismida aniqlangan. Sferaning nuqtalari funksiyaning aniqlanish sohasiga tegishli emas.

Yuqoridagi kabi mulohaza yuritib to‘rt o‘zgaruvchining funksiyasi

$u=f(x,y,z,t)$ ga kelamiz. Bu holda haqiqiy sonlarning tartiblangan to‘rtligi (x,y,z,t) to‘rt o‘lchovli fazo deb ataluvchi fazo nuqtasining koordintalarini tashkil etadi. To‘rt o‘zgaruvchi funksiyasining aniqlanish sohasi butun to‘rt o‘lchovli fazodan yoki uning biror qismidan iborat bo‘ladi.

Shunga o‘xshash besh va undan ortiq o‘zgaruvchilarning funksiyasi tushunchasiga kelamiz. Masalan, $u=f(x_1, x_2, \ldots x_n)$-$n$ o‘zgaruvchining funksiyasi.

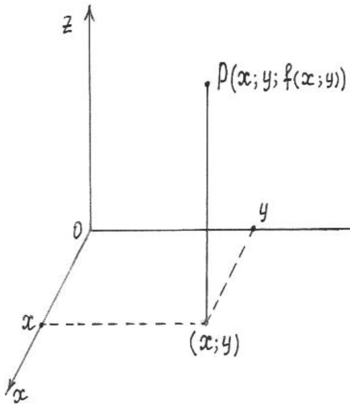
To‘rt va undan ortiq o‘zgaruvchilarning funksiyalarini aniqlanish sohalarini geometrik tasvirlab bo‘lmaydi.

2. Ko‘p o‘zgaruvchili funksiyaning geometrik tasviri.

0XY tekislikdagi *D* sohada aniqlangan

$$z=f(x,y) \qquad (1)$$

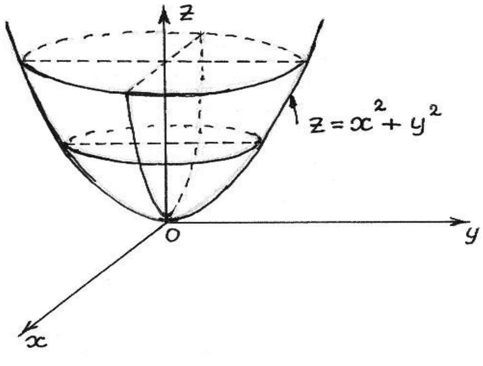
funksiyani va *0XZ* to‘g‘ri burchakli Dekart koordinatalari sistemasini qaraymiz.



1-chizma.

(1-chizma), bunda *D* soha butun *0XY* tekislikdan iborat bo‘lishi ham mumkin. *D* sohaning har bir *(x,y)* nuqtasidan *0XY* tekislikka perpendikulyar to‘g‘ri chiziq o‘tkazamiz va unda *f(x,y)* ga teng kesma ajratamiz. U holda fazoda koordinatalari *x,y*, *z=f(x,y)* bo‘lgan *P* nuqtani hosil qilamiz.

Shuningdek, $x^2+y^2+z^2=R^2$ tenglama markazi koordinatalar boshida bo‘lgan radiusi *R* ga teng sferani tenglamasi edi. Demak sfera $z=\sqrt{R^2-x^2-y^2}$ va $z=-\sqrt{R^2-x^2-y^2}$ funksiyalarni grafiklarini birlashmasidan iborat, ya’ni sferaning *0XY* tekisligidan pastda yotgan yarmi $z=-\sqrt{R^2-x^2-y^2}$ funksiyaning grafigini, sferaning *0XY* tekislikdan yuqorida yotgan yarmi $z=\sqrt{R^2-x^2-y^2}$ funksiyaning grafigini ifodalaydi.



2-chizma.

Umuman olganda ikki o‘zgaruvchili funksiyaning grafigini chizish unchalik oson ish emas. Uni ikkinchi tartibli sirtlarni chizishda foydalanilgan parallel kesimlar usulidan foydalanib, ya’ni sirtni koordinatalar tekisligiga parallel tekisliklar bilan kesilganda kesimda hosil bo‘lgan chiziqqa qarab sirt haqida biror to‘xtamga kelish mumkin.

Masalan, $z=x^2+y^2$ funksiyaning grafigini chizish uchun sirtni $0XY$ tekislikka parallel $z=h$ $(h>0)$ tekislik bilan kesganda kesimda aylana va $0YZ$, $0XZ$ tekisliklariga parallel tekisliklar bilan kesganda kesimda parabola hosil bo‘lishiga qarab $z=x^2+y^2$ funksiyaning grafigi 2–chizmada tasvirlangan aylanish paraboloiddan iborat ekanligi chiqadi.

3. Ko‘p o‘zgaruvchili funksiyaning limiti

Funksiyaning limiti tushunchasini qarashdan oldin, berilgan nuqtaning ***δ-atrofi*** tushunchasini kiritamiz. `Bir o‘zgaruvchili funksiya qaralganda $x=x_0$ nuqtaning δ -atrofi deganda $(x_o-\delta, x_o+\delta)$ interval tushunilar edi.

3-Ta’rif. $0XY$ tekislikning koordinatalari

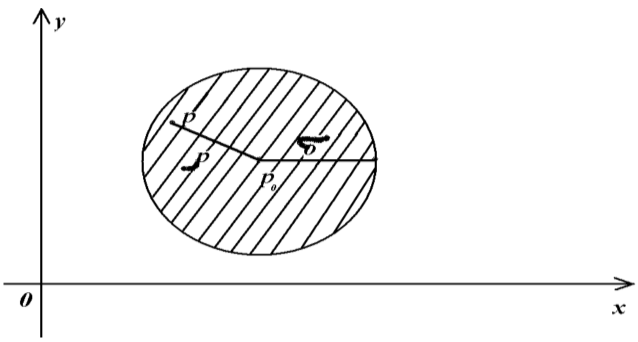
$$\rho(P;P_o)=\sqrt{(x-x_o)^2+(y-y_o)^2}<\delta$$

tengsizlikni qanoatlantiruvchi $P(x,y)$ nuqtalari to‘plami $P_o(x_o,y_o)$ nuqtaning

δ ***-atrofi deyiladi***.

Boshqacha aytganda P_o nuqtaning δ -atrofi bu markazi P_o nuqtada bo‘lgan

δ radiusli doiraning ichki nuqtalaridir (3-chizma).



3-chizma.

Fazodagi $P_o(x_o,y_o,z_o)$ nuqtaning δ -atrofi, markazi P_o nuqtada bo‘lib radiusi δ ga teng

$$\rho(P;P_o)=\sqrt{(x-x_o)^2+(y-y_o)^2+(z-z_o)^2}<\delta$$

sharning $p(x,y,z)$ nuqtalaridan iborat bo‘ladi. Sharning sirti-sferaning nuqtalari nuqtaning atrofiga tegishli bo‘lmaydi. Bunda C(P,P_o) orqali P_o va P nuqtalar orasidagi masofa belgilangan.

n o‘lchovli ($n>3$ da) fazoda $P_o(x_{10}, x_{20},... x_{no},)$ nuqtaning δ -atrofi ham shunga o‘xshash ta’riflanadi.

Faraz qilaylik $z=f(x,y)$ funksiya $P_o(x_0, y_0)$ nuqtaning biror atrofida aniqlangan bo‘lsin (P_o nuqtaning o‘zida aniqlanmagan bo‘lishi ham mumkin).

Funksiyaning limiti. Agar xarakatdagi P nuqta xar qanday usul bilan P_0 nuqtaga yaqinlashganda , ya’ni $P=P_0$, P nolga intilganda ($P=P_0$, $P\to 0$) $F(P)$ - A ayirma cheksiz kichik bo’lsa, $\lim_{p\to P_0} F(P)=A$ deyiladi.

1-Misol. $z=\frac{x-y}{x+y}$ funksiyaning $0(o;o)$ nuqtada limitga ega emasligi ko‘rsatilsin.

Yechish**.** Bu funksiya $x+y=o$ to‘g‘ri chiziq nuqtalaridan tashqari $0XY$ tekisligining barcha nuqtalarida aniqlangan. Uning $(0,0)$nuqtada limitga ega emasligini ko‘rsatish uchun $P(x,y)$nuqta $P_o(0,0)$ nuqtaga ikki xil yo‘nalish bo‘yicha intilgan xollarni kuzatamiz. P nuqta $0y$ o‘q bo‘ylab $P_o(0,0)$ nuqtaga yaqinlashsa $x=0$ bo‘lganligi sababli

$$\lim_{p\to p_o}\frac{x-y}{x+y}=\lim_{y\to 0}\frac{-y}{y}=-1$$

bo‘ladi. P nuqta $0x$ oq bo‘ylab $P_o(0,0)$ nuqtaga intilganda $y=0$ bo‘lganligi sababli

$$\lim_{p\to p_o}\frac{x-y}{x+y}=\lim_{x\to x_o}\frac{x}{x}=1$$

bo‘ladi. Shunday qilib $P(x,y)$ nuqta $P_o(0,0)$nuqtaga ikki xil yo‘nalish bo‘yicha yaqinlashganda funksiya ikki xil limitga ega bo‘ldi. Bu berilgan funksiya $P_o(0,0)$ nuqtada limitga ega emas degan so‘z.

4. Ko‘p o‘zgaruvchili funksiyasining uzluksizligi

$z=f(x,y)$ funksiya $P_o(x_o\ ,y_o)$ nuqtada va uning biror atrofida aniqlangan bo‘lsin.

4-**Ta’rif.** Agar $\lim_{p\to po} f(P)=f(P_o)$ bo‘lsa, u holda $z=f(x,y)$ funksiya $P_o(x_o\ ,y_o)$ nuqtada uzluksiz deb ataladi.

5-**Ta’rif.** Agar $z=f(x,y)=f(P)$ funksiya $P_o(x_o\ ,y_o)$ nuqtada va uning biror atrofida aniqlangan bo‘lsa, hamda agar argumentning $\Delta\ x$ va $\Delta\ y$ cheksiz kichik orttirmalariga funksiyaning Δz cheksiz kichik to‘liq orttirmasi mos kelsa, ya’ni

$$\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\Delta z=0$$

bo‘lsa, u holda bu funksiya $P_o(x_o\ ,y_o)$ nuqtada uzluksiz deb ataladi.

Biror to‘plamning har bir nuqtasida uzluksiz funksiya shu to‘plamda uzluksiz deb ataladi.

1) $z=f(x,y)$ funksiya $P_o(x_o\ ,y_o)$ nuqtaning biror atrofida aniqlangan, lekin $P(x_o\ ,y_o)$ nuqtaning o‘zida aniqlanmagan;
2) $z=f(x,y)$ funksiya $P_o(x_o\ ,y_o)$ nuqtada va uning biror atrofida aniqlangan, lekin

$$\lim_{\substack{x\to x_o\\ y\to y_o}} f(x,y)$$

mavjud emas.

3) $z=f(x,y)$ funksiyaning $P_o(x_o, y_o)$ nuqtada va uning biror atrofida aniqlangan va $\lim\limits_{\substack{x\to x_o\\ y\to y_o}} f(x,y)$ limit ham mavjud , lekin $\lim\limits_{\substack{x\to x_o\\ y\to y_o}} f(x,y)\neq f(x_o,y_o)$.

1-Misol. $z=x^2+y^2$ funksiyaning $0XY$ tekislikning istalgan nuqtasida uzluksizligini ko‘rsating.

Yechish. $\Delta z=[(x+\Delta x)^2+(y+\Delta y)^2]-(x^2+y^2)=[(x^2+2x\cdot\Delta x+\Delta x^2+y^2+2y\cdot\Delta y+\ +\Delta y^2)]-(x^2+y^2)=2x\cdot\Delta x+2y\cdot\Delta y+\Delta x^2+\Delta y^2$,

demak $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \Delta z=0$,

va $z=x^2+y^2$ funksiya $0XY$ tekislikning ixtiyoriy $P(x,y)$ nuqtasida uzluksiz ekan.

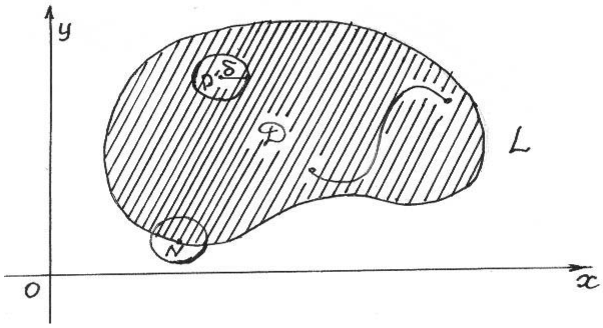
2-Misol. Ushbu $z=\dfrac{1}{x-y}$ funksiyaning uzilish nuqtalarini toping.

Yechish**.** Funksiya $0XY$ tekislikning koordinatalari x-$y=0$ ($x=y$) tenglamani qanoatlantiruvchi nuqtalardan tashqari barcha nuqtalarida aniqlangan va uzluksiz. $y=x$ to‘g‘ri chiziq birinchi va uchinchi koordinata burchaklarining bissektrisasidir. Ana shu bissektrisaning har bir nuqtasi berilgan funksiyaning uzilish nuqtasi bo‘ladi. Shunday qilib, uzilish nuqtalari funksiyaning uzilish to‘g‘ri chizig‘ini hosil qilar ekan.

5. Yopiq sohada uzluksiz funksiyaning xossalari

6-Ta’rif. Agar tekislik nuqtalarining D to‘plamiga tegishli ixtiyoriy ikki nuqtani shu to‘plam nuqtalaridan tashkil topgan uzluksiz chiziq bilan tutashtirish mumkin bo‘lsa, u holda D to‘plam ***bog‘lamli to‘plam*** deb ataladi.

7-Ta’rif. Agar D to‘plamning P nuqtasi shu to‘plamning nuqtalaridan tashkil topgan biror ∂ - atrofi bilan shu to‘plamga tegishli bo‘lsa, u holda u D to‘plamning ***ichki nuqtasi*** deyiladi (4-chizma).



4-chizma.

8-Ta'rif. Faqat ichki nuqtalaridan tashkil topgan *D* to'plam ***ochiq to'plam*** deyiladi.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1. Quyidagi limitlar hisoblansin.

a). $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{y}{x-y}$. J: mavjud emas. b). $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{xy+9}-3}{xy}$. J: $\frac{1}{6}$.

v). $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x+y)}{x+y}$. J: *1*. g). $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^2 y)}{xy}$. J: *0*

2. $z=\frac{5}{x+y}$ funksiyaning uzilish nuqtalari topilsin.

3. $z=\sqrt{1-\frac{x^2}{4}-y^2}$ funksiyaning uzluksizlik sohasi topilsin.

4. $z=\frac{x+y}{(x-1)^2+(y+2)^2}$ funksiyaning uzilish nuqtasi topilsin.

5. $z=f(x,y)=\begin{cases} 2, & agar \quad xy>0 \quad bo'lsa, \\ o, & agar \quad xy=0 \quad bo'lsa, \\ -2, & agar \quad xy<0 \quad bo'lsa. \end{cases}$ funksiyaning grafigi yasalsin va uzilish chizig'i ko'rsatilsin.

6. $u=\sqrt{x^2+y^2+z^2-4}$ funksiyaning uzluksizlik sohasi topilsing.

6. Ko‘p o‘zgaruvchi funksiyasining xususiy hosilalari

9-Ta’rif. Agar

$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x}\left(\lim_{\Delta y\to 0}\frac{\Delta_y z}{\Delta y}\right)$$

limit mavjud bo‘lsa, u holda bu limit *z=f(x,y)* funksiyaning *P(x,y)* nuqtadagi *x (y)* ***o‘zgaruvchi bo‘yicha xususiy hosilasi*** deb ataladi va

$$z'_x,\ f'_x,\ \frac{\partial z}{\partial x},\ \frac{\partial f}{\partial x}\ (z'_y,\ f'_y,\ \frac{\partial z}{\partial y},\ \frac{\partial f}{\partial y})$$

simvollarning biri orqali belgilanadi.

1-Misol. $z=x^3\cos y$ funksiyaning xususiy hosilalari topilsin.

Yechish.

$$z'_x=(x^3\cos y)'_x=(x^3)'_x\cos y=3x^2\cos y,$$
$$z'_y=(x^3\cos y)'_y=x^3(\cos y)'_y=-x^3\sin y.$$

2-Misol. $z=\ln(x^2+y^2)$, $\frac{\partial z}{\partial x}-?\frac{\partial z}{\partial y}-?$

$$\frac{\partial z}{\partial x}=\left(\ln(x^2+y^2)\right)'_x=\frac{(x^2+y^2)'_x}{x^2+y^2}=\frac{(x^2)'_x+(y^2)'_x}{x^2+y^2}=\frac{2x+0}{x^2+y^2}=\frac{2x}{x^2+y^2},$$

$$\frac{\partial z}{\partial y}=\frac{2y}{x^2+y^2}.$$

Bunda $(\ln u)'=\frac{u'}{u}$ formuladan foydalanildi.

3-Misol. $z=\sin(x^3y)$, $\frac{\partial z}{\partial x}-?\frac{\partial z}{\partial y}-?$

$$\frac{\partial z}{\partial x}=\left(\sin(x^3y)\right)'_x=\cos(x^3y)\cdot(x^3y)'_y=\cos(x^3y)\cdot(x^3)'_x\cdot y=3x^2y\cos(x^3y);$$
$$\frac{\partial z}{\partial y}=\left(\sin(x^3y)\right)'_y=\cos(x^3y)\cdot(x^3y)'_y=\cos(x^3y)\cdot x^3y'_y=x^3\cos(x^3y).$$

Bu yerda $(\sin u)'=\cos u\cdot u'$ va $(x^n)^!=nx^{n-1}$ formulalardan foydalanildi.

4-Misol. $z=arctg\frac{x}{y},\frac{\partial z}{\partial x}-?\frac{\partial z}{\partial y}-?$

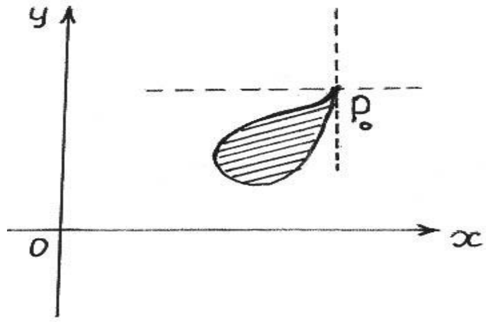
$$\frac{\partial z}{\partial x}=\left(arctg\frac{x}{y}\right)'_x=\frac{\left(\frac{x}{y}\right)'_x}{1+\left(\frac{x}{y}\right)^2}=\frac{\frac{1}{y}}{1+\frac{x^2}{y^2}}=\frac{y}{y^2+x^2};$$
$$\frac{\partial z}{\partial y}=\left(arctg\frac{x}{y}\right)'_y=\frac{\left(\frac{x}{y}\right)'_y}{1+\left(\frac{x}{y}\right)^2}=\frac{-\frac{x}{y^2}}{1+\frac{x^2}{y^2}}=-\frac{x}{y^2+x^2}.$$

Bu yerda $(arctgu)'=\frac{u'}{1+u^2}$ formuladan foydalanildi.

5-Misol. $z=x^y$, $\frac{\partial z}{\partial x}-?\frac{\partial z}{\partial y}-?$

$$\frac{\partial z}{\partial x}=(x^y)'_x=yx^{y-1},\frac{\partial z}{\partial y}=(x^y)'_y=x^y\ln x.$$

Biz bu yerda $(x^\alpha)'=\alpha x^{\alpha-1}va(a^x)'=a^x\ln a$ formuladan foydalandik. Ikki o‘zgaruvchi funksiyasining xususiy hosilasini keltirilgan ta’rifi funksiyaning aniqlanish sohasini ichki nuqtalari uchun to‘g‘ri keladi. Agar *P(x,y)* nuqta funksiyani aniqlanish sohasini chegara nuqtasi bo‘lsa, u holda Δx □ Δyz) xususiy orttirmalar aniqlanmagan bo‘lishlari ham mumkin, chunki bu holda $P_1(x+\Delta x,y)$, $P_2(x,y+\Delta y)$ nuqtalar $\Delta x\neq0$, $\Delta y\neq0$ orttirmalarning hech bir qiymatlarida funksiyaning aniqlanish sohasiga tegishli bo‘lmasligi mumkin. Bu 7-chizmadagi P_0 nuqta uchun o‘rinli.



7-chizma.

Bunday holda sohaning ichki *P(x,y)* nuqtalarida z'_x xususiy hosila hamda $\lim\limits_{p\to p_0} z'_x(P)$ limit mavjud bo‘lsa, u holda shu limit funksiyasining P_0 nuqtadagi *x* bo‘yicha xususiy hosilasi deb qabul qilinadi, ya’ni

$$z'_x(P_0) = \lim_{p\to p_0} z'_x(P).$$

$z'_y(P_0)$ ham shunga o‘xshash aniqlanadi.

Uch va undan ortiq o‘zgaruvchi funksiyasining xususiy hosilasi ham shunga o‘xshash ta’riflanadi va hisoblanadi. Masalan, *u=f(x,y,z)* uch o‘zgaruvchining funksiyasini xususiy hosilalari

$$u'_x = \lim_{\Delta x\to 0}\frac{\Delta_x u}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x, y, z) - f(x,y,z)}{\Delta x},$$
$$u'_y = \lim_{\Delta y\to 0}\frac{\Delta_y u}{\Delta y} = \lim_{\Delta y\to 0}\frac{f(x, y+\Delta y, z) - f(x,y,z)}{\Delta y},$$
$$u'_z = \lim_{\Delta z\to 0}\frac{\Delta_z u}{\Delta z} = \lim_{\Delta z\to 0}\frac{f(x, y, z+\Delta z) - f(x,y.z)}{\Delta z}$$

kabi aniqlanadi. Sohaning barcha nuqtalarida *x* (yoki *y*) bo‘yicha xususiy hosilaga ega funksiya shu sohada *x* (yoki *y*) bo‘yicha ***xususiy hosilaga*** ega deyiladi.

7. **Ko‘p o‘zgaruvchili funksiyasining differensiallanuvchanligi.**

z=f(P) funksiya *P(x,y)* nuqtaning biror atrofida aniqlangan bo‘lsin.

Ta’rif. Agar *z=f(P)* funksiyaning *P(x,y)* nuqtadagi to‘la orttirmasi

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y),$$

$$\Delta z = A\Delta x + B\Delta y + D(\Delta x,\Delta y)\Delta x + E(\Delta x,\Delta y)\Delta y \qquad (1)$$

Ko‘rinishda tasvirlansa, u holda *z=f(P)* funksiya *P(x,y)* nuqtada ***differensiallanuvchi*** deb ataladi, bunda *A* va *B*, Δx, Δy ga bog‘liq bolmagan sonlar, $D(\Delta x,\Delta y)$ va $E(\Delta x,\Delta y)$ $\Delta x\to 0$, $\Delta y\to 0$ da yoki $\rho=\sqrt{\Delta x^2+\Delta y^2}\to 0$ da cheksiz kichik funksiyalar.

8. Ko‘p o‘zgaruvchili funksiyaning to‘la differensiali va uning tatbiqi

Ta’rif. $\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$, to‘la orttirmaning $\Delta x, \Delta y$ larga nisbatan chiziqli bosh bo‘lagi $f_x'(x,y)\,\Delta x + f_y'(x,y)\,\Delta y$ differensiallanuvchi *z=f(x,y)* funksiyaning ***to‘la differensiali*** deb ataladi va *dz* yoki *df* bilan belgilanadi.

1-Misol. *z=xy* funktsyaining (3,4) nuqtada $\Delta x=0,1$, $\Delta y=0,2$ bo‘lganda to‘la orttirmasi va uning to‘la differensiali topilsin.

Yechish**.** $\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = (x+\Delta x)(y+\Delta y) - xy = xy + y\Delta x + x\Delta y + \Delta x\Delta y - xy = y\Delta x + x\Delta y + \Delta x\Delta y,$

$$df = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = ydx + xdy = y\Delta x + x\Delta y.$$

Demak, $\Delta z=4\cdot 0,1+3\cdot 0,2-0,1\cdot 0,2=1,02;$

$$dz=4\cdot 0,1+3\cdot 0,2=1;$$

$$\Delta z-\partial z=1,02-1=0,2.$$

2-Misol. $z=\arcsin\frac{x}{y}$ funksiyaning to‘la differensiali topilsin.

Yechish. $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$ xususiy hosilalarni $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$ formulaga asoslanib topamiz. $\frac{\partial z}{\partial x}$ ni hisoblashda *y* ni, $\frac{\partial z}{\partial y}$ ni hisoblashda *x* ni o‘zgarmas deb olamiz:

$$\frac{\partial z}{\partial x}=\left(\arcsin\frac{x}{y}\right)'_x=\frac{\left(\frac{x}{y}\right)'_x}{\sqrt{1-\left(\frac{x}{y}\right)^2}}=\frac{\frac{1}{y}\cdot 1}{\sqrt{1-\frac{x^2}{y^2}}}=\frac{1}{y\sqrt{\frac{y^2-x^2}{y^2}}}=\frac{1}{y\sqrt{\frac{y^2-x^2}{|y|}}}=\frac{|y|}{y\sqrt{y^2-x^2}},$$

$$\frac{\partial z}{\partial y}=\left(\arcsin\frac{x}{y}\right)'_y=\frac{\left(\frac{x}{y}\right)'_y}{\sqrt{1-\left(\frac{x}{y}\right)^2}}=\frac{-\frac{x}{y^2}}{\sqrt{\frac{y^2-x^2}{y^2}}}=-\frac{x}{y^2\cdot\frac{\sqrt{y^2-x^2}}{|y|}}=-\frac{|y|\cdot x}{y^2\sqrt{y^2-x^2}}.$$

Demak,

$$dz=\frac{\partial z}{\partial x}\partial x+\frac{\partial z}{\partial y}\partial y=\frac{|y|\cdot dx}{y\sqrt{y^2-x^2}}-\frac{|y|\cdot x}{y^2\sqrt{y^2-x^2}}dy=\frac{|y|\cdot ydx-|y|\cdot xdy}{y^2\sqrt{y^2-x^2}}=\frac{|y|(ydx-xdy)}{|y|^2\sqrt{y^2-x^2}}=$$
$$=\frac{ydx-xdy}{|y|\sqrt{y^2-x^2}}.$$

3-Misol. Uch o‘zgaruvchi *x,y,z* ning funksiyasi $u=e^{x^3+y^3}\cos z$ funksiyaning to‘la differensiali topilsin.

Yechish**.** Xususiy hosilalarni $(e^u)'=e^u\cdot u'$ formuladan foydalanib topamiz. $\frac{\partial u}{\partial x}$ ni hisoblaganda *y,z* ni, $\frac{\partial u}{\partial y}$ hisoblaganda *x,z* ni va $\frac{\partial u}{\partial z}$ ni hisobalganda *x,y* ni o‘zgarmas sanaymiz.

$$\frac{\partial u}{\partial x}=e^{x^3+y^3}3x^2\cos z,\ \frac{\partial u}{\partial y}=e^{x^3+y^3}3y^2\cos z,\ \frac{\partial u}{\partial z}=-e^{x^3+y^3}\sin z$$

xususiy hosilalar istalgan *P(x,y,z)* nuqtada uzluksiz bo‘lganligi sababli

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz=e^{x^3+y^3}(3x^2\cos zdx+3y^2\cos zdy-\sin zdz).$$

4-Misol. Agar $u=e^x$, $v=cosx$ bo‘lsa, $z=u^3+v^2$ murakkab funksiyaning $\frac{dz}{dx}$ hosilasini toping.

Yechish. Bu masalani ikki usul bilan hal etishimiz mumki.

1. z ni x orqali ifodalasak $z=e^{3x}+cos^2x$ bitta x o‘zgaruvchining murakkab funksiyasiga ega bo‘lamiz.

Uni differensiallab $\frac{dz}{dx}=3e^{3x}+2\cos x(\cos x)'=3e^{3x}+2\cos x(-\sin x)=3e^{3x}-\sin 2x$

ni hosil qilamiz.

2. $\frac{dz}{dx}$ hosilani z ni bevosita x orqali ifodalamasdan formuladan foydalanib topamiz: $\frac{du}{dx}=e^x, \frac{dv}{dx}=-\sin x, \frac{\partial z}{\partial u}=3u^2, \frac{\partial z}{\partial v}=2v$ bo‘lgani uchun

$$\frac{dz}{dx}=3u^2\cdot e^x+2v\cdot(-\sin x)=3(e^x)^2\cdot e^x-2\cos x\sin x=3\cdot e^{3x}-\sin 2x$$

5-Misol. Agar $y=asinx$, $u=cosx$ bo‘lsa, $z=\frac{e^{ax}(y-u)}{a^2+1}$ funksiyaning $\frac{dz}{dx}$ to‘la hosilasi topilsin.

Yechish**.** To‘la hosilani $\frac{dz}{dx}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dx}+\frac{\partial z}{\partial u}\cdot\frac{du}{dx}$ formuladan foydalanib topamiz:

$$\frac{\partial z}{\partial x}=\frac{\left(e^{ax}\right)_x'(y-u)}{a^2+1}=\frac{ae^{ax}(y-u)}{a^2+1}=\frac{ae^{ax}(a\sin x-\cos x)}{a^2+1},$$
$$\frac{\partial z}{\partial y}=\frac{e^{ax}(y-u)_y'}{a^2+1}=\frac{e^{ax}}{a^2+1}, \frac{\partial z}{\partial u}=\frac{e^{ax}(y-u)_u'}{a^2+1}=-\frac{e^{ax}}{a^2+1},$$
$$\frac{dy}{dx}=(a\sin x)'=a\cos x, \frac{du}{dx}=(\cos x)'=-\sin x$$

ekanini hisobga olsak

$$\frac{dz}{dx}=\frac{ae^{ax}(a\sin x-\cos x)}{a^2+1}+\frac{ae^{ax}}{a^2+1}\cdot a\cos x+\frac{e^{ax}}{a^2+1}\cdot\sin x=$$
$$=\frac{e^{ax}(a^2\sin x-a\cos x+a\cos x+\sin x)}{a^2+1}=e^{ax}\sin x$$

6-Misol. $z = arctg\frac{u}{v}$, bu yerda $u=x+y$, $v=x-y$ funksiya uchun $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\frac{x-y}{x^2+y^2}$

munosabatning bajarilishi ko‘rsatilsin.

Yechish. $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}=\left(arctg\frac{u}{v}\right)'_u\cdot(x+y)'_x+\left(arctg\frac{u}{v}\right)'_v\cdot(x+y)'_x=$$

$$=\frac{\left(\frac{u}{v}\right)'_u}{1+\left(\frac{u}{v}\right)^2}\cdot 1+\frac{\left(\frac{u}{v}\right)'_v}{1+\left(\frac{u}{v}\right)^2}\cdot 1=\frac{\frac{1}{v}}{1+\frac{u^2}{v^2}}+\frac{-\frac{u}{v^2}}{1+\frac{u^2}{v^2}}=\frac{v}{u^2+v^2}-\frac{u}{u^2+v^2}=\frac{v-u}{u^2+v^2};$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}=\left(arctg\frac{u}{v}\right)'_u\cdot(x+y)'_y+\left(arctg\frac{u}{v}\right)'_v\cdot(x-y)'_y=$$

$$=\frac{\frac{1}{v}}{1+\left(\frac{u}{v}\right)^2}\cdot 1+\frac{-\frac{u}{v^2}}{1+\left(\frac{u}{v}\right)^2}\cdot(-1)=\frac{v}{v^2+u^2}+\frac{u}{v^2+u^2}=\frac{v+u}{v^2+u^2}.$$

Demak,

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\frac{v-u}{u^2+v^2}+\frac{v+u}{u^2+v^2}=\frac{2v}{u^2+v^2}=\frac{2(x-y)}{(x+y)^2+(x-y)^2}=\frac{2(x-y)}{2(x^2+y^2)}=\frac{x-y}{x^2+y^2}$$

Shuni isbotlash talab etilgan edi.

7-Misol. Ushbu $z=u^3\ v^3$, $u=x^2\ siny$, $v=x^2\ e^y$ murakkab funksiyaning to‘la differensiali topilsin.

Yechish. Formulaga ko‘ra:

$$dz=(u^3v^3)'_u\cdot du+(u^3v^3)'_v\,dv=3u^2v^3(u'_xdx+u'_ydy)+3u^3v^2(v'_xdx+v'_ydy)=$$
$$=3u^2v^3(2\sin y dx+x^2\cos y dy)+3u^3v^2(2xe^ydx+x^2e^ydy).$$

Bu ifodani bunday yozish ham mumkin:

$$dz=\left(3u^2v^3 2x\sin y+3u^3v^2 2xe^y\right)dx+\left(3u^2v^3x^2\cos y+3u^3v^2x^2e^y\right)dy=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy.$$

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

Quyidagi funksiyalarning xususiy hosilalari topilsin.

1. $z=y^2 \sin^2 x$ $\quad j: \frac{\partial z}{\partial x} = y^2 sin2x; \quad \frac{\partial z}{\partial x} = 2ysin^2 \sim;$

2. $u = e^{x^2+y^2+z^2}$. J: $\frac{\partial u}{\partial x} = 2xe^{x^2+y^2+z^2}; \quad \frac{\partial u}{\partial y} = 2ye^{x^2+y^2+z^2}; \quad \frac{\partial u}{\partial z} = 2ze^{x^2+y^2+z^2}$

3. $z=ln(xy)$. J: $\frac{\partial u}{\partial x} = \frac{1}{x}; \quad \frac{\partial u}{\partial y} = \frac{1}{y};$

4. $z = \ln \frac{\sqrt{x^2+y^2} - x}{\sqrt{x^2+y^2} + x}$.

5. $u = e^{\frac{x}{y}} + e^{\frac{z}{y}}$.

6. $z=arcsin\,(x+y)$.

7. $f(x,y) = x^4y^2 + 8x^2y$

8. $u = \frac{x}{(x+y)^2}$

9 $u = tg(xy)$

10. Agar $u=lnx$, $v=sinx$ bo‘lsa, $z=u^2+v$ murakkab funksiyaning $\frac{dz}{dx}$ hosilasi topilsin.

11. Agar $u=x^3$, $v=sin3x$, $t=arctg\,\frac{1}{x}$ bo‘lsa, $z=e^{u-2v+3t}$ murakkab funksiyaning $\frac{dz}{dx}$ hosilasi topilsin.

12. Agar $y=x^3$ bo‘lsa, $z=ln(e^x+e^y)$ funksiyaning to‘la hosilasi topilsin..

13. Agar $u = \frac{x}{y}$, $v=3x-2y$ bo‘lsa, $z=u^2\, lnv$ murakkab funksiyaning $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ xususiy hosilalari topilsin.

14. Agar $u = e^{x+y^2}$, $v=x^2+y$ bo‘lsa, $z=ln(u^2+v)$ murakkab funksiyaning $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ xususiy hosilalari topilsin.

15. Agar $y=\sin x$ bo‘lsa, $z=x^2+\sqrt{y}$ murakkab funksiyaning to‘la hosilasi topilsin.

17. Agar $u=-\cos x$, $v=\cos x$, bo‘lsa, $z=\sqrt{\dfrac{1+u}{1+v}}$ murakkab funksiyaning $\dfrac{dz}{dx}$ to‘la hosilalari topilsin.

18. Agar $u=x^2+\sin y$, $v=\ln(x+y)$ bo‘lsa, $z=u+v^2$ murakkab funksiyaning $\dfrac{\partial z}{\partial x}$ va $\dfrac{\partial z}{\partial y}$ xususiy hosilalari topilsin.

9. Murakkab funksiyalarni differensiallash.

1. Agar $z=F(u,v)$ bo’lib, $u=f(t)$, $v=\varphi(t)$ bo’lsa, z t ning murakkab funksiyasi deyiladi. U holda, agar F, f, φ differensiallanuvchi funksiyalar bo’lsa,

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\frac{du}{dt}+\frac{\partial z}{\partial v}\frac{dv}{dt}$$

2. Agar $z=f(u,v)$ funksiya argumentlari $u=\varphi(x;y)$, $v=\psi(x;y)$ ko’rinishda va F, f, φ differensiallanuvchi funksiyalar bo’lsa,

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x};\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}.$$

bo’ladi.

3. Agar $u=x$, $v=y(x)$ bo‘lsa, u holda formula quyidagi ko‘rinishni oladi:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial x}$$

bu formula funksiyaning to‘la hosilasinia ifodalaydi.

1-Misol. Agar $z=\cos(uv)$ funksiya $u=2x+3y$, $v=xy$ bo‘lsa, uning xususiy hosilalarini toping.

Yechish:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x};\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

Formulaga asosan:

$$\frac{\partial z}{\partial x}=-v\sin(uv)\cdot 2-u\sin(uv)\cdot y=-(4xy+3y^2)\cdot\sin(2x^2y+3xy^2),$$
$$\frac{\partial z}{\partial y}=-v\sin(uv)\cdot 3-u\sin(uv)\cdot x=-\left(6xy+2x^2\right)\cdot\sin(2x^2y+3xy^2).$$

2-Misol. Agar $u=x+y^2+z^3$

funksiyada *y=sinx, z=cosx* bo‘lsa , uning to‘la hosilasini toping.

Yechish:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

formulaga ko‘ra topamiz:

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial x}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}=1+2y\cos x+3z^2\left(-\sin x\right)=$$
$$=1+2\sin x\cos x-3\cos^2 x\sin x.$$

9. Oshkormas funksiyalarni differensiallash.

1. $(x_0,\ y_0)$ yechimga ega bo’lgan $F(x,y)=0$ tenglama, uning F'_y hosilasi $(x_0,\ y_0)$ nuqtaning qandaydir atrofida uzluksiz va $F'_y(x;y)\neq 0$ shartlarni qanoatlantirgandagina, x_0 atrofida y ni x ning uzluksiz funksiyasi sifatida aniqlab beradi. $(x_0,\ y_0)$ nuqta atrofida, yuqoridagi shartlardan tashqari F'_x hosila ham mavjud va uzluksiz bo’lsa , u holda oshkormas funksiya hosilaga ega bo’lib, bu hosila ushbu

$$\frac{dy}{dx}=-\frac{F'_x(x;y)}{F'_y(x;y)}$$

formula bilan aniqlaniladi.

2. $F(x;y;z)=0$ tenglama uchun yuqoridagiga o'xshash shartlar (ya'ni F'_z hosila (x_0, y_0, z_0) nuqta atrofida noldan farqli va uzluksiz, F'_x, F'_y hosilalar bu nuqta atrofida mavjud va uzluksiz bo'lsa), bu tenglama z ni x va y ning oshkormas funksiyasi sifatida aniqlaydi va u quyidagi hususiy hosilalarga ega bo'ladi:

$$\frac{dz}{dx}=-\frac{F'_x(x;y;z)}{F'_z(x;y;z)},\quad \frac{dz}{dy}=-\frac{F'_y(x;y;z)}{F'_z(x;y;z)}.$$

3-Misol. $x^3+y^3-e^{xy}-5=0$ tenglama bilan berilgan oshkormas funksiyaning hosilasini toping.

Yechish.

$$\frac{dy}{dx}=-\frac{F'_x(x;y)}{F'_y(x;y)}$$

formulaga asosan :

$$\frac{dy}{dx}=-\frac{F'_x(x;y)}{F'_y(x;y)}=-\frac{3x^2-ye^{xy}}{3y^2-xe^{xy}}.$$

4-Misol. $xyz+x^3-y^3-z^3+5=0$ tenglama bilan berilgan oshkormas funksiyaning xususiy hosilasini toping.

Yechish. Quyidagi formulaga asosan:

$$\frac{dz}{dx}=-\frac{F'_x(x;y;z)}{F'_z(x;y;z)}=-\frac{yz+3x^2}{xy-3z^2},$$

$$\frac{dz}{dy}=-\frac{F'_y(x;y;z)}{F'_z(x;y;z)}=-\frac{xz-3y^2}{xy-3z^2}.$$

7.2. YUQORI TARTIBLI XUSUSIY HOSILALAR.

MAVZUGA OID NAZARIY MATERIALLAR

1.Ikkinchi tartibli xususiy hosila.

Birinchi tartibli xususiy hosiladan olingan xususiy hosila *ikkinchi tartibli xususiy hosila* deyiladi va uni quyidagicha yoziladi:

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\cdot\left(\frac{\partial z}{\partial x}\right)=f''_{xx}(x;y),\ \frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial}y\cdot\left(\frac{\partial z}{\partial y}\right)=f''_{yy}(x;y),$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\cdot\left(\frac{\partial z}{\partial x}\right)=f''_{xy}(x;y),\ \frac{\partial^2 z}{\partial y\partial x}=\frac{\partial}{\partial x}\cdot\left(\frac{\partial z}{\partial y}\right)=f''_{yx}(x;y),$$

Xuddi shuningdek,uch va undan yuqori tartibli xususiy hosilalar ham yuqoridagi kabi aniqlaniladi.

$\frac{\partial^n z}{\partial x^k \partial y^{n-k}}$ yozuv *z* funksiya *x* o‘zgaruvchi bo‘icha *k* marta va **y** o‘zgaruvchi bo‘yicha *n-k* marta differensiallanganligini bildiradi.

$f''_{xx}(x;y)$ *va* $f''_{yx}(x;y)$xususiy hosilalar $z=f(x;y)$, funksiyaning *aralash hosilalari deyiladi.*

1-Misol. $z=e^{x^2y^2}$ funksiyaning ikkinchi tartibli xususiy hosilalarini toping.

Yechish. Dastlab birinchi tartibli xususiy hosilalarini toping:

$$\frac{\partial z}{\partial x}=e^{x^2y^2}\cdot 2xy^2,\ \ \frac{\partial z}{\partial y}=e^{x^2y^2}\cdot 2x^2y$$

Bularni yana differensiallasak, quyidagilarga ega bo‘lamiz:

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\cdot\left(\frac{\partial z}{\partial x}\right)=f''_{xx}(x;y)=e^{x^2y^2}\cdot 4x^2y^4+e^{x^2y^2}\cdot 2y^2=2y^2e^{x^2y^2}(2x^2y^2+1),$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\cdot\left(\frac{\partial z}{\partial y}\right)=f''_{yy}(x;y)=e^{x^2y^2}\cdot 4x^4y^2+e^{x^2y^2}\cdot 2x^2=2x^2e^{x^2y^2}(2x^2y^2+1),$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \cdot \left(\frac{\partial z}{\partial x} \right) = f''_{xy}(x;y) = e^{x^2y^2} \cdot 4x^3y^3 + e^{x^2y^2} \cdot 4xy = 4xye^{x^2y^2}(x^2y^2+1),$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x} \cdot \left(\frac{\partial z}{\partial y} \right) = f''_{yx}(x;y) = e^{x^2y^2} \cdot 4x^3y^3 + e^{x^2y^2} \cdot 4xy = 4xye^{x^2y^2}(x^2y^2+1)$$

Oxirgi ikki ifodani solishtirib, ularning o'zaro teng ekanligiga ishonch hosil qilamiz:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

Demak, bitta funksiyaning faqat differensiallash tartibi bilan farq qiladigan aralash xususiy hosilalari uzluksiz bo'lsa, ular o'zaro teng bo'lar ekan.

2-Misol. $z = arctg\frac{y}{x}$ funksiya $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ Loplas tenglamasini qanoatlantirishini isbot qiling.

Yechish: Berilgan funksiyaning birinchi va ikkinchi tartibli xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x} = \frac{y}{x^2+y^2}, \quad \frac{\partial^2 z}{\partial x^2} = \frac{2xy}{\left(x^2+y^2\right)^2},$$
$$\frac{\partial z}{\partial y} = \frac{x}{x^2+y^2}, \quad \frac{\partial^2 z}{\partial y^2} = \frac{2xy}{\left(x^2+y^2\right)^2}.$$

Bularni Loplas tenglamasiga qo'yamiz:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2xy}{\left(x^2+y^2\right)^2} - \frac{2xy}{\left(x^2+y^2\right)^2} \equiv 0.$$

$z = f(x;y)$ funksiyaning ikkinchi tartibli to'la differensiali *(d^2z)*

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2\frac{\partial^2 z}{\partial x \partial y} dxdy + \frac{\partial^2 z}{\partial y^2} dy^2$$

Formula bilan ifodalanadi.

3-Misol. $z = x^3 + y^3 + x^2y^2$ funksiyaning ikkinchi tartibli to‘la differensialini topamiz.

Yechish. Berilgan funksiyaning ikkinchi tartibli xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x} = 3x^2 + 2xy^2, \quad \frac{\partial^2 z}{\partial x^2} = 6x + 2y^2,$$
$$\frac{\partial z}{\partial y} = 3y^2 + 2x^2y, \quad \frac{\partial^2 z}{\partial y^2} = 6y + 2x^2, \frac{\partial^2 z}{\partial x \partial y} = 4xy.$$

Demak, $d^2z = (6 + 2y^2)dx^2 + 8xydxdy + (6y + 2x^2)dy^2.$

Agar sirt $z=f(x;y)$ tenglama bilan berilgan bo‘lsa, u holda $M_o(x_o;y_0;z_o)$ nuqtadan berilgan sirtga o‘tkazilgan urinma tekislik tenglamasi

$$z - z_o = f'_x(x_o, y_o)(x - x_o) + f'_y(x_o, y_o)(y - y_o)$$

formula bilan , sirtga $M_o(x_o;y_0;z_o)$ nuqtadan o‘tkazilgan normalning kanonik tenglamasi esa

$$\frac{x - x_o}{f'_x(x_o, y_o)} = \frac{y - y_o}{f'_y(x_o, y_o)} = \frac{z - z_o}{f'_z(x_o, y_o)}$$

formula bilan aniqlaniladi.

Agar sirt tenglamasi $F(x;y;z)=0$ oshkormas ko‘rinishzda bo‘lib,$F(x_o;y_o;z_o)=0$ bo‘lsa, u holda $M_o(x_o;y_0;z_o)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi

$$F'_x(x_o; y_o; z_o)(x - x_o) + F'_y(x_o, y_o, z_o)(y - y_o) + F'_z(x_o, y_o, z_o)(z - z_o) = 0$$

normal tenglamasi esa

$$\frac{x - x_o}{F'_x(x_o, y_o, z_o)} = \frac{y - y_o}{F'_y(x_o, y_o, z_o)} = \frac{z - z_o}{F'_z(x_o, y_o, z_o)}$$

ko‘rinishda bo‘ladi.

4-Misol. $x^3+y^3+z^3+xyz=0 \quad sirtga \ M_o(1;2;-1)$ nuqtada o‘tkazilgan urinma tekislik va normal tenglamalarini toping.

Yechish. Xususiy hosilalarni *M*$M_o(1;2;-1)$ nuqtadagi qiymatlarini hisoblaymiz.

$$F'_x(x_o;y_o;z_o)=\left.(3x^2+yz)\right|_{M_o}=1, \qquad F'_y(x_o;y_o;z_o)=\left.(3x^2+xz)\right|_{M_o}=11,$$,

Bularni formulalarga qo‘yib , urinma tekislik tenglamasini va normal tenglamasini hosil qilamiz.

$$(x-1)+1\cdot(y-2)+5\cdot(z+1)=0, \quad \frac{x-1}{1}=\frac{y-2}{11}=\frac{z+1}{5}.$$

2. Ikki o‘zgaruvchili funksiyaning ekstremumlari.

Agar $M_o(x_o;y_o)$ nuqtaning shunday kichik atrofida mavjud bo‘lsaki, bu atrofning M_o dan farqli barcha $M(x;y)$ nuqtalari uchun

$$f(x_o;y_o)\geq f(x;y)\big(f(x_o;y_o)\leq f(x;y)\big)$$

tengsizliklar bajarilsa, $M_o(x_o;y_o)$ nuqta z=f(x;y) funksiyaning *lokal maksimumi* (mi nimumi) deyiladi. Funksiyaning maksimumi yoki minimumi uning *ekstremumi* deyiladi. Funksiyaning ekstremumga erishadigan nuqtasi uning *ekstremum nuqtasi* deyiladi.

1-**Teorema**.(ekstremum mavjudligini yetarli sharti) Agar $M_o(x_o;y_o)$ nuqta $z=f(x;y)$ funksiyaning ekstremum nuqtasi bo‘lsa, u holda uning xususiy hosilalari $f'_x(x_o;y_o)=f'_y(x_o;y_o)=0$ bo‘ladi yoki bu hosilalardan birortasi mavjud bo‘lmaydi.

Bu shartni qanoatlantiradigan nuqtalar *statsionar* yoki *kritik* nuqtalar deyiladi. Ekstremum nuqtalar har doim statsonar nuqta bo‘ladi, ammo statsionar nuqtalar ekstremum nuqtasi bo‘lishi ham, bo‘lmasligi ham mumkin. Ststsionar nuqta ekstremum nuqtasi bo‘lishi uchun ekstremum mavjud bo‘lishining zaruriy sharti ham bajarilishi kerak.

Ikki o‘zgaruvchili funksiya ekstremumi mavjud bo‘lishining zaruriy shartini ta’riflash uchun quyidai belgilarni kiritamiz:

$$A = f''_{xx}(x_o; y_o),\ B = f''_{xy}(x_o; y_o),$$
$$C = f''_{yy}(x_o; y_o),\ \Delta = AC - B^2.$$

2-Teorema. (ekstremum mavjudligini zaruriy sharti).

$M_o(x_o;y_o)$ statsionar nuqtaga ega bo‘lgan biror sohada $z=f(x;y)$ funksiya uzluksiz va uchinchi tartibli xususiy hosilaga ega bo‘lsin. U holda:

a) agar $\Delta > 0$ $bo'lsa$, $M_0(x_0, y_0)$ nuqta berilgan funksiya uchun *ekstremum nuqtasi* bo‘ladi*;*

b) agar $\Delta < 0$ bo‘lsa, $M_{0}(x_0;y_0)$ nuqta ekstremum nuqtasi bo‘lmaydi;

v) agar $\Delta = 0$ bo‘lsa, $M_o(x_o;y_o)$ nuqta *ekstremum nuqtasi* bo‘lishi mumkin, bo‘lmasligi ham mumkin.

Uchinchi holda qo‘shimcha tekshirish o‘tkazish zarurligini eslatib o‘tamiz.

1-Misol. $z = x^3 + y^3 - 3xy$ Funksiyaning ekstremumini toping.

Yechish. Berilgan funksiya uchun $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial x}$ har doim mavjud va bu xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x} = 3x^2 - 3y,\quad \frac{\partial z}{\partial y} = 3y^2 - 3x.$$

Endi quyidagi sistemani tuzamiz:

$$\begin{cases} x^2 - y = 0 \\ y^2 - x = 0 \end{cases}$$

Bunda $x_1=0$, $x_2=1$, $y_1=0$, $y_2=1$.Shunday qilib , $M_1(0;0)$ va $M_2(1;1)$ ikkita statsonar nuqtaga ega bo‘ldik.Quyidagilarni topamiz:

$M_1(0;0)$ nuqtadan $\Delta = -9 < 0$ bo‘lgani uchun bu nuqtada ekstremum yo‘q.

$M_2(1;1)$ nuqtada $\Delta = 27 > 0$ *va* $A = 6 > 0$ bo `lgani uchun bu nuqtada berilgan funksiya lokal minimumga erishadi: $z_{min}=-1$.

$\varphi(x;y) = 0$ funksiya yordamida *z=f(x;y)* funksiyaning topilgan ekstremumihi shartli ekstremum deyiladi. $\varphi(x;y) = 0$ tenglama bog‘lovchi tenglama deyiladi.

Geozmetrik masalalarda shartli ekstremumlarni aniqlash *z=f(x;y)* sirtning $\varphi(x;y) = 0$ slindr bilan kesishishidan hosil bo‘lgan egri chiziqning ekstremum nuqtalarini topishga keltiriladi.

Agar bog‘lovchi tenglamadan *y=y(x)* ni topib (agar uni topish mumkin bo‘lsa), uni *z=f(x;y)* funksiyaga qo‘ysak, shartli ekstremumni topish masalasi *z=(x;y(x))* bir o‘zgaruvchili $\varphi(x;y)$ funksiyaning ekstremumini topishga keltiriladi.

2-Misol. $z = x^2 - y^2$ funksiyaning *y=2x-6* shart bo‘yicha ekstremumini toping.

Yechish. *y=2x-6* ni berilgan funksiyaga qo‘yib , *x* o‘zgaruvchiga nisbatan bir o‘zgaruvchili quyidagi funksiyani hosil qilamiz:

$$z = x^2 - (2x-6)^2, \qquad z = -3x^2 + 24x - 36$$

Ikkinchi tartibli hosila $z'' = -6 < 0$ bo‘lgani uchun *M(4;2)* nuqtada berilgan funksiya shartli maksimumga erishadi: $z_{max}=12$.

Differensiallanuvchi funksiya chegaralangan yopiq $\overline{D}$ sohada o‘zining eng katta (eng kichik) qiymatiga $\overline{D}$ soha ichida yotuvchi stastsionar nuqtasida yoki shu sohaning chegarasida erishadi; *z=f(x;y)* funksiyaning chegaralangan yopiq $\overline{D}$ sohadagi eng katta va eng kichik qiymatlarini topish uchun funksiyaning bu sohaga tegishli kritik nuqtalardagi qiymatlarini hamda uning $\overline{D}$ sohaning chegarasidagi eng katta va eng kichik qiymatlar aniqlaniladi. Bu qiymatlarning orasidagi eng katta va eng kichigi berilgan funksiyaning $\overline{D}$ sohadagi mos ravishda eng katta va eng kichik qiymatlari bo‘ladi.

Masalan. To‘la sirtning yuzi *S*, hajmi esa eng katta bo‘lgan to‘g‘ri burchakli parallelopipedning o‘lchamlarini aniqlang.

Yechish. To‘g‘ri burchakli parallelopipedning hajmi V =xyz ga teng, bunda *x,y,z*- parallelopipedning o‘lchamlari.To‘la sirtning yuzi : $S = 2(xy + xz + yz)$ bundan

$$z = \frac{S - 2xy}{2(x+y)}, \quad V = xyz = \frac{Sxy - 2x^2y^2}{2(x+y)} = V(x;y).$$

V=V(x;y) funksiyaning ekstremumlarini topamiz:

$$\left.\begin{aligned} \frac{\partial V}{\partial x} &= \frac{y^2(S - 2x^2 - 4xy)}{2(x+y)^2} = 0 \\ \frac{\partial V}{\partial y} &= \frac{x^2(S - 2y^2 - 4xy)}{2(x+y)^2} = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} S - 2x^2 - 4xy = 0, \\ S - 2y^2 - 4xy = 0 \end{aligned}\right\}$$

Masalaning shartiga ko‘ra *x>0, y>0* bo‘lgani uchun oxirgi sistemadan

$$x = y = \sqrt{\frac{S}{6}}$$

ni topamiz . Demak, yagona statsionar nuqtaga ega. U V=V(x;y) funksiya uchun maksimum nuqtasi bo‘ladi.

Shunday qilib , hajmi eng katta bo‘lgan parallelopipid , ya’ni qirrasi $\sqrt{\frac{S}{6}}$ ga teng kubga ega bo‘lamiz.

AUDITORIYADA TAHLIL QILINADIGAN MISOLLAR .

1.Quyidagi funksiyalarning ekstremumini toping:

a) $z = x^2 - xy + y^2 + 9x - 6y + 20$ j: $z_{\min} = -1$, bunda x=-4, y=1

b) $z = x^3 + 8y^3 - 6xy + 1$ j: $z_{\min} = 0$, *bunda x=1, y=-1/2 bo‘lganda.*

v) $z=2xy-4x-2y$ j: *Ekstremumi yo'q.*

g) $z=3x+6y-x^2-xy-y^2$ j: $z_{\min}=9$, *bunda x=0, y=3*

d) $z=y\sqrt{x}-y^2-x+6y$ j: $z_{\min}=12$, *bunda x= y=4*

e) $z=x^2+y^2-2x-4\sqrt{xy}-2y+8$ j: $z_{\min}=0$, *bunda x= y=2 bo'lganda*

2. $z=\frac{1}{x}+\frac{1}{y}$ funksiyag *x+y=2* shart bo'yicha ekstremumini toping.

J: $z_{\min}=2$, *x=y=1*

3. $z=xy-y^2+3x+4y$ funksiyaning *x=0, y=0. x+y-1=0* chiziqlar bilan chegaralangan sohadagi eng katta va eng kichik qiymatlarini toping.

j: $z_{eng.kat.}=z\left(\frac{1}{2};\frac{1}{2}\right)=3{,}5;\ z_{eng.kich.}=z(0;0)=0$

MUSTAQIL YECHISH UCHUN MASHQLAR.

Berilgan funksiyaning xususiy hosilasini toping.

1.1. $z=2(x+y)-x^2-y^2$

1.2. $z=x(12-x-y)$

1.3. $z=(x-5)+y^2+1$

1.4. $z=x^2-xy+y^2+x-y+1$

1.5. $z=3(y+2)^2+x^2$

1.6. $z=x^2-xy+y^2+3x-6y+20$

1.7. $z=(x-2)^2+2y^2-10$

1.8. $z=3x^2+3y^2-9xy+10$

1.9. $z=1+6x-x^2+y^2-xy-y^2$

1.10. $z=xy-3x^2-2y^2$

1.11. $z=y\sqrt{x}-y^2-x+6y$

1.12. $z=2xy-5x^2-3y^2+2$

1.13. $z=2x^3+2y^3-6xy+5$

1.14. $z=y\sqrt{x}-2y^2-x+14y$

1.15. $z=(x-1)^2+2y^2$

1.16. $z=x^3+8y^3-6xy+1$

1.17. $z=x\sqrt{y}-x^2-y+6x+3$

1.18. $z=x^2+xy+y^2-6x-9y$

1.19. $z=x^3+y^2-6xy-39x+18y+20$

1.20. $z=x^2+xy+y^2-2x-y$

1.21. $z=2xy-3x^2-2y^2+10$

1.22. $z=2xy-2x^2-4y^2$

1.23. $z=6(x-y)-3x^2-3y^2$

1.24. $z=1-2x^2-xy-y^2-15x$

1.25. $z=x^2+y^2-xy+x+y$

1.26. $z=xy-x^2-y^2+9$

1.27. $x^2+y^2-2z-2y=0$

1.28. $z=4(x-y)-x^2-y^2$

1.29. $z=x^3+8y^3-6xy+5$

1.30. $z=xy(6-x-y)$

Berilgan funksiyaning xususiy hosilasini va xususiy differensialini toping.

1.31. $z=\sqrt{x^2+y^2-5}$

1.32. $z=tg(x^2+y^2)$

1.33. $z=\arccos(x+y)$

1.34. $z=ctg\sqrt{xy^3}$

1.35. $z=e^{-x^2+y^2}$

1.36. $z=3x+\dfrac{y}{2-x+y}$

1.37. $z=\sqrt{9-x^2-y^2}$

1.38. $z=\ln(3x^2-y^4)$

1.39. $z=\ln(x^2+y^2-3)$

1.40. $z=\arccos\dfrac{y}{x}$

1.41. $z=\sqrt{2x^2-y^2}$

1.42. $z=arcctg(xy^2)$

1.43. $z=\dfrac{4xy}{x-3y+1}$

1.44. $z=\cos\sqrt{x^2+y^2}$

1.45. $z=\dfrac{\sqrt{xy}}{x^2+y^2}$

1.46. $z=\sin\sqrt{x-y^3}$

1.47. $z = \arcsin\dfrac{x}{y}$

1.48. $z = tg\left(x^3y^4\right)$

1.49. $z = \ln\left(x^2 - y^2\right)$

1.50. $z = ctg(3x - 2y)$

1.51. $z = \dfrac{x^3y}{3 + x - y}$

1.52. $z = e^{2x^2 - y^2}$

1.53. $z = \arccos(x + 3y)$

1.54. $z = \ln\left(\sqrt{xy} - 1\right)$

1.55. $z = \arcsin(2x + y)$

1.56. $z = \arcsin\left(2x^3y\right)$

1.57. $z = \ln\left(9 - x^2 - y^2\right)$

1.58. $z = arctg\dfrac{x^2}{y^2}$

Berilgan *z=f(x;y)* murakkab funksiyaning ko‘rsatilgan xususiy hosilasini toping.

1.59. $z = e^{y-2x},\ y = \ln\sin t,\ x = \cos t.\ \dfrac{dz}{dt} = ?$

1.60. $z = \arccos(3x - y),\ y = 3t^2,\ x = 4t.\ \dfrac{dz}{dt} = ?$

1.61. $z = u^2\ln v,\ u = \dfrac{x}{y},\ v = 2x - 3y.\ \dfrac{dz}{dx} = ?$

1.62. $z = arcctgxy,\ y = e^{\cos^3 x}.\ \dfrac{dz}{dx} = ?$

1.63. $z = arctg\dfrac{y}{x},\ y = x^2.\ \dfrac{dz}{dx} = ?$

1.64. $z = e^{x-2y},\ y = \sin t,\ x = \cos t.\ \dfrac{dz}{dt} = ?$

1.65. $z = \ln\left(e^x - e^{-y}\right),\ y = t^2,\ x = t^3.\ \dfrac{dz}{dt} = ?$

1.66. $z = u^2\ln v,\ u = \dfrac{y}{x},\ v = x^2 + y^2.\ \dfrac{dz}{dy} = ?$

1.67. $z = x^y,\ y = \ln\sin x.\ \dfrac{dz}{dx} = ?$

1.68. $z = \ln\left(e^x + e^y\right),\ y = \dfrac{1}{3}x^3 + x.\ \dfrac{dz}{dx} = ?$

1.69. $z = \dfrac{x^2}{y+1},\ y = arctgt,\ x = 1 - 2t.\ \dfrac{dz}{dt} = ?$

1.70. $z = \sqrt{x + y^2 + 3},\ y = \ln t,\ x = t^2.\ \dfrac{dz}{dt} = ?$

1.71. $z = \dfrac{x^2 - y}{x^2 + y},\ y = 3x + 1.\ \dfrac{dz}{dx} = ?$

1.72. $z = \arcsin\dfrac{x^2}{y},\ y = \sin t,\ x = \cos t.\ \dfrac{dz}{dt} = ?$

1.73. $z = u^2 + v^2,\ u = x - y^2, v = x^2 + y.\ \dfrac{dz}{dx} = ?$

1.74. $z = arcctgxy,\ y = e^{2x}.\ \dfrac{dz}{dx} = ?$

1.75. $z = \arcsin\frac{x}{y},\ y = \sqrt{x^2+1},\ \frac{dz}{dx} = ?$

1.76. $z = x^2 e^y,\ y = \sin t,\ x = \cos t.\ \frac{dz}{dt} = ?$

1.77. $z = x^y,\ y = \ln t,\ x = e^t.\ \frac{dz}{dt} = ?$

1.78. $z = \ln\left(u^2 - v^2\right),\ u = xy, v = \frac{x}{y}.\ \frac{dz}{dy} = ?$

1.79. $z = \ln\left(e^x - e^y\right),\ y = x^3 + 1.\ \frac{dz}{dx} = ?$

1.80. $z = e^{y-2x},\ y = \sin t,\ x = t^4.\ \frac{dz}{dt} = ?$

1.81. $z = \ln\left(e^{-x} + e^y\right),\ y = t^2,\ x = t^3.\ \frac{dz}{dt} = ?$

1.82. $z = \arcsin\frac{x}{y},\ y = \sin t,\ x = \cos t.\ \frac{dz}{dt} = ?$

1.83. $z = u^2 \ln v,\ y = \frac{x}{y},\ v = 3y - 2x.\ \frac{dz}{dy} = ?$

1.84. $z = xy,\ y = \sin x.\ \frac{dz}{dx} = ?$

1.85. $z = \arccos\frac{2x}{y},\ y = \sin t,\ x = \cos t.\ \frac{dz}{dt} = ?$

1.86. $z = \frac{x^2}{y+1},\ y = arctgt,\ x = 1 - 2t.\ \frac{dz}{dt} = ?$

Oshkormas ko‘rinishda berilgan *z(x;y)* funksiyaning hususiy hosilalarini toping.

1.87. $z^2 = xy - z + x^2 - 4$

1.88. $x^2 - xy - xz + y^2 - 3x = 11$

1.89. $x^2 + y^2 - 2z - 2y = 0$

1.90. $2x^2 + 2y^2 + z^2 - 8xz - z + 6 = 0$

1.91. $\ln z = x + 2y - z + \ln 3$

1.92. $z^3 - 3xyz - z = 0$

1.93. $x^2 + y^2 + z^2 - 2xy - 2xz - 2yz = 17$

1.94. $x^2 + 2y^2 + 3z^2 = 59$

1.95. $\sqrt{x^2 + y^2} + z^2 - 3z = 3$

1.96. $x^2 - y^2 - z^2 - 4y - 6z + 2x - 4 + 12 = 0$

1.97. $x^2 + y^2 + z^2 + 6z - 4x + 9 = 0$

1.98. $x^2 + y^2 - xz - yz = 0$

1.99. $x^3 + 2y^3 + z^3 - 3xyz - 2y = 0$

1.100. $e^x - xyz - x + 1 = 0$

Berilgan funksiyaning ikkinchi tartibli xususiy hosilalarini toping va

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ ekanini tekshiring.

2.1. $z = x \cdot e^{\frac{y}{x}}$

2.2. $z = \ln\left(x + e^{-y}\right)$

2.3. $z = arctg\frac{x}{y}$

2.4. $z = e^{xy}$

2.5. $z = e^{-x-3y} \cdot \sin(x+3y)$

2.6. $z = \frac{\sin(x-y)}{x}$

2.7. $z = e^{\frac{x}{y}}$

2.8. $z = e^{-\frac{x^2+y^2}{2}}$

2.9. $z = \sqrt{\frac{x}{y}}$

2.10. $z = tg\,\frac{x}{y}$

2.11. $z = ctg(x+y)$

2.12. $z = \sin(x^2 - y)$

2.13. $z = \arcsin(x-y)$

2.14. $z = \ln(3x^2 - 2y^2)$

2.15. $z = tgxy^2$

2.16. $z = \ln(3xy - 3)$

2.17. $z = arc(x-4y)$

2.18. $z = \ln(x^2 + y^2 - 3)$

2.19. $z = arcctg(x-4y)$

2.20. $z = \sin\sqrt{xy}$

2.21. $z = \arccos(x-5y)$

2.22. $z = e^{x^2-y^2}$

2.23. $z = \arcsin(4x+y)$

2.24. $z = \ln(4x^2 - 5y^2)$

2.25. $z = arctg(2x-y)$

2.26. $z = \cos(x^2y^2 - 3)$

2.27. $z = e^{\sqrt{x+y}}$

2.28. $z = ctg\,\frac{y}{x}$

2.29. $z = tg\sqrt{xy}$

2.30. $z = e^{2x^2+y^2}$

Quyidagi chiziqlar bilan chegaralangan yopiq sohada *z=f(x;y)* funksiyaning eng kichik va eng kata qiymatini toping:

2.31. $z = x^2 - y^2 - x + y$
$x = 0,\ x = 2,\ y = 0,\ y = 1.$

2.32. $z = -3x^2 + 2y^2 + 12x - 4y$
$x = 0,\ y = 0,\ 3x + 4y = 12.$

2.33. $z = x^2 + 2xy - 4x + 8y$
$x = 0, \ y = 0, 5x - 3y + 45 = 0.$

2.34. $z = x^2 2xy - y^2 - 4x$
$x = 3, \ y = 0, \ y = x + 1.$

2.35. $z = 2xy - 3x^2 - 2y^2 + 5$
$x = -1, \ y = -1, x + y = 5.$

2.36. $z = x^2 - xy + 5$
$x^2 + y = 1, \ y = 0,.$

2.37. $z = 3y - 2x - xy$
$x = 0, \ y = 0, 3x - 4y = 12.$

2.38. $z = x^2 - 4xy + y^2 + 6y$
$x = 4, \ y = x, \ y = 0.$

2.39. $z = x^2 - 2xy + \frac{5}{2}y^2 - 2x$
$x = 0, \ x = 2, \ y = 0, \ y = 2.$

2.40. $z = 3xy - 6x^2 - 6y^2 + 15x$
$x = 0, \ x = 2, \ y = 0, \ y = 1.$

2.41. $z = x^2 + 6xy - x + 3y$
$x = 0, \ x = 3, \ y = 0, \ y = 3.$

2.42. $z = 5xy - y^2$
$x = 4, \ y^2 = 5x + 5.$

2.43. $z = x^2 - 2xy - 10$
$y = 0, \ y = x^2 - 4.$

2.44. $z = x^2 y$
$x = 4 \ , y = 1 - x.$

2.45. $z = x^2 - 2xy - y^2 + 4x + 1$
$x = -3, \ x + y + 1 = 0, \ y = 0.$

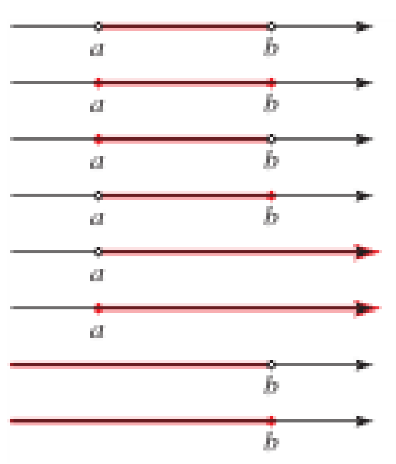
2.46. $z = x^2 + 2xy + 4x - y^2$
$x = 0, \ \ y = 0, x + y + 2 = 0.$

2.47. $z = xy - 2x - y$
$x = 0, \ x = 3, \ y = 0, \ y = 4.$

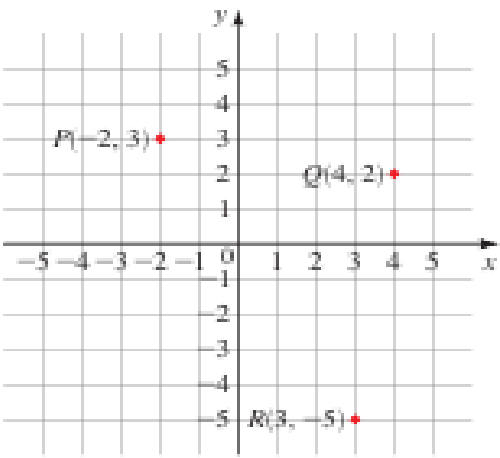
2.48. $z = x^2 + 2xy + 4x - y^2$
$x = 0, \ \ y = 0, x + y + 2 = 0.$

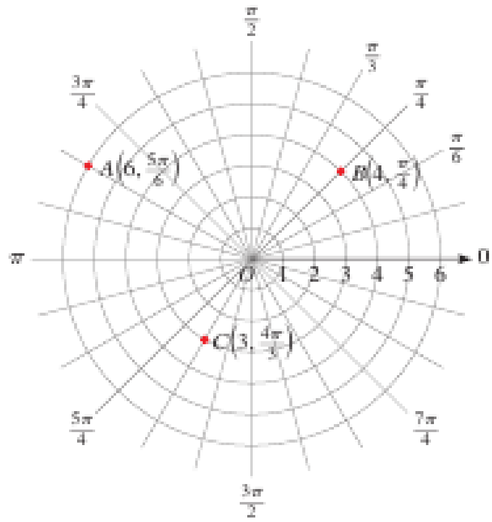
Sonli to‘plamlarning ba’zi turlari

(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	



Dekart va qutb koordinatalarida nuqtalarning o‘rni





Ba’zi algebraik formulalar

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

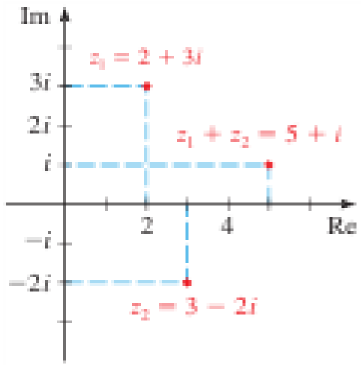
$$\log_a a^x = x \qquad a^{\log_a x} = x$$

$$\log_a 1 = 0 \qquad \log_a a = 1$$

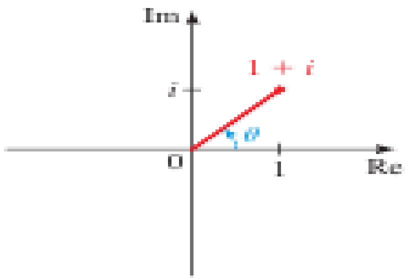
$$\log x = \log_{10} x \qquad \ln x = \log_e x$$

$$\log_a xy = \log_a x + \log_a y \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^b = b\log_a x \qquad \log_b x = \frac{\log_a x}{\log_a b}$$



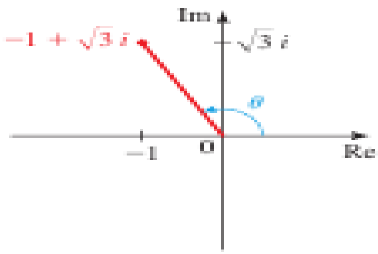
$$\begin{aligned} z_1 z_2 &= r_1 r_2(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) \\ &= r_1 r_2[\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)] \\ &= r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)] \end{aligned}$$



$$\begin{aligned} \frac{z_1}{z_2} &= \frac{2}{5}\left[\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)\right] \\ &= \frac{2}{5}\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right] \\ &= \frac{2}{5}\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right) \end{aligned}$$

$$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

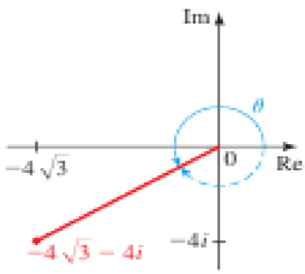


$$\begin{aligned} \sqrt[n]{z} &= [r(\cos\theta + i\sin\theta)]^{1/n} \\ &= r^{1/n}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right) \end{aligned}$$

$$-1 + \sqrt{3}i = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -2i$$

$$2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = \sqrt{3} - i$$



$$2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3} + i$$

$$2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$$

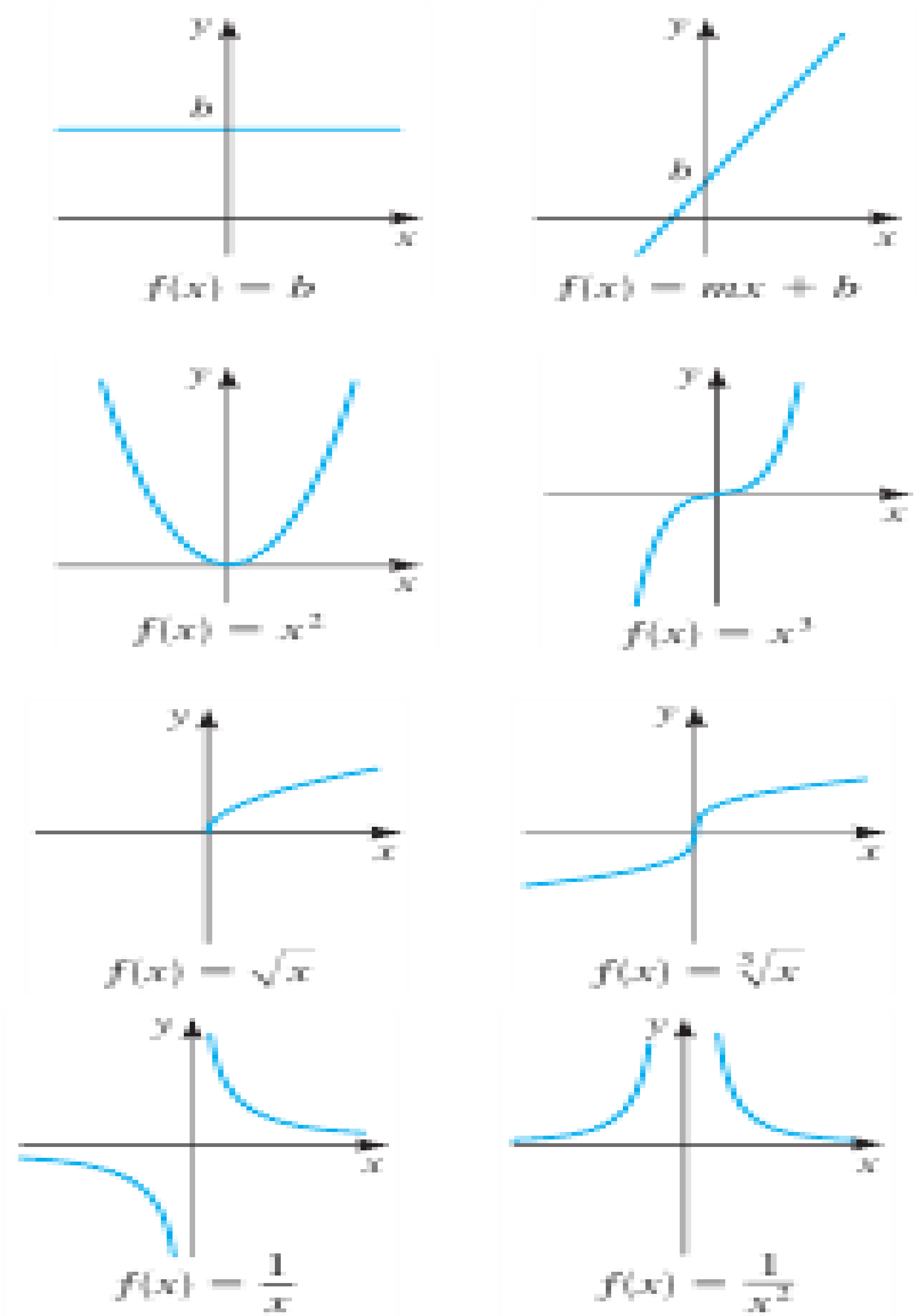
$$2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = -\sqrt{3} + i$$

$$2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = -\sqrt{3} - i$$

$$-4\sqrt{3} - 4i = 8\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$3 + 4i = 5\left[\cos\left(\tan^{-1}\tfrac{4}{3}\right) + i\sin\left(\tan^{-1}\tfrac{4}{3}\right)\right]$$

Chiziqli va chiziqsiz funksiyalar



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