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«Karxona servisi » yo’nalishining 4 kurs talabasi
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« Dinamik tizimning echimining qiymatini kompyuterda modellashtirish»
mavzusidagi

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Mavzuning aktualligi: Lyapunovning to’g’ri usuli yoki Lyapunov funkciyasi usullarning biri bo’lib hisoblanadi. XIX-asrning oxiridagi buyuk rus olimlarining ko’rsatmalari yangilanish davriga ega bo’ldi. Agarda uning yordami bilan olingan birinchi natija oddiy differencial tenglamalar tizimining turg’unlik yoki turg’un emaslik shartlarining mavjudligi nazariy xarakterga ega bo’lsa, unda kelajakda uning rivojlanishi ikki yo’nalishda bo’ladi.

Xalanay A., Bromberg P.V., Bellman R., Ko’ncevich V.M., Chexovogo Yu.N., Martinok D.I., larning ishlarida ajiralmali tenglamalar tizimi qaraladi. O`zining xarakteri bo’yicha ular differencial hisobiga yaqin bo’lib va ularga tabiyiy umumlashgan Lyapunov funkciyalar metodi ishlatiladi.

Zubov V.I, Krasovskiy N.N., Korobov V.I. larning ishlarida dinamik tizimni boshqarish masalasi uchun Lyapunov funkciya metodin foydalanish g’oyalari keng rivojlandi.

Oxirgi vaqtda Lyapunovning optimal funkciyalar metodi keng rivojlanishga yo’l oldi. Shuning bilan birga bu metod konkret texnik xarakteristik tizimni olishga imkoniyot beradi.

Shu vaqtgacha matematikaning ko’bchilik sohalarida ajiralmali tenglamalar tizimi har xil shakldа o’rganildi.

Texnik ilmlarning rivojlanish tendenciyalariga bog’liqli bu soha bo’yicha har xil yangi muammoalar kelib chiqdi. Elektron mashinalarin diskret boshqarishning impulsli tizimlarin ifodalarni modellashtirishda ajiralmali tizimlar keng miqyosda foydalaniladi.

Ajiralmali tizimni ifodaluvchi matematik apparat differenciallash apparatiga o’xshash bo’lib keladi. Asosan Cipkin Ya.Z., Barbashin E.A., Veksler D., Bellman. R h.t.b. larning ishlarida diskret tizimning turg’unlik nazariyasi to’liq o’rganildi. Oxirgi vaqtlarda hisoblash mashinalari yordamida ajiralmali tizimning turg’unligi haqidagi muammoga qiziqish ko’chaydi.

Ishning maqsadi va vazifalari: Chiziqli ayirmali tizimning turgunligin Lyapunovning ikkinchi usulidan foydalanib echamiz. Chiziqli ayirmali tizim uchun shu funkciya yordamida baxo olamiz. Yani chiziqli ayirmali tizimning turgunligin aniqlash matricalik tenglamani echishga olib keladi.

Aniqlash obekti va pridmeti: Optimallashtirish usullari,differencial tenglamalar.

Ishning tuzilishi: Birinchi paragrafda chiziqli tizimlar qaraladi. Bu paragrafda doimiy chiziqli diferencial tenglamalar tizimi ko’riladi. Va kvadratli shaklning tengsizligiga asoslanib tizimning echimining bahosi olindi.

Ikkinchi paragrafda ajiralmali tizimning echimining integral bahosini optimallashtirish masalasi qaraladi.

Uchunchi paragrafda Lyapunov funkciyasining algoritmlari ko’riladi va tushunchalar beriladi.

To’rtinchi paragrafda Lyapunovning optimal funkciyasin topish algoritmi ko’riladi.

Beshinchi paragrafda vaqt bo’yicha o’tish jarayonining bahosi va uni optimallashtirish masalasi ko’rib chiqiladi

Oltinchi paragrafda ajiralmali tizimlarni optimallashtirishda gradient usuldan foydalanib echish masalasi qaraladi.

Ettinchi paragrafda integral sifat kriteriyasi va uni optimallashtirish masalasi ko’riladi.

§1. Chiziqli tizimlar.

Ajiralmali tenglamalar tizimi umumiy shaklda qo'ydagi ko'rinishga ega:

$$x(k+1) = f(k, x(k)), \quad k = 0,1,2\ldots \quad x(k) \in R^n$$

Tarif-1 $x(k) = 0, \ (k = 0,1,2\ldots)$ echimi Lyapunov bo'yicha turg'un deb nomlanadi,agarda hohlagan $\varepsilon > 0$ soni uchun, shunday $\delta(\varepsilon) > 0$ soni mavjud bo'lib,

$$\|x(0)\| < \delta(\varepsilon)$$

tengsizligi bajaruvchining, hohlagan $x(k)$-echim (ajiralmali tenglamalar tizimin, echimi) uchun

$$\|x(k)\| < \varepsilon$$

tengsizligi bajarilsa.

Tarif-2. $x(k) \equiv 0$ echimi asimptotik turg'un deb nomlanadi, agarda u Lyapunov bo'yicha turg'un bo'lsa, va

$$\lim_{k \to \infty} \|x(k)\| = 0$$

bo'lsa.

Yakka holatda, tizimning echimining, turg'unligin aniqlashda A.M.Lyapunovning, ikkinchi metodi foydalaniladi, va ayrim musbat aniqlangan $V(x)$-funkciyasin topish talab qilinadi.

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$$

Bu yuqorida ayirma musbat aniqlangan bo'ladi, unda tizimning nollik echimi asimptotik turg'un bo'ladi. Doimiy koefficientlar chiziqli ajiralmali tizimin qaraymiz:

$$x(k+1) = Ax(k), \ x(k) \in R^n, \ k = 1,2,.... \qquad (7^1)$$

Bizga ma’lum, (7^1) tizimning asimptotik turg’un bo’lishi uchun, A matricasining xarakteristik sonlari birlamchi doiraning ichida yotishi zarur va etarli ya’ni A matricasining hususiy sonlari uchun)

$$|\lambda_i(A)| < 1, \quad \left(i = \overline{1,n}\right)$$

tengsizligi bajarilishi zarur va etarli.

Chiziqli tizim uchun, musbat aniqlangan

$$V(x) = x^T Hx$$

Kvadratli shakl olinadi va uning birinchi ayirmasi (7^1)-tizimdan foydalanib qo’ydagi ko’rinishda bo’ladi:

$$\Delta V(x(k)) = x^T(k)\left[A^T HA - H\right]x(k)$$

Agarda bu ayirma manfiy aniqlangan

$$\Delta V(x(k)) = x^T(k)Cx(k)$$

Kvadratli shakl teng bo’lsa, unda (7^1)-tizim asimptotik turg’un deyiladi.

Shunday qilib (7^1)-chiziqli ajiralmali tizimning turg’unligin aniqlash masalasi, qo’ydagi matricali tenglamani echishga olib keladi:

$$A^T HA - H = C \qquad (8)$$

Bu erda H va C matricalari musbat aniqlangan bo’lishi kerak. (8)-matricali tenglamaning echimining mavjudligining va birdan birligining zarur va etarli sharti qo’ydagi ko’rinishga ega

$$\lambda_i(A) \cdot \lambda_i \neq 1, \quad 1 \le i \le j \le n$$

Tizimning asimptotik turg’unligi haqida Lyapunovning ikkinchi metodi, tizimning ayrim sifatli xarakteristik echimiga olib keladi.

Agarda H va C matricalari musbat aniqlangan bo’lsa, unda qo’ydagi $V(x)$-kvadratli shaklni baholashlar o’rinli:

$$\begin{aligned}&\lambda_{\min}(H)\|x\|^2 \le V(x) \le \lambda_{\min}(H)\|x\|^2\\&\Delta V(x) \le -\lambda_{\min}(C)\|x\|^2\end{aligned} \qquad (9)$$

Bunda $\lambda_{\min}(H), \lambda_{\max}(H),\ H$ matricasining mos eng kichik va eng katta hususiy soni.

Normani qo'ydagicha aniqlaymiz:

$$\|x\| = \left\{\sum_{i=1}^{n} x_i^2\right\}^{1/2}$$

matricasining normasin

$$\|D\| = \left\{\lambda_{\max}\left(D^T D\right)\right\}^{1/2}$$

bundan

$$V(x(k)) - V(k\ ) \le -\frac{\lambda_{\min}(C)}{\lambda_{\max}(H)} V(x(k)).$$

$$V(x(k+1)) \le \left[1 - \frac{\lambda_{\min}(c)}{\lambda_{\max}(H)}\right]^k V(x(0))$$

Hosil bo'lgan tengsizlikni echib qo'ydagi tengsizlikga ega bo'lamiz:

$$V(x(k)) \le \left[1 - \frac{\lambda_{\min}(c)}{\lambda_{\max}(H)}\right]^k V(x(0))$$

(9) Kvadratli shaklning tengsizligiga asoslanib (7^1) tizimning echimi $x(k)$ uchun bahoni qo'ydagicha ko'rinishda olamiz:

$$\|x(k\ )\| \le \sqrt{\frac{\lambda_{\max}(H)}{\lambda_{\max}(H)}} \left[1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(C)}\right]^{k/2} \|x(0)\|$$

Boshlang'ich qo'zdirish bo'yicha optimal Lyapunov funkciyasining mavjud bo'lish sharti.

Biz $\varphi_1(H)$ funkciyasining optimizaciyaviy masalasin qaraymiz, $H \in W_H$, ya’ni boshlang’ich qo’zdirishning optimal bahosin topish masalasin qaraymiz.

Odatda $1 \le \varphi_1(H) < \infty$, unda Lyapunov funkciyasining eng yaxshisi, yoki $\lambda_{\max}(H)/\lambda_{\min}(H_1) = 1$ bo’ladigan funkciya $V(x) = x^T H_1 x$ ko’rinishida bo’ladi.

Lemma-1. Agarda A-asimptotik turg’un matrica bo’lsa, unda Hi C matricalarining hususiy ma’nolari uchun (2) chi tenglamaga kiruvchi ushbu tengsizlik o’rinli

$$0 < \lambda_{\min}(C) < \lambda_{\max}(H)$$

Isbotlash. (2) chi tenglamadan qo’ydagiga ega bo’lamiz

$$0 < \lambda_{\min}(C) = \lambda_{\min}\left(H - A^T HA\right) \le \lambda_{\max}(H) - \lambda_{\min}\left(A^T HA\right)$$

Agarda H-musbat aniqlangan bo’lsa va A- asimptotik turg’un, unda $A^T HA$-musbat aniqlangan bo’ladi va bundan

$$0 < 1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)} < 1$$

Mayli x$_0$ kichik bo’lishi shart emas boshlang’ich qo’zdirish ya’ni $\|x_k\| \to 0$ asimptotik, unda konkret tizimni aniqlashda $\|x_k\| < \delta$ bo’lishi etarli, bunda δ-etarli kichik fikserlangan chama. Endi $N(\varepsilon)$ sonin topamiz, $\|x_k\| < \delta$ bo’lganday, faqatgina $k > N(\varepsilon)$ uchun. Bu sonni vaqt bo’yicha o’tish jarayoni deb nomlaymiz. Olingan bahoni foydalanib qo’ydagiga ega bo’lamiz

$$N(H, x_0, \varepsilon) \le \left[\frac{\ell n\left(\frac{\varepsilon^2}{\|X_0\|^2} \cdot \frac{\lambda_{\min}(H)}{\lambda_{\max}(H)} \right)}{\ell n\left(1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)} \right)} \right] + 1$$

Bu erda $[\cdot]$ sonning butun bo’lagining funkciyasi. Vaqt bo’yicha o’tish jarayonin aniq baholash qiziquvchilik uyg’otadi, ya’ni H matricasin topish, N minimal bo’ladi.

Tarif. $\vartheta_0(x) = x^T H_0 x$ Lyapunov funkciyasi,

$$H_0 = \arg \inf_{H \in W_H} \{\varphi(H)\} \quad (*)$$

bunda

$$\varphi(H) = \ell n\left(\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}\right) / \ell n\left(1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)}\right)$$

W_H-bo'lsa, H simmetrik matricaning ko'bligi, ya'ni $A^T H - HA -$manfiy aniqlangan, vaqt bo'yicha o'tish jarayonining bahosi uchun optimal deyiladi.

Shunday qilib, (1) chi tizimning vaqt bo'yicha o'tish jarayonining optimalliligin hisoblash ushbu $\vartheta(x) = x^T Hx$ kvadratli funkciya yordamida, $\varphi(H)$ funkciyasin optimallashtirish masalasiga olib keladi, W_H -ko'bligida aniqlangan.

Lemma 2. Mayli W_H yoyiq bo'lsin.

Isbotlash. Musbat aniqlangan matricalar ko'bligi yoyiq bo'lsin, unda cheklashlar chiziqli. Shuning uchun W_H ko'bligi yoyiq. Bundan boshqa, ixtiyoriy d uchun: $0 < d < +\infty,\ H \in W_H$ dan kelib chiqadi, ya'ni $dH \in W_H$. Shunday qilib, W_H-ochiq yoyiq konus (ya'ni yon tarafga ega emas).

Optimallashtirish masalasin echganda funkciyasining aniqlanish sohasining yoyiqligi talab qilinadi va funkciyaning o'zining yoyiqligi. Bu talablarning bajarilishining natijasida echimning mavjudligi va birdan birligi kafolatlanadi. Sonli hisoblashlarda gradient usullar foydalaniladi. Bizning holatimizda $\varphi(H)$ funkciyasi yoyiq bo'lib hisoblanadi.

Endi (*) masalasining echimining mavjudligi va birdan birligin qaraymiz. Shuni ko'rish qiyin emas, ya'ni

$$0 < \lambda_{\min}(H) / \lambda_{\max}(H) \le 1,\ 0 < 1 - \lambda_{\min}(C) / \lambda_{\max}(H) < 1$$

Shuning uchun $\varphi(H) \ge 0$ va eng yaxshi optimal Lyapunov funkciyasi bo'lib, agarda $\lambda_{\min}(H) / \lambda_{ma}(H) = 1,$ ya'ni $H_0 = \lambda E,\ \lambda > 0$.

Teorema. $\vartheta_0(x) = x^T H_0^x \in W_H$ Lyapunov funkciyasi vaqt bo'yicha o'tish jarayonining bahosi uchun optimal bo'lgan, $\varphi(H_0) = 0$ bo'ladigan, shunda faqatgina shunda mavjud, agarda $E - A^T A$ musbat aniqlangan matrica bo'lsa. Bu holatda $H_0 = \lambda E,\ \lambda > 0,\ \varphi(H_0) = 0$.

Zarurliligi. Mayli $\vartheta_0(x) = x^T H_0 x$ optimal funkciyasi mavjud bo'lsin, $H_0 \in W_H$ va

$$\inf_{H \in H_H} \{\varphi(H)\} = \varphi(H_0) = d\,;$$

shart bo'yicha H_0, W_H sohasining ichida bo'ladi, shuning uchun, shuni hisobga olib, W_H-konus, $\varphi(H)$ funkciyasi H bo'yicha bir xil, H_0 nurda joylashadi, konusning tepasi orqali o'tishi. Shuning uchun $H_0/\|H_0\|$ deb olishga bo'ladi va

$$\inf_{H \in H_H} \{\varphi(H)\} = \min_{H \in W_H} \{\varphi(H)\} = 2$$

Ya'ni $0 \le \varphi(H) < +\infty$, bo'lsa, unda $d = 0$. Bundan kelib chiqadi, ya'ni $H_0 = E$. Shuning uchun $E - A^T A = H_0 - A = C_0$ matrica musbat aniqlangan.

Etarliligi. Agarda $E - A^T A$ musbat aniqlangan bo'lsa, unda Lyapunov funkciyasi sifatida $\vartheta_0(x) = x^T H_0 x$ olib $\varphi(E) = 0$ ega bo'lamiz. Shunday qilib $\varphi(H) \ge 0$, unda tuzilgan Lyapunov funkciyasi optimal bo'lib hisoblanadi.

Mayli $E - A^T A$ musbat aniqlangan bo'lmasin. Bu holatda $\vartheta_0(x) = x^T H_0 x$ optimal Lyapunov funkciyasi mavjud bo'ladi, lekin $\varphi(H_0) > 0$

TEOREMA -1(8)-tenglama $\lambda_{\max}(H_1)/\lambda_{\min}(H_1) = 1$ bo'ladigan echimi $H_1 \in W_H$ matricaga ega bo'ladi, faqatgina shunda, agarda $(E - A^T A)$ matricasi musbat aniqlangan bo'lsa, bunda E-birlik matrica.

Isbotlash: Zarurliligi: Mayli $\lambda_{\max}(H_1)/\lambda_{\min}(H_1)$ bo'ladigan $H_1 \in W_H$ matricasi uchun C_1 matricasi mavjud bo'lsin. Bizga ma'lum $U^T H_1 U$-matricasi dioganalli tuzilishga ega bo'ladigan U-ortogonal matricasi mavjud bo'ladi.

$$\lambda_{\max}(H_1) = \lambda_{\min}(H_1) = \lambda > 0$$

Unda

$$U^T H_1 U = \lambda E$$

bundan

$$U^T A^T U U^T H_1 U U^T - U^T H_1 U = -U^T C_1 U$$

yoki

$$U^T A^T A U - E = -\frac{1}{\lambda} U^T C_1 U$$

shunday qilib, $E - A^T A = \frac{1}{\lambda} C_1$ musbat aniqlangan matrica bo'ladi.

Etarliligi: Mayli $(E - A^T A)$-musbat aniqlangan matrica bo'lsin, $C_1 = (E - A^T A)$ deb olib

$$H_1 = E, \qquad \lambda_{\max}(H_1)/\lambda_{\min}(H_1) = 1$$

Tengliligiga ega bo'lamiz.

TEOREMA-2 Boshlang'ich qo'zdirishning bahosi uchun optimal

$V(x)=x^T H_1 x, \quad H_1 \in W_H$ Lyapunov funkciyasi mavjud bo'ladi shunda, faqatgina shunda, agarda $E-A^T A$ matricasi musbat aniqlangan bo'lsa.
Bu holatda $H_1=\lambda E, \quad \lambda>0, \ \varphi_1(H_1)=1$

Isbotlash: Zarurliligi: Mayli optimal $V_1(x)$-funkciyasi mavjud bo'lsin, ya'ni (8)-tenglamaning qanoatlantiruvchi musbat aniqlangan $H_1 \in W_H$ -va C_1 matricalari mavjud bo'lsin.

$$\inf_{H\in H_H}\left\{\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}\right\}=\frac{\lambda_{\max}(H_1)}{\lambda_{\min}(H_1)}=a$$

$a=1$ ekanligin ko'rsatamiz. Mayli $a>1$bo'lsin
H_1 matricasin diagonallashtiruvchi U ortogonal matricasi mavjud bo'lsin.

$$U^T H_1 U=\begin{bmatrix} \lambda_1(H & 0 & \dots & 0 \\ 0 & \lambda_2(H) & \dots & 0 \\ -- & -- & -- & -- \\ 0 & 0 & \dots & \lambda_n(H) \end{bmatrix}=\Lambda_1$$

Mayli $\lambda_{\min}(H_1)=\lambda_{i2}(H_1)=\ldots=\lambda_{is}(H)$, $s<n$bo'lsin. Qo'ydagi matricani kiritamiz:

$$S=\begin{bmatrix} \mu_1 & 0 & \dots & 0 \\ 0 & \mu_2 & \dots & 0 \\ 0 & 0 & \dots & \mu_n \end{bmatrix}$$

Bu erda $\mu_{i1}=\mu_{i2}=\ldots=\mu_{is}, \quad \mu_j=0, \quad j\neq i, i_2,\ldots,i_s$
C_1-matricasi musbat aniqlangan, unda etarli darajada kichik bo'lgan δ uchun qo'ydagi matrica aniqlangan bo'ladi.

$$C_1^\delta=C_1-\delta\left[A^T USU^T A-USU^T\right]$$

$$A^T H_1^\delta A-H_1^\delta=-C$$

Tenglamaning olingan H_1^δ matricasi va H_1teng bog'langan:

$$H_1=H_1^\delta-\delta USU^T$$

Shuning uchun

$$\frac{\lambda_{\max}\left(H_1^{\delta}\right)}{\lambda_{\min}\left(H_1^{\delta}\right)} = \frac{\lambda_{\max}\left(H_1 + \delta USU^T\right)}{\lambda_{\min}\left(H_1 + \delta USU^T\right)} = \frac{\lambda_{\max}\left[U^T\left(H_1 + \delta USU^T\right)U\right]}{\lambda_{\min}\left[U^T\left(H_1 + \delta USU^T\right)U\right]}$$

$$= \frac{\lambda_{\max}\left[U^T H_2 U + \delta\right]}{\lambda_{\min}\left[U^T H_1 U + \delta\right]} = \frac{\lambda_{\max}\left[U^T H_2 U\right]}{\lambda_{\min}\left[U^T H_1 U\right] + \delta} = \frac{\lambda_{\max}(H_1)}{\lambda_{\min}\left(H_1\right) + \delta} < a$$

Shunday qilib, $\varphi_1(H_1^{\delta}) < a$ tengsizligi bajariladigan H_1^{δ} matricasin tuzdik. Bu bizning taxminimizga qarama-qarshi keladi. Demak $a > 1$ tengsizligi o'rinli emas, shunday qilib $\varphi_1(H_1) = 1$ va teorema -1 dan $E - A^T A$ matricasining musbat ekanligi qo'llab chiqadi. Etarlilik sharti teorema-1 dan chiqadi.

TEOREMA-3. C_1 matricasining manfiy emas aniqlanish, ya'ni

$$\lambda_{\min}(C_1) = 0, \;\; H_1 \in \partial W_H$$

funkciya optimal bo'lishining sharti bo'lib hisoblanadi

Isbotlash: Mayli, aksincha C matricasi musbat aniqlangan bo'lsin, ya'ni

$$\lambda_{\min}(C_1) > 0$$

tenglamani qo'ydagi ko'rinishga keltiramiz:

$$A^T\left(H_1 + \delta E\right)A + \left(H_1 + \delta E\right) = -\left[C_1 + \delta\left(E - A^T A\right)\right]$$

Simmetrik matricaning hususiy sonlarining hossasidan foydalanib qo'ydagi tengsizlikga ega bo'lamiz:

$$\lambda_{\min}\left[C_1 + \delta\left(E - A^T A\right)\right] \geq \lambda_{\min}(C_1) + \delta\lambda_{\min}(E - A^T A)$$

Agarda $0 < \delta < \lambda_{\min}(C_1)/\left[\lambda_{\min}(E - A^T A)\right]$ deb olsak, unda $C_1\delta(E - A^T A)$ matricasida musbat aniqlangan bo'ladi, aksincha $H_1 + \delta E$ uchun qo'ydagi o'rinli:

$$\varphi_1(H_1 + \delta E) = \frac{\lambda_{\max}(H_1 + \delta E)}{\lambda_{\min}(H_1 + \delta E)} = \frac{\lambda_{\max}(H_1) + \delta}{\lambda_{\min}(H_1) + \delta} < \frac{\lambda_{\max}(H_1)}{\lambda_{\min}(H_1)} = \varphi(H_1)$$

Shunday qilib , agarda $\lambda_{\min}(C_1) > 0$ bo'lsa, unda

$$\varphi_1(H_1+\delta E)<\varphi_1(H_1)$$

tengsizligi bajariladigan $H+\delta E$, bunda

$$\delta=\lambda_{\min}(C_1)/\left|\lambda_{\min}(E-A^T A)\right|$$

matricasin topishga bo’ladi.

Demak $V(x)=x^T H_1 x$ funkciyasining optimal bo’lishining zarur sharti

$$\lambda_{\min}(C_1)=0,\ \ H_2\in\partial W_H$$

yoki tengligining bajarilishi bo’lib hisoblanadi.

Boshlang’ich qo’zdirishni baholashning optimizaciyaviy algoritmi.

Agarda $(E-A^T A)$ matricasi manfiy emas aniqlangan bo’lsa, unda boshlang’ich qo’zdirishni optimal baholuvchi Lyapunov funkciyasi qo’ydagi ko’rinishga ega bo’ladi:

$$V_1(X)=x^T x$$

ya’ni $H_1=E$

Agarda bu shart bajarilmasa, unda optimal funkciya $\varphi_1(H_1)>1$ uchun mavjud bo’ladi va $H_1\in\partial W_H$, ya’ni

$$\lambda_{\min}(E-A^T A)>0$$

H_1 matricasin topish masalasi, manfiy emas va yoyiq emas maqsad funkciyaga ega murakkab matematik dasturlash masalasiga olib keladi.

Optimal Lyapunov funkciyasin topishning ikki algoritmin ko’rib o’tamiz.

§2 Ajiralmali tizimning echimining integral bahosini optimallashtirish.

Chiziqli ajiralmali stacionar tizimni qaraymiz, A asimptotik turg’un matrica bilan. Endi x=x(k),k=1,2, … echimni topish uchun kvadratli ko’rinishdagi Lyapunov funkciyasidan foydalanamiz va matricali tenglamaga ega bo’lamiz. Shu Lyapunov funkciyasi yordamida bahoga ega bo’lamiz.

Aniqlama. V (x)=x^{T}Hx Lyapunov funkciyasi

$$H = arg\square\ \text{EMBED Equation.3}\ \square\square\square\{\varphi(H)\},\ \varphi(H) = \frac{\sqrt{\frac{\lambda_{\max(H)}}{\lambda_{\min(H)}}}}{\sqrt{\frac{1-\lambda_{\min(C)}}{\lambda_{\max(H)}}}}$$

bunda W_H - musbat aniqlangan matricalarning ko’bligi, manfiy aniqlangan bo’ladigan, integral ma’noda optimal deyiladi.

Endi eng yaxshi integral bahoni olishni qaraymiz, A matricasining ko’rinishiga bog’liq.

Lemma. Agarda tizim asimptotik turg’un va E-A^{T}A musbat aniqlangan matrica bo’lsa, unda

Isbotlash. Endi S=E-A^{T}A deb olib, N=E ga ega bo’lamiz va

Simmetrik matricaning hossasidan foydalanib, qo’ydagiga ega bo’lamiz

Lemma. Agarda berilgan tizim asimptotik turg'un bo'lsa va A normal matrica bo'lsa, yoki $A^TA=AA^T$, unda $E-A^TA$ matricasi musbat aniqlangan bo'ladi.

Isbotlash. $E-A^TA=N$ deb belgilab olamiz va uni Lyapunov tenglamasiga qo'yib, qo'ydagiga ega bo'lamiz

$$AN-H=$$

Ixtiyoriy simmetrik matricaning kvadrati musbat aniqlangan matrica bo'ladi, unda S musbat aniqlangan matrica bo'ladi. Shunday qilib, Lyapunov tenglamasi $S=(E-A^TA)^2$ musbat aniqlangan matricasi uchun, musbat aniqlangan matricasining echimiga ega bo'ladi, unda $N=E-A^TA$ bo'lsa, musbat aniqlangan matrica bo'ladi.

Soldar. Agarda berilgan tizim asimptotik turg'un va A-normal bo'lsa, unda ushbu baho o'rinli.

Haqiyqatdan ham, agarda A-normal bo'lsa, unda $E-A^TA$-musbat aniqlangan va qo'ydagi tengsizlik o'rinli

Shunday qilib, A-normal bo'lsa, unda

$$\lambda_{\max\left(A^{T}A\right)}=\left[\frac{max}{i=1,n}\left|\lambda_i(A)\right|\right]^{\frac{1}{2}}$$

Bundan

Aytilgan lemmalarni hisobga olib, uch holatni qaraymiz.

1) A-matricasi normal. Shuni ko'rsatamiz, ya'ni bu holatda eng aniq baho bo'lib (*) cha hisoblanadi. Hamma vaqtda x=x(k) yakka echim mavjud bo'ladi, ya'ni tengsizlik tenglikka o'tadigan

Bu holatda Lyapunov funkciyasi, berilgan boshlangich ma'lumotlari bo'yicha optimal bo'lgan va integral baho ma'nosida mos keladi.

2) E-A^{T}A matricasi musbat aniqlangan, unda A normal hisoblanmaydi. Bu holatda Lyapunov funkciyasi, boshlangich ma'lumotlari bo'yicha optimal bo'lgan va integral baho ma'noda mos kelmasligi mumkin.

3) E-A^{T}A matricasi musbat aniqlangan bo'lmasin. Bu holatda N$_1$ va S$_1$ matricalari, berilgan baho ma'lumotlari uchun optimal bo'lgan, integral bahoni cheksizlikka aylantiradi.

TEOREMA. Agarda berilgan tizim asimptotik turg'un bo'lsa va A normal

matrica bo'lsa, unda ixtiyoriy $\delta > 0$ uchun x= yakka echim

mavjud bo'ladi, , ya'ni

Isbotlash. Ya’ni A normal bo’lsa, unda U ortaganal matricasi mavjud bo’ladi, ya’ni

U^{-}
^{1}AU=
$(\propto_1 \beta_1 \cdots 0 \quad 0 \quad 0 \quad \cdots \quad 0@ - \beta_1 \alpha_1 \cdots 0 \quad 0 \quad 0 \quad \cdots \quad 0@ \quad \cdot \quad \cdot \cdots 0 \quad 0 \quad 0 \quad \cdots \quad 0@ \quad 0 \quad 0$

Bunda $\lambda_p = \alpha_p \pm i\beta_p \; ; \; \lambda_q = \gamma_K, P = \overline{1,K,}$ q=$\overline{1,s,}$ $2l + s = n -$ bo’lsa A matricaning hususiy ma’nolari. Endi x=Uy o’zgarishlar kiritib chiziqli tizimga ega bo’lamiz.

y=$(k+1) = \Lambda \quad y(k).$ $(**)$

1) Mayli $\frac{max}{i=1,n} |\lambda_i(A)| = \gamma_j$ bo’lsin. Shunda berilgan tizimning yakka echimi sifatida $\bar{x} = (k)U \; (0,\cdots,0,\gamma_j^K,0,\cdots,0)^T \cdot \delta$ olib, qo’ydagiga ega bo’lamiz, ya’ni

2) Mayli $\overset{max}{i=1,n} |\lambda_i(A)| = \sqrt{\alpha_j^2 + \beta_j^2}$ shunda berilgan tizimning yakka echimi sifatida

, bunda $\rho_j = \sqrt{\alpha_j^2 + \beta_j^2}$, $\omega_j = arctg \frac{\beta_j}{\alpha_j}$ bo’lsa, qo’ydagiga ega bo’lamiz.

TEOREMA. Berilgan tizimning echimi uchun = bajarilishi uchun, $|\lambda_i(A)| = r, i = 1,n$ bo’lishi zarur va etarli.

Isbotlash: A-normal bo’lganligdan, ortogonal U matricasi mavjud bo’ladi, berilgan tizimni (* *) ga olib keladigan. Bu (* *) cha tizimning umumiy echimi bo’lib ushbu vektor funkciya hisoblanadi.

y(k) = [█(■(ρ↓1↑k cos 〖ω↓1 k〗 &ρ↓1 sin 〖ω↓1 k ⋯ 〗 0 0 0 ⋯&0@ρ↓1↑k sin 〖ω↓1 k 〗 &ρ↓1 k c

Bu erda

;

$$\omega_p = arctg\frac{\beta_p}{\alpha_p};\ p = 1,l;\ \lambda_q(A) = \gamma_q, q = \overline{1,s};$$

ortogonal matrica, unda

,

Bu erda r=ρ_p =γ_q r=

TEOREMA. Agarda tizim asimptotik turg’un bo’lsa va normal matrica bo’lsa, unda Lyapunov funkciyasi, berilgan ma’lumotlari bo’yicha optimal bo’lgan, integral ma’noda mos keladi.

N=E, S=E-A^{T}A, va

Isbotlash: Agarda A-normal bo’lsa, unda, ko’rsatilgan lemmadan kelib chiqadi, E-A^TA musbat aniqlanganligi, Lyapunov funkciyasi, dastlabki qo’zdirish bahosi uchun optimal bo’lgan, ushbu ko’rinishga ega bo’ladi.

$$\vartheta_1 = x^T x, H_1 = E, C_1 = E - A^T A$$

Masalan: Ushbu tizimni qaraymiz

$$\begin{cases} x(k+1) = ax(k) + by(k) \\ y(k+1) = -bx(k) + ay(k) \end{cases}$$

Bu erda normal matrica, a^2+b^2 asimptotik turg’un. Agarda

deb olsak, unda

Unda qo’ydagiga ega bo’lamiz

optimal bo’lib

hisoblanadi.

§3 Lyapunov funkciyasining algoritmlari.
Chegaraga chiqish algoritmi.

Mayli C matricasi hohlagan aniqlangan matrica bo’lsin, H matricasi bo’lsa (7^1) tenglamaning echimi bo’lsin. Matricali tenglamani qo’ydagi ko’rinishga keltiramiz:

$$A^T(H+\delta E)A-(H+\delta E)=-\left[C+\delta\left(E-A^T A\right)\right]$$

Agarda C matricasi musbat aniqlangan bo’lsa, unda etarli darajada kichik bo’lgan $\delta > 0$ uchun $C+\delta\left[E-A^T A\right]$ matricasi manfiy emas aniqlangan bo’ladi.

Musbat aniqlangan simmetrik matricalarning hossasi bo’yicha

$$0 < \delta \le \lambda_{\min}(C) / \left|\lambda_{\min}\left[E-A^T A\right]\right.$$

Tengsizligi bajariladigan barcha δ uchun

$$\lambda_{\min}\left[C+\delta\left(E-A^{T}A\right)\right]\geq\lambda_{\min}(C)+\delta\lambda_{\min}\left[E-A^{T}A\right]\geq 0$$

tengsizligi bajariladi.

Shunday qilib, agarda C matricasi musbat aniqlangan bo’lsa, unda

$$\delta_{1}=\lambda_{\min}(C)/\left|\lambda_{\min}\left[E-A^{T}A\right]\right.$$

ko’rinishida olib, musbat aniqlangan matricaga ega Lyapunov tenglamasining echimi bo’ladigan

$$H_{1}=H+\delta_{1}E,\qquad C_{1}=C+\delta\left(E-A^{T}A\right)$$

matricalar uchun

$$\varphi(H)=\frac{\lambda_{\max}(H_{1})}{\lambda_{\min}(H_{1})}=\frac{\lambda_{maz}(H+\delta_{1}E)}{\lambda_{\min}=\left(H+\delta_{1}E\right)}=\frac{\lambda_{\max}(H)+\delta_{1}}{\lambda_{\min}(H)+\delta_{1}}<\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}=\varphi(H)$$

shunga ega bo’lamiz va boshlang’ich qo’zdirishni Lyapunov funkciyasi

$$V(x(k))=\left(x^{T}(k)Hx(k)\right)$$

qaraganda

$$V_{1}(x(k))=\left(x^{T}(k)H_{1}x(k)\right)$$

funkciyasi baholaydi.

O`qni cho’zish algoritmi.

Mayli C matricasi hohlagan musbat aniqlangan bo’lsin. H matricasi unga mos keluvchi Lyapunov tenglamasining echimi bo’lsin. H matricasin diagonallashtiruvchi U ortogonal matricasi mavjud bo’lsin, ya’ni

$$U^{T}HU=\Lambda(0)=\begin{bmatrix}\lambda_{1}(H) & 0 & \dots & 0\\ 0 & \lambda_{2}(H) & \dots & 0\\ --- & -- & -- & --\\ 0 & 0 & & \lambda_{n}(H)\end{bmatrix}$$

Bunda matricasining hususiy soni.

Lyapunov tenglamasin chap tarafidan U^{T} matricasiga, o’ng tarafidan U matricasiga ko’paytamiz.

$$U^T A^T U U^T H U U^T A U - U^T H U = -U^T C U$$

$$A_1 = U^T A U, \qquad C_1 = U^T C U$$

deb olib, qo'ydagi tenglamaga ega bo'lamiz

$$A_1^T \Lambda(0) A_1 - \Lambda(0) = -C_1$$

olingan tenglamani qo'ydagicha turlandiramiz

$$A_1^T \Lambda(\varepsilon) A_1 - \Lambda(\varepsilon) = A_1^T \begin{bmatrix} \lambda_1(H)+\varepsilon & 0 & \dots & 0 \\ 0 & \lambda_2(H) & \dots & 0 \\ -- & -- & -- & -- \\ 0 & 0 & & \lambda_n(H) \end{bmatrix} A_1$$

$$-\begin{bmatrix} \lambda_1(H)+\varepsilon & 0 & \dots & 0 \\ 0 & \lambda_2(H) & \dots & 0 \\ -- & -- & -- & -- \\ 0 & 0 & \dots & \lambda_n(H) \end{bmatrix} = -\left\{ C_1 - A_1^T \begin{bmatrix} \varepsilon & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ - & - & - & - \\ 0 & 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} \varepsilon & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ - & - & - & - \\ 0 & 0 & \dots & 0 \end{bmatrix} \right\} = -C_1(\varepsilon)$$

Demak, bizga ma'lum ε chamasining o'sish bilan ε ga bog'liqli funkciyaga keltiriladigan $\varphi_1(H)$ funkciyasi kamayadi.

Endi $C_1(\varepsilon)$ matricasi manfiy emas aniqlangan bo'ladigan, ixtiyoriy ε maksimal ma'nosin topamiz.

Qo'ydagi belgilashni kiritamiz:

$$C_1(\varepsilon) = C_1 + \varepsilon \begin{bmatrix} 1-a_{11}^2 & -a_{11}a_{12} & \cdots & -a_{11}a_{1i} & \cdots -a_{11}a_{1i} & \cdots -a_{11}a_{1n} \\ -a_{12}a_{11} & -a_{12}^2 & \cdots & -a_{12}a_{1i} & \cdots -a_{12}a1i & \cdots -a_{12}a_{1n} \\ . & . & \cdots & . & \cdots \quad . & \cdots \quad . \\ -a_{ij}a_{11} & -a_{ij}a_{12} & \cdots & a_{ij}^2 & \cdots \quad a_{1i}a_{1j} & \cdots -a_{1i}a_{1n} \\ 0a_{ij}a_{11} & -a_{ij}a_{12} & \cdots & -a_{1j}a_{1i} & \cdots -a_{1j}^2 & \cdots -a_{1j}a_{1n} \\ -a_{1n}a_{11} & -a_{1n}a_{12} & \cdots & -a_{1n}a_{1j} & \cdots -a_{1n}a_{1j} & \cdots -a_{1n}^2 \end{bmatrix} = C_1 + \varepsilon \cdot \Delta_1$$

Δ_1 matricasi aynigan matrica, uning rangisi ikkiga teng, ya'ni $rang\Delta_1 = 2$ uning xarakteristik tenglamasi qo'ydagi ko'rinishga ega

$$\det(\Delta_1 - \lambda E) = (-1)^n \lambda^{n-2}(\lambda^2 + \rho_2\lambda + \rho_1) = 0$$

bunda

$$\rho_1 = -S_\rho \Delta_1 = \sum_{i=1}^{n} a_{1i}^2 - 1$$

$$\rho_2 = \sum_{i=2}^{n} \begin{bmatrix} 1 - a_{11}^2 & -a_{11}a_{11} \\ -a_{1i}a & -a_{1i}^2 \end{bmatrix} + \sum_{i,j=2}^{n} \begin{bmatrix} -a_{1i}^2 & -a_{1i}a_{1j} \\ -a_j a_{1i} & -a_{1j}^2 \end{bmatrix} = -\sum_{i=2}^{n} a_{1i}^2$$

Shunday qilib, Δ_1 matricasining xarakteristik sonlari qo'ydagi ko'rinishga ega:

$$\lambda_i = 0, \qquad i - \overline{1, n-2}:$$

$$\lambda_{n-1,n-2} = \frac{1}{2}\left[\left(\sum^{n} i - 1a_{1i} - 1\right) - \sqrt{\left(\sum_{i=1}^{n} a_{1i}^2\right) + 4\sum_{i=2}^{n} a_{1i}^2}\right]$$

Agarda $\varepsilon > 0$ bo'lsa, unda qo'ydagiga ega bo'lamiz.

§4 Lyapunovning optimal funkciyasin topish algoritmi.

Lyapunov funkciyasining algoritmin topish masalasin qaraymiz, integral ma'noda optimal bo'lgan. Shuni ham aytib o'tish kerak, $\varphi_2(H)$ maqsad funciyasi umumiy holatda yoyiq emas, H matricasining elementlar funkciyalarda aniqlanmagan. Umumiy holatda masala echilmaydi. Endi kvaziopatimal funkciyasin topishning ikki algoritmin qaraymiz.

Nur bo'yicha optimizaciyalash algoritmi.

Endi deb olamiz, bunga mos ushbu matricali tenglamaning echimi bo'lib hisoblanadi.

$$A^T H_E A - H_E = -E \tag{10}$$

Integral kriteriyalar bahosi ushbu ko'rinishga ega bo'ladi.

$$\varphi_2(H_E) = \frac{\sqrt{\lambda_{\max}(H_E)/\lambda_{\min}(H_E)}}{1-\sqrt{1-1/\lambda\max(H_E)}}$$

Endi tenglamani qo'ydagi ko'rinishga keltiramiz

$$A^T(H_E + \delta E)A - (H_E + \delta E) = -\left[E + \delta\left(EA^T A\right)\right] \tag{11}$$

φ_2 funkciyasi H_E fiksirlangan ma'tricada skalyar parametrli δ funkciyasiga aylanadi:

$$\varphi_2(\delta) = \frac{\sqrt{\dfrac{\delta + \lambda_{\max}(H_E)}{\delta + \lambda_{\min}(H_E)}}}{1-\sqrt{1-\dfrac{\lambda_{\min}\left[E + \delta\left(E - A^T A\right)\right]}{\delta + \lambda_{\max}(H_E)}}}$$

Endi $\varphi_2(\delta)$ funkciyasining minimumin δ o'zgaruvchisi orqali qidiramiz. Ya'ni matricalar musbat aniqlangan bo'lishi kerak, unda cheklashlar tabiy

$$\delta + \lambda_{\min}(H_E) > 0,\ 1 + \lambda_{\min}\left[\delta(E - A^T A)\right] > 0$$

Uchta holatni qaraymiz.

I-Agarda A matricasi normal bo'lsa, unda, $H_0 = E,\ C_0 = E - A_T A$ (bu sonni ko'rsatadi, (11) tenglamasin δ bo'lib va $\delta \to \infty$ yo'naltiramiz.

II-Mayli A matricasi normal bo'lmasin, lekin $E - A^T A$ musbat aniqlangan. Bu holatda $\lambda_{\min}(E - A^T A) > 0$ va

$$\lambda_{\min}\left[\delta(E - A^T A\right] = \begin{cases} \delta_1 \lambda_{\min}(E - A^T A) agarda \delta > 0 \\ \delta + \lambda_{\max}(E - A^T A) agarda < 0 \end{cases}$$

$\varphi_2(\delta)$ funkciyasi qo'ydagi ko'rinishga keladi

$$\varphi_2(\delta)=\begin{cases}\sqrt{\dfrac{\delta+\lambda_{\max}(H_E)}{\delta+\lambda_{\min}(H_E)}/\left[1-\sqrt{1-\dfrac{1+\delta\cdot\lambda_{\min}(E-A^TA)}{\delta+\lambda_{maz}(H_E)}}\right]} \\ \qquad agarda\ \ \delta>0 \\ \sqrt{\dfrac{\delta+\lambda_{\max}(H_E)}{\delta+\lambda_{\min}(H_E)}/\left[1-\sqrt{1-\dfrac{1+\delta\lambda_{\max}(H_E)}{\delta+\lambda_{\max}(H_E)}}\right]}, \\ agarda\quad \max\left\{-\lambda_{\min}(H_E),-[\dfrac{1}{\lambda_{\max}(E=-A^TA)}\right\}<\delta\le 0\end{cases}$$

Endi

$$\varphi(\delta)=\left(\sqrt{\frac{\delta+a}{\delta+\varphi}}/\left[1-\frac{1+\delta c}{\delta+a}\right]\right)$$

Qaraymiz va buni ekstremumga aniqlaymiz. Al δ_0 ma’nosin, $\varphi^1(\delta)$ bo’ladigan, qo’ydagi ushbu shartdan topamiz

$$\left[1-\frac{1+\delta c}{\delta+a}\right]-\sqrt{1-\frac{1+\delta c}{\delta+a}}+\frac{(1-ac)(\delta b)}{(a-b)(\delta+a)}=0 \qquad (12)$$

$\varphi(\delta)$ funkciyasi $\delta=\delta_0$ minimumga erishish uchun,

$$\varphi^1(\delta_0)=\frac{(a-b)\sqrt{\dfrac{\delta_0+b}{\delta_0+a}(1-ac)\left[4\sqrt{\dfrac{1+\delta_0 c}{\delta_0+a}}\right.}}{4(\delta_0+a)^2(\delta_0+b)^2\left[1-\dfrac{1+\delta_0 c}{\delta_0+a}\right]\left[1-\dfrac{1+\delta_0 c}{\delta_0+a}\right]}>0$$

bo’lishi etarli.

Shuni eska olish kerak,tengsizlikning ayrim a’zolari manfiy, ikkinchi hosilaning manfiy bo’lish shartin olamiz

$$(1-ac)\left[4\sqrt{\frac{1+\delta_0 c}{\delta_0+a}}-1\right]>0 \qquad (13)$$

(***) tenglamasi ikkiga bo’linadi

$$\left[1-\frac{1+\delta\lambda_{\min}(E-A^TA)}{\delta+\lambda_{\max}(H_E)}\right]-\sqrt{1-\frac{1+\delta\lambda_{\min}(E-A^TA)}{\delta+\lambda_{\max}(H_E)}+}$$
$$+\frac{\left[1-\lambda_{\max}(H_E)\cdot\lambda_{\min}\left(E-A^TA\right)\right]\left[\delta+\lambda_{\min}(H_E)\right]}{\left[\lambda_{\max}(H_E)-\lambda_{\min}(H_E)\right]\left[\delta+\lambda_{\max}(H_E)\right]}=0 \qquad (14)$$

$$\left[1-\frac{1+\delta\cdot\lambda_{\max}\left(E-A^TA\right)}{\delta+\lambda_{\max}(H_E)}\right]-\sqrt{1-\frac{1+\delta\lambda_{\max}(E-A^TA)}{\delta+\lambda_{\max}(H_E)}}+ \qquad (15)$$
$$+\frac{\left[1-\lambda_{\max}(H_E)\cdot\lambda_{\min}(E-A^TA)\right]\left[\delta+\lambda_{\min}(H_E)\right]}{\left[\lambda_{\max}(H_E)-\lambda_{\min}\left(H_E\right)\right]\left[\delta+\lambda_{\max}(H_E)\right]}=0$$

Mayli δ_1, (14) tenglamaning echimi, δ_2, bo’lsa (15) tenglamaning echimi. $\varphi_2(\delta)$ funkciyasining aniqlanish sohasining sharti qo’ydagi ko’rinishda bo’ladi

$$\delta_1>0,\ MAX\left\{-\lambda_{\min}(H_E),-\frac{1}{\lambda_{\max}(E-A^TA)}\right\}<\delta_1<0 \qquad (16)$$

ikkinchi hosilaning manfiy bo’lishi shart

$$\left[1-\lambda_{\max}(H_E)\lambda_{\min}(E-A^TA)\right]\left[4\sqrt{1-\frac{1+\delta_{1\lambda\min}(E-A^TA)}{\delta_1+\lambda_{\max}(H_E)}}\right]>0$$
$$\left[1-\lambda_{\max}(H_E)\lambda_{\max}(E-A^TA)\right]\left[4\sqrt{\frac{1+\delta_2\cdot\lambda_{\max}(E-A^TA)}{\delta_2+\lambda_{\max}(H_E)}}\right]>0 \qquad (17)$$

Shunday qilib, Mayli $E-A^TA$ matricasi musbat aniqlangan, u δ_1, va δ_2 (14), (15), ning echimi (7^1),(8^1) Bu holatda
$\delta_0=\arg\min\{\varphi_2(\delta_1),\varphi_2(\delta_2)\}$
Agarda hech bir shart bajarilmasa, unda $\delta_0=0$
III. Mayli A matricasi shunday, $E-A^TA$manfiy aniqlangan bo’lmaydigan. A-asimptotik turg’un, unda $\lambda_{\max}(E-A^TA)>0$, lekin ishlangan taxmin bo’yicha $\lambda_{\min}(E-A^TA)>0$
Shuning uchun

$$\lambda_{\min}\left[\delta(E-A^TA\right]=\left\{\frac{\delta\cdot\lambda_{\min}(E-A^TA)agarda\delta>0}{\delta\cdot\lambda_{\max}(E-A^TA)agarda\leq 0}\right.$$

$\varphi_2(\delta)$ funkciyasi qo'ydagi ko'rinishda bo'ladi.

$$\varphi_2(\delta)=\begin{cases}\sqrt{\dfrac{\delta+\lambda_{\min}(H_E)}{\delta+\lambda_{\min}(H_E)}\Big/\left[1-\sqrt{1-\dfrac{1+\delta\cdot\lambda_{\min}(E-A^TA)}{\delta+\lambda_{maz}(H_E)}}\right]} \\ \qquad\qquad\qquad agarda\,0<\delta<-\dfrac{1}{\lambda_{\min}(E-A^TA)} \\ \sqrt{\dfrac{\delta+\lambda_{\max}(H_E)}{\delta+\lambda_{\min}(H_E)}\Big/\left[1-\sqrt{1-\dfrac{1+\delta\cdot\lambda_{\min}(E-A^TA)}{\delta+\lambda_{\max}(H_E)}}\right]}, \\ agarda\quad \max\left\{-\lambda_{\min}(H_E),-[\dfrac{1}{\pi_{\max}(H_E)}\right\}\end{cases}$$

(18)

Mayli, δ_1, va δ_2 (14) va (15) tenglamaning echimi bo'lsin. $\varphi_2(\delta)$ funkciyasining aniqlanish sohasi uchun qo'ydagi tengsizliklar bajariladi

$$\begin{gathered}0<\delta_1<-\frac{1}{\lambda_{\min}(E-A^TA)} \\ \max\left\{-\lambda_{\min}(H_E),-\frac{1}{\lambda_{\min}(E-A^TA)}\right\}<\delta_2\leq 0\end{gathered}\qquad(19)$$

Qo'ydagiga ega bo'lamiz. Mayli $E-A^TA$ musbat aniqlangan bo'lsin, δ_1, va δ_2 (14) va (15) echimi bajariladigan. Bu holatda

$$\delta_0=\arg\min\{\varphi_2(\delta_1),\varphi_2(\delta_2)\}$$

Agarda shart bajarilmasa, unda $\delta_0=0$ ya'ni kvazioptimal sifatida H_E olishga bo'ladi.

O`qni cho'zish algoritmi.

Yanada $C=E$ deb olib matricali tenglamani qaraymiz.

$$A^TH_EA-H_E=-E$$

Mayli U ortogonal turlandirish, H_E dioagonal ko'rinishga olib keluvchi, ya'ni

$$U^T H_E U = \Lambda = \begin{bmatrix} \lambda_1(H_E) & 0 & \cdots & 0 \\ 0 & \lambda_2(H_E) & \cdots & 0 \\ . & . & \cdots & 0 \\ 0 & 0 & \cdots & \lambda_n(H_E) \end{bmatrix}$$

Lyapunov tenglamasin chapdan emas o’ngdan ko’paytamiz

$$\left(U^T AU\right)^T (U^T H_E U)\left(A^T UA\right) - \left(U^T H_E U\right) = -E$$

yoki

$$A_1^T \Lambda A_1 - \Lambda = -E \qquad (20)$$

bunda $A_1 = U^T AU$ Endi (19) tenglamani ushbu ko’rinishga keltiramiz.

$$A_1^T \begin{bmatrix} \lambda_1(H_E)+\varepsilon & 0 & \cdots & 0 \\ 0 & \lambda_2(H_E) & \cdots & 0 \\ . & . & \cdots & . \\ 0 & 0 & \cdots & \lambda_n(H_E) \end{bmatrix} A_1 -$$

$$- \begin{bmatrix} \lambda_1(H_E)+\varepsilon & 0 & \cdots & 0 \\ 0 & \lambda_2(H_E) & \cdots & 0 \\ . & . & \cdots & . \\ 0 & 0 & \cdots & \lambda_N(H_E) \end{bmatrix} = -E +$$

$$+\varepsilon \begin{bmatrix} \left(a_{11}^1\right)^2 - 1 & a_{11}^1 a_{12}^1 & \cdots & a_{11}^1 a_{1n}^1 \\ a_{12}^1 a_{11}^1 & \left(a_{11}\right)^2 & \cdots & a_{12}^1 a_{1n}^1 \\ . & . & \cdots & . \\ a_{1n}^1 a_{11}^1 & a_{1n}^1 a_{12}^1 & \cdots & \left(a_{1n}^1\right)^2 \end{bmatrix}$$

Bunda $a^1 ij$-bu A_1 matricasining elementlar.
shuning uchun maqsad funkciyasi qo’ydagi ko’rinishga keladi

$$\varphi_2(\varepsilon) = \sqrt{\frac{\lambda_{\max}(H_E)}{\varepsilon + \lambda_{\min}(H_E)} \Big/ \left[1 - \sqrt{1 - \frac{\lambda_{\min}(E - \varepsilon \cdot \Delta_1)}{\lambda_{\max}(H_E)}}\right]}$$

bunda

$$\Delta = \begin{bmatrix} a_{11}^{1} - 1 & a_{11}a_{12} & \cdots & a_{11}a_{1n} \\ a_{12}a_{11} & a_{12}^{2} & \cdots & a_{12}a1n \\ a_{13}a_{11} & a_{13}a_{12} & \cdots & a_{13}a_{1n} \\ a_{1n}a_{11} & a_{1n}a_{12} & \cdots & a_{1n}^{2} \end{bmatrix}$$

$E - \varepsilon \cdot \Delta_1$ matricasining hususiy ma’nosi bo’lib

$$\lambda_i = 1, \qquad i = \overline{0, n-2}$$

$$\lambda n - 1,\ n - 2 = 1 + \frac{\varepsilon}{2}\left[\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right) \pm \sqrt{\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right)^{2} + 4\sum_{i=1}^{n} a_{1i}^{2}}\right]$$

Belgilashlar kiritamiz

$$a = \lambda_{\max}(H_E), \quad b = \lambda_{\min}(H_E)$$

$$C_1 = \frac{1}{2}\left[\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right) + \sqrt{\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right)^{2} + 4\sum_{i=1}^{n} a_{1i}^{2}}\right] \tag{21}$$

$$C_2 = \frac{1}{2}\left[\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right) - \sqrt{\left(\sum_{i=1}^{n} a_{1i}^{2} - 1\right)^{2} + 4\sum_{i=1}^{n} a_{1i}^{2}}\right]$$

shunda

$$\lambda_{\min}(E - \varepsilon \cdot \Delta_1) = \begin{cases} 1 - \varepsilon \cdot C_1 \ \ agarda \ \ \varepsilon > 0 \\ \hline 1 - \varepsilon \cdot C_2 \ \ agarda \ \varepsilon > 0 \end{cases}$$

$\varphi_2(\varepsilon)$ funkciyasi qo’ydagi ko’rinishda bo’ladi.

$$\varphi_2(\varepsilon) = \begin{cases} \sqrt{\dfrac{a}{\varepsilon + b}} / \left[1 - \sqrt{1 - \dfrac{1 - \varepsilon \cdot c_1}{a}}\right] & agarda \ \ 0 < \varepsilon < \dfrac{1}{c1} \\ \sqrt{\dfrac{a}{\varepsilon + b}} / \left[1 - \sqrt{1 - \dfrac{1 - \varepsilon \cdot c_2}{a}}\right] & agarda \min\left\{-b, \dfrac{1}{c}\right\} < \varepsilon \le 0 \end{cases}$$

Funkciyasining minimum shartin qaraymiz

$$\varphi(E) = \sqrt{\frac{a}{\varepsilon + b}} / \left[1 - \frac{1 - \varepsilon c}{a}\right]$$

Ekstremumin zarur sharti $\varphi^{1}(\varepsilon) = 0$ qo’ydagicha bo’ladi

$$\varphi^{1}(\varepsilon) = -\frac{a}{2} \cdot \frac{\sqrt{a} \cdot \sqrt{(a-1)+\varepsilon\, c} - (a+b-1) - 2\varepsilon\, c}{\varepsilon + b^{3/2} \cdot \sqrt{(a-1)\varepsilon c \cdot \left[\sqrt{a} - \sqrt{(a-1)+\varepsilon\, c}\right]^{2}}} = 0$$

Shuning uchun chamasi qo'ydagi tenglamadan topamiz

$$\sqrt{a}\sqrt{(a-1)+\varepsilon_0 \cdot c} - (a+bc-1) - 2\varepsilon_0 \cdot = 0 \qquad (22)$$

Minimumning etarliligi sharti $\varphi^{1}(\varepsilon_0) = 0$ ya'ni

$$\varphi^{11}(\varepsilon) = \frac{a\,c}{4} \cdot \frac{4\sqrt{(a-1)+\varepsilon_0 c} - \sqrt{a}}{\left(\varepsilon_0 + b^{3/2}[a-1+\varepsilon_0 c]\left[\sqrt{a}-1)\right] + \varepsilon_0 c\right)} > 0$$

yoki, (22) tengsizlikni yodga olsak minimumning etarliligi sharti

$$C(3a+4bc-5+8\varepsilon_0 c) > 0 \qquad (23)$$

Mayli ε_1 va ε_2 mos tenglamalarning echimi

$$\sqrt{a} \cdot \sqrt{(a-1)+\varepsilon_1 c_1} - (a+bc_1-1) - 2\varepsilon_1 c_1 = 0 \quad (24)$$

$$\sqrt{a} \cdot \sqrt{(a-1)+\varepsilon_2 c_2} - (a+bc_2-1) - 2\varepsilon_2 c_2 = 0 \quad (25)$$

tengsizliklarni qanoatlantiradi

$$0 < \varepsilon < \frac{1}{c1}, \quad \min\left\{-b\frac{1}{c_2}\right\} < \varepsilon_2 < 0$$

va quydagi tenglik o'rinli

$$c_1(3a+4bc_1-5+8\varepsilon_0 c_1) > 0 \qquad (26)$$

$$c_2(3a+4bc_2-5+8\varepsilon_0 c_2) > 0 \quad (27)$$

Shundan bunday tenglik o'rinli

$$\varepsilon_0 = \arg\min\{\varphi_2(\varepsilon_1), \varphi_2(\varepsilon_2)\}$$

Agarda shart bajarilmasa, unda $\varepsilon_0 = 0$

§5. Vaqt bo'yicha o'tish jarayonining bahosi va uni optimallashtirish.

O'tish jarayonining eng ahmiyatli xarakteristikalarining biri vaqt bo'lib hisoblanadi, echim ε isbotlashga erishiladi, ya'ni ε-boshlang'ich koordinata aylanasi uni tashlamaydi. Bu chama vaqt bo’yicha o’tish jarayoni deyiladi.

O`tgan paragrafdagi tengsizlikni foydalansak u qo’ydagi bog’liqlilik bilan baholanadi

$$N(H_0, X_0, \varepsilon) \le \ell n\left(\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)} \cdot \frac{\varepsilon^2}{|x(o)|^2}\right) / \ell n\left(1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)}\right)$$

Agarda boshlang’ich holati $x(o)$ berilsa va mumkin bo’lgan nuqta ε, unda vaqt bo’yicha o’tish jarayonining bahosin $H \in W(H)$ ni o’zgartirish orqali olishga bo’ladi.

<u>Tarif.</u> Lyapunov funkciyasi $V_0(x) = x^T H_0 x,$

$$H_0 = \arg \inf_{H \in w(H)} \{\varphi_3(H)\}$$

bunda

$$\varphi_3(H) = \ell n\left(\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}\right) / \ell n\left(1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)}\right)$$

vaqt bo’yicha o’tish jarayoni bahosi uchun optimal deyiladi.

Al $0 \le \lambda_{\min}(C)/\lambda_{\max}(H) < 1,$ unda $0 \le \varphi_3(H) < \infty$

TEOREMA. Lyapunov funkciyasi, vaqt bo’yicha, $\varphi_3(H_0) = 0 \quad H \in W(H)$ bo’ladigan shunda faqatgina shunda mavjud bo’ladi, agarda $E - A^T A$ musbat aniqlangan matrica bo’lsa.

Isbotlash. $\varphi_3(H_3) = 0$ bo’lishining zarurliligi va etarliligi, $\lambda_{\max}(H_0)/\lambda_{\min}(H_0) = 1$. Agarda $E - A^T A$ musbat aniqlangan matrica bo’lsa, unda

$$\lim_{H \to \lambda E} \ell n\left(\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}\right) = 0, \; \lim_{H \to \lambda E} \ell n\left(1 - \frac{\lambda_{\min}(C)}{\lambda_{\max}(H)}\right) = 0$$

$\varphi_3(H)$ funkciyasi chegarasi uning $\partial W(H)$ nuqtasida bo’laklanishga o’chiraydi.

Mayli $E - A^T A$ manfiy yarim aniqlangan matrica bo’lmasin. Bu holatda optimal $V_0(x)$ Lyapunov funkciyasi mavjud bo’ladi, $\varphi_3(H_0) > 0$

TEOREMA. Agarda A matricasi blokli-diogonal tuzilishga ega bo’lsa, unda H_0 matricasi, $\varphi_3(H)$ funkciyasiga optimal ma’no beruvchi, blokli diagonal tuzilishga ega bo’ladi.

Isbotlash. O`tgan paragraflardagi teoremaning isbotlashlariga asoslansak, qo’ydagiga ega bo’lamiz.

$$
\begin{gathered}
0 < \lambda_{\min}\left(H_0\right) \le \lambda_{\min}(H_0^1) \le \lambda_{\min}(H_0^1) \le \lambda_{\min}(H_0) \\
0 < \lambda_{\min}(C_0) \le \lambda_{\min}\left(C_0^1\right) \le \lambda_{\max}\left(C_0^1\right) \le \lambda_{\max}\left(C_0\right)
\end{gathered}
$$

Shuning uchun

$$
\varphi_3(H_0) = \ell n\left(\frac{\lambda_{\max}(H_0)}{\lambda_{\min}(H_0)}\right) / \ell n\left(\frac{\lambda_{\max}(H_0)}{\lambda_{\max}(H_0) - \lambda_{\min}(C_0)}\right) \ge \ell n\left(\frac{\lambda_{\max}(H_0^1)}{\lambda_{\min}(H_0^1)}\right) / \ell n \cdot
$$

$$
\cdot\, \ell n\left(\frac{\lambda_{\max}(H_0^1)}{\lambda_{\max}(H_0^1) - \lambda_{\min}(C_0^1)}\right) = \varphi_3(H_0^1)
$$

Nur bo’yicha cho’zish algoritmi.

Mayli $C = E$ va $H - H_E$ berilgan tenglamaning echimi bo’lsin. Uni ushbu ko’rinishga turlandiramiz

$$A^T(H+\delta E)A-(A+\delta E)=-\left[C+\delta\left(E-A^TA\right)\right]$$

shunda $\varphi_3(H)$ funkciyasi δ parametrli funkciyaga aylanadi, ya’ni

$$\varphi_3(\delta)=\ell n\left(\frac{\delta+\lambda_{\min}(H_E)}{\delta+\lambda_{\max}(H_E)}\right)/\,\ell n\left(1-\frac{\lambda_{\min}\left[E+\delta\left(E-A^TA\right)\right]}{\delta+\lambda_{\max}(H_E)}\right)$$

Agarda $E-A^TA$ matricasi musbat aniqlangan bo’lsa, unda $H_0=E$. Manfiy aniqlangan bo’lishi mumkin emas, sababi tizim turg’un emas. Shuning uchun

$$\lambda_{\max}(E-A^TA)>0,\quad \lambda_{\min}(E-A^TA)<0$$

$$\lambda_{\min}\left[E+\delta\left(E-A^TA\right)\right]=\begin{cases}1+\delta\cdot\lambda_{\min}(E-A^TA), & \delta>0,\\ 1+\delta\,\lambda_{\max}\left(E-A^TA\right), & \delta\le 0\end{cases}$$

Bundan

$$\varphi_3(\delta)=\begin{cases}\ell n\left(\dfrac{\delta+\lambda_{\min}\left(H_E\right)}{\delta+\lambda_{\max}(H_E)}\right)/\,\ell n\left(1-\dfrac{1+\delta\cdot\lambda_{\min}\left(E-A^TA\right)}{\delta+\lambda_{\max}(H_E)}\right),\\ \qquad\qquad 0<\delta<-1/\lambda_{\min}(E-A^TA),\\ \ell n\left(\dfrac{\delta+\lambda_{\min}(H_E)}{\delta+\lambda_{\max}(H_E)}\right)/\,\ell n\left(1-\dfrac{1+\delta\cdot\lambda_{\max}(E-A^TA)}{\delta+\lambda_{\max}(H_E)}\right),\\ \max\left\{-\lambda_{\min}(H_E),-1/\lambda_{\max}(E-A^TA)\right\}<\delta\le 0\end{cases}$$

Endi funkciyani minimizaciyalash masalasin qaraymiz

$$\psi(\delta)=\ell n\left(\frac{\delta+b}{\delta+a}\right)/\,\ell n\left(1-\frac{1+\delta c}{\delta+a}\right)$$

Minimumning zarur sharti $\psi^1(\delta)=0$. Yoki

$$\frac{a-b}{\delta+b}\,\ell n\left(1-\frac{1+\delta c}{\delta+a}\right)-\frac{1-ac}{(1-c)\delta+(a-1)}\,\ell n\left(\frac{\delta+b}{\delta+a}\right)=0\ .(*)$$

Minimumning etarliligi sharti $\psi^{11}(\delta_0)>0.$ yoki

$$\frac{1-ac}{\left[(1-c)\delta_0+(a-1)\right]^2}\,\ell n\left(1-\frac{1+\delta_0 c}{\delta_0+a}\right)>0$$

Shuni hisobga olsak, ya’ni δ_0 berilgan (*) berilgan tenglamaning echimi ekanin, unda

$$\delta_0 + b > 0, \ (1 - ac)[\delta_0 c + b - a + 1] < 0$$

Endi $\varphi_3(\delta)$ funkciyasin qaraymiz. Belgilashlar kiritamiz

$$a = \lambda_{\max}(H_E), \ b = \lambda_{\min}(H_E), \ C_1 = \lambda_{\min}(E - A^T A), \ C_2 = \lambda_{\max}(E - A^T A)$$

Mayli δ_1, δ_2 bo’lsa, (*) tenglamaning echimi bo’lsin, C_1 va C_2 ga mos keluvchi

$$0 < \delta_1 < -1/c_1, \ \max\{-b, -1/c_2\} < \delta_2 \le 0,$$

shunda

$$\delta_0 = \arg\min\{\varphi_3(\delta_1), \varphi_3(\delta_2)\}$$

Agarda hech bir shart bajarilmasa, unda $\delta_0 = 0$ va kvazioptimal sifatida $H_0 = H_E$ olish kerak.

O`qni cho’zish algoritmi.

Mayli U ortogonal matrica, H_E diagonal ko’rinishga keltiruvchi. $\lambda_{\min}(H_E)$ ga ε qo’zdirish kiritib, qo’ydagi tenglamaga ega bo’lamiz.

$$A_1^T \Lambda(\varepsilon) A_1 - \Lambda(\varepsilon) = -E + \varepsilon\Delta$$

Maqsad funkciyasi, fiksirlangan H_E qo'ydagi ko'rinishga ega bo'ladi

$$\varphi_3(E) = \ell n\left(\frac{\lambda_{\min}(H_E) + \varepsilon}{\lambda_{\max}(H_E)}\right) / \ell n\left(1 - \frac{\lambda_{\min}(E - \varepsilon\Delta)}{\lambda_{\max}(H_E)}\right)$$

Al $E - \varepsilon \cdot \Delta$ matricasining hususiy sonlari bo'lib

$$a = \lambda_{\max}(H_E), b = \lambda_{\min}(H_E)$$

$$C_{1,2} = \frac{1}{2}\left[\left(\sum_{i=1}^{n}\left(a_{1i}^{1}\right)^2 - 1\right) \pm \sqrt{\left(\sum_{i=1}^{n}\left(a_{1i}^{1}\right)^2 - 1\right)^2 + 4\sum_{i=1}^{n}(a_{1i}^{1})^2}\right]$$

Shunda

$$\lambda_{\min}(E - \varepsilon\Delta) = \begin{cases} 1 - \varepsilon c_1, & \varepsilon > 0 \\ 1 - \varepsilon c_2, & \varepsilon \le 0 \end{cases}$$

Shunda $\varphi_3(\varepsilon)$ funkciyasi qo'ydagi ko'rinishga ega bo'ladi.

$$\varphi_3(\varepsilon) = \begin{cases} \ell n\left(\frac{b + \varepsilon}{a}\right) / \ell n\left(1 - \frac{1 - \varepsilon c_1}{a}\right), 0 < \varepsilon < 1/c_1, \\ \ell n\left(\frac{b + \varepsilon}{a}\right) / \ell n\left(1 - \frac{1 - \varepsilon c_2}{a}\right), \min\{-b, -1/c_2\} < \varepsilon \le 0 \end{cases}$$

$\varphi_3(\varepsilon)$ funkciyasining minimumining zarur sharti bo'lib $\varphi^1(\varepsilon) = 0$ hisoblanadi.

Bundan ε_0 ni qo'ydagi shartdan topamiz

$$\frac{1}{b + \varepsilon}\ell n\left(1 - \frac{1 - \varepsilon c}{a}\right) - \frac{c}{a - 1 + \varepsilon c}\ell n\left(\frac{b + \varepsilon}{a}\right) = 0 \qquad (**)$$

Minimumning etarliligi sharti bo'lib $\varphi^{11}(\varepsilon_0) > 0$. U qo'ydagi ko'rinishga ega bo'ladi.

$$\varphi^{11}(\varepsilon_0) = \frac{-\frac{1}{(b + \varepsilon_0)^2} \cdot \ell n\left(1 - \frac{1 - \varepsilon c_0}{a}\right) + \frac{c^2}{(a - 1 + \varepsilon_0 c)^2}\ell n\left(\frac{b + \varepsilon_0}{a}\right)}{\left[\ell n\left(1 - \frac{1 - b_0 c}{a}\right)\right]^2}$$

ε_0 bo’lsa (**) tenglamaning echimi, unda bu ekvivalent

$$c(bc+1-a)>0$$

Mayli $\varepsilon_1,\varepsilon_2$ bo’lsa, (**) tenglamaning echimi , mos C_1 va C_2 bo’lgan va shartlar bajariladi

$$0<\varepsilon_1<1/c_1,\ \min\{-b,-1/c_2\}<\varepsilon_2\le 0$$

Unda bunday olish kerak

$$\varepsilon_0=\arg\min\{\varphi_3(\varepsilon_1),\varphi_3(\varepsilon_2)\}$$

Agarda hech bir shart bajarilmasa, unda $\varepsilon_0=0$ ya’ni $H_0=H_E$

§6. Ajiralmali tizimlarni optimallashtirishda gradient usulidan foydalanish.

Lagrajning aniq emaslik usulidan foydalanib, ajiralmali tizimning optimizaciyaviy masalalarin echamiz

1) Funkciyani minimizaciyalashni qaraymiz, takroran rejalashtiruvchi chamasin xarakterluvchi

$$\varphi_1(H) \to \inf_{H \in W(H)}$$

Bunda

$$\varphi_1(H) = \lambda_{\max}(H) / \lambda_{\min}(H), \ \overline{W}(H) = \{H : \lambda_{\min}(H - A^T HA) = 0\}$$

$\varphi_1(H)$ funkciyasining bir hilligidan sharning bir bo’lagi bilan cheklanamiz $\lambda_{\max}(H) \leq 1$, qo’ydagi $\overline{W}(H)$ ning ichida joylashgan va funkciyani minimizaciyalash masalasin qaraymiz

$$f_0^1(H) \to \min_{H \in \Omega(H)}$$

Qo’ydagi cheklashlarda $f_1(H) \leq 0 \ f_2(H) \leq 0$, bunda $f_0^1(H) = -\lambda_{\min}(H), f_1(H) = -\lambda_{\min}(H - A^T HA), f_2(H) = \lambda_{\max}(H) - 1$

$\Omega(H)$ -simmetrik musbat aniqlangan matricalarning ko’bligi. Lagranj funkciyasi qo’ydagi ko’rinishga ega bo’ladi

$$L_1(H, \beta) = f_0^1(H) + \sum_{i=1}^{2} \beta_i f_i(H)$$

Al H bo’yicha uning gradienti

$$\tilde{g}\tilde{r}\tilde{a}dL_1(H, \beta)\tilde{g}\tilde{r}\tilde{a}f_0^1(H) + \sum_{i=1}^{2} \beta_i \cdot \tilde{g}\tilde{r}\tilde{a}df_i(H),$$

$$\tilde{g}\tilde{r}\tilde{a}df_0^1(H) = -x_{\min} x_{\min}^T, \tilde{g}\tilde{r}\tilde{a}df_a(H) = x_{\max} x_{\max}^T,$$

$$\tilde{g}\tilde{r}\tilde{a}df_1(H) = R[H] = \begin{bmatrix} y^T R[\Delta_n]y & \cdots & y^T R[\Delta_{1n}]y \\ \cdot & \cdots & \cdot \\ y^T R[\Delta_n]y & \cdots & y^T R[\Delta_{nn}] \end{bmatrix},$$

$x_{\min}, x_{\max}$-vektorlar, birlik sferada joylashgan, $x^T Hx$ kvadratli shakl minimal va maksimal ma’noga erishadigan, $R[\Delta_j] = \Delta_{ij} - A^T \Delta_{ij} A, \ \Delta_{ij}$ matrica , i-chi yo’lda va j-chi ustunda birlardan iborat bo’lgan, u qolgan elementlar nollar, $y_{\min}$-vektor, birlik sferada yotuvchi, $y^T(H - A^T HA)y$ kvadratli shakli minimal ma’noga erishadigan.

Al $f_0^1(H), f_i(H), i = \overline{1,2}$ funkciyalari yoyiq, Sleyter sharti bajariladi. $L_1(H, \beta)$ Lagranj funkciyasi erlik nuqtaga ega.

2) Integral kriteriyali optimallastirish masalasin qaraymiz.

$$\varphi_2(H) \to \inf_{H \in W(H)},$$

bunda

$$f_0^2(H) = -\ell n\left[1 - \sqrt{1 - \lambda_{\min}\left(H - A^T HA\right)}\right] - \frac{1}{2}\ell n\left[\lambda_{\min}\left(H\right)\right]$$

Lagranj funkciyasi qo’ydagi ko’rinishda bo’ladi

$$L_2\left(H,\beta\right) = f_0^2(H) + \sum_{i=1}^{2}\beta_i f_i(H)$$

H bo’yicha uning gradienti

$$\tilde{g}\tilde{r}\tilde{a}dL_2\left(H,\beta\right) = \tilde{g}\tilde{r}\tilde{a}f_0^2(H) + \sum_{i=1}^{2}\beta_i\tilde{g}\tilde{r}\tilde{a}df_i(H),$$

bunda

$$\tilde{g}\tilde{r}\tilde{a}df_0^2\left(H\right) = \frac{-R\left[H\right]}{2\left[1 - \sqrt{1 - \lambda_{\min}\left(H - A^T HA,\right)}\right]\cdot\left[1 - \lambda_{\min}\left[A^T HA\right]\right]} - \frac{x_{\min}x_{\min}^T}{2\lambda_{\min}(H)}$$

3) O`tish jarayoni bo’yicha optimallashtirish masalasin qaraylik

$$\lambda_3(H) \to \inf_{H\in W(H)},$$

bunda

$$\varphi_3(H) = \frac{\ell n\left[\lambda_{\min}(H)/\lambda_{\max}(H)\right]}{\ell n\left[1 - \lambda_{\min}\left(H - A^T HA\right)/\lambda_{\max}(H)\right]}$$

Bu masalani qo’ydagicha olmashtiramiz

$$f_0^3(H) \to \min_{H\in\Omega(H)},\ f_1(H) \le 0,\ f_2(H) \le 0$$

bunda

$$f_0^3(H) = \ell n\left\{\ell n\left[\lambda_{\min}(H)\right]\right\} - \ell n\left\{\ell n\left[1 - \lambda_{\min}\left(H - A^T HA\right)\right]\right\}$$

Lagranj funkciyasi qo’ydagi ko’rinishga ega bo’ladi

$$\tilde{g}\tilde{r}\tilde{a}dL_3(H,\beta) = \tilde{g}\tilde{r}\tilde{a}df_0^3(H) + \sum_{i=1}^{2}\beta_i\tilde{g}\tilde{r}\tilde{a}df_i(H),$$

$$\tilde{g}\tilde{r}\tilde{a}df_0^3(H) = \frac{x_{\min}\cdot x_{\min}}{\ell n\left[\lambda_{\min}\left(H\right)\right]\cdot\lambda_{\min}(H)} + \frac{R\left[H\right]}{\ell n\left[1 - \lambda_{\min}\left(H - A^T HA\right)\right]\cdot\lambda_{\min}\left(H - A^T HA\right)}$$

§7. Integral sifat kriteriyasi va uni optimallashtirish.

Ko’bchilik holatda tizimning sifati (rejalashtirish tizimida) barcha kesmada integral bo’yicha baholandi. (3) chi tengsizlikdan qo’ydagi kelib chiqadi

$$\sum_{\kappa=0}^{\infty}|x(k)| \le \sqrt{\frac{\lambda_{\min}(H)}{\lambda_{\min}(H)}}\,|x(0)|\sum_{k=0}^{\infty}\left(1-\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}\right)^{k} =$$

$$= \frac{\sqrt{\lambda_{\max}(H)/\lambda_{\min}(H)}}{1-\sqrt{1-\lambda_{\min}(C)/\lambda_{\max}(H)}}\,|x(0)|.$$

Tarif: $V_0(x)=x^T H_0 x$ Lyapunov funkciyasi

$$H_0 = \arg \inf_{H\in\varpi(H)}\{\varphi_2(H)\},$$

bunda

$$\varphi_2(H)=\sqrt{\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}}\left(1-\sqrt{1-\frac{\lambda_{\min}(C)}{\lambda_{\max}(H)}}\right)^{-1},$$

integral ma'noda optimal deyiladi.
Endi $\varphi_2(H)$ funkciyasin qaraymiz integral o'tish jarayonin xarakterlaydigan. Buning $\varphi_1(H)$ dan o'zgachaligi bu murakkab ko'rinishga ega va uni o'rganib chiqish qiyin. Shunday qilib $\varphi_1(H)$ uchun A matricaga bog'liqligin qaraymiz, parametri uchun. Endi A matricasining uch holatin urganishni qaraymiz, A normal, A normal emas lekin $E-AA^t$ musbat aniqlangan, eng oxirida $E-AA^t$ matricasi manfiy aniqlanmagan.
Agarda (1) tizim asimptotik turg'un bo'lsa, ya'ni $|\lambda\ (A)|\angle 1, i=1,\overline{n}$ $E-AA^t$ musbat aniqlangan, unda $C=E-A^tA$ deb olib, $H=E$ ega bo'lamiz.
shuning uchun

$$|x(k)|\le[1-\lambda_{\min}(E-A^tA)]^{k/2}\cdot|x(0)|=[\lambda_{\max}(A^tA)]^{k/2}\cdot|x(0)|=|A|^k\cdot|x(0)|. \qquad (1)$$

Lemma. Agarda (1) chi tizim asimptotikali turg'un bo'lsa va A normal matrica bo'lsa, unda $E-A^tA$ musbat aniqlangan.
Isbotlash. Shart bo'yicha $\max\limits_{i=1,n}|\lambda_i|(A)\langle 1$

shunda

$$\lambda_{\min}(E-A^tA)=1\cdot\lambda_{\max}(A^tA)=1-\max_{i=1,n}|\lambda_i(A)|^2\rangle 0$$

Soldar. Agarda (1) tizim asimptotikali turg'un va A normal matrica bo'lsa, unda

$$|x(k)| \le \left(\max_{i=1,n} |\lambda_i(A)|^k \cdot |x(k)| \right) \qquad (2)$$

(1). Mayli A normal, asimptotik matrica bo’lsin. Shuni ko’rsatamiz eng aniq baho bo’lib (2) ning to’g’ri ekanligin. U yaxshilanmaydigan, ya’ni $5c(k)$ echim mavjud bo’ladi, ya’ni tengsizlik tenglikga aylanadi, ya’ni

$$\left|\bar{x}\right|(k) = \max_{i=1,n} \lambda_i(A)^k \cdot |x(0)|$$

Bu holatda optimal Lyapunov funkciyasi bo’lib $V_0(x) = x^t H_0 x, H_0 = \lambda E$.

TEOREMA. Agarda (1) chi tizim asimptotik turg’un va A normal matrica bo’lsa, unda

$$\left|\bar{x}\right|(k) = \max_{i=1,n} \lambda_i(A)^k \cdot |x(0)| .$$

bo’ladigan.

Isbotlash. Ya’ni A normal, unda V ortogonal matricasi mavjud bo’ladi, A matricasin normal ko’rinishga keltiradigan

$$U^T A U = \begin{bmatrix} \alpha_1 \;\; \beta_1 .....0.......0.......0......0 \\ -\beta_1 \;\; \alpha_1 .....0.......0.......0......0 \\ 0....\; 0.......\alpha_2 .....0.......0......0 \\ \bullet \quad \bullet \qquad\qquad \bullet \quad \bullet \quad \bullet \\ 0.....0.......0.......\alpha_k \quad \beta_k .....0 \\ 0.....0.......0...-\beta_k \quad \alpha_k .....0 \\ 0.....0......0.......0.......\gamma_1 ......0 \\ \bullet \quad \bullet \quad \bullet \quad \bullet \qquad \bullet \\ 0......0......0.......0.......0.......\gamma_3 \end{bmatrix} = \Delta$$

Bunda $\lambda_p = \alpha_p + i\beta_h, \; p = \overline{1,n}, \; \lambda_q = \gamma_q, q = \overline{1,s}, 2k + s = n$ A matricasining hususiy ma’nolari o’rin olmashtirishlar yasab $x = Uy,$ chiziqli tizimga ega bo’lamiz

$$y(k+1) = \Delta y(k).$$

Mayli $\max_{i=1,n} |\lambda_i(A)| = \chi_j$ bo’lsin. Yakka echimi sifatida $\overrightarrow{x(k)} = U\left(0,..,0,\gamma_j{}^n, 0, ...,0\right)^T |x(0)|$ olib teoremaning tastiqlashiga ega bo’lamiz. Agarda $\max_{i=\overline{1,n}} |\lambda_i(A)| = \sqrt{\alpha_j{}^2 + \beta_j{}^2}$, bo’lsa unda bunday olish etarliligi

$$\overline{x(k)} = U\left(0,...,0,\rho_j{}^k, \sin \varpi k, \rho_j{}^k \cos \varpi k, 0,...,0\right)^T |x(k)|,$$

bunda

$$\rho_j = \sqrt{\alpha_j{}^2 + \beta_j{}^2},\ \omega_j = arctg\left(\beta_j / \alpha_j\right).$$

TEOREMA. Agarda (1) chi tizim asimptotik turg'un bo'lsa va A normal matrica bo'lsa, unda Lyapunov funkciyasi berilgan ma'lumotlari bo'yicha va integral ma'nolari bo'yicha mos keladi, ya'ni $H_0 = E.$

Isbotlash. Mayli A normal bo'lsin. Shunda o'tgan lemmadan kelib chiqadi, $E - AA^T$ ning manfiy aniqlanganligi. Va Lyapunov funkciyasi berilgan ma'lumotlarni bahosi bo'yicha qo'ydagi ko'rinishga ega bo'ladi, $V_0(x) = x^T Hx,\ H_0 = E,\ C_0 = E - AA^T.$

2. Mayli $E - AA^E$ matricasi manfiy aniqlangan bo'lsin, lekin A matricasi normal bo'lmaydi. Bu holatda boshqacha bo'lishi mumkin, ya'ni optimal funkciya mos kelmaydi.

Masalan (1) chi tizimnы qaraymiz, qo'ydagi matrica bilan

$$A = \begin{bmatrix} 1/2 & \delta \\ 0 & 1/2 \end{bmatrix}.$$

A matricasi $\delta \neq 0$ da normal bo'lmaydi, lekin $|\delta| < 3/4$ manfiy aniqlangan. Bunday olamiz $0 < \delta < 3/4.$

Lyapunov funkciyasi boshlang'ich qo'zdirishni optimal baholuvchi $H_0 = E$ hisoblanadi. Agarda uni integral sifat kriteriyasining bahosi uchun ishlacak, qo'ydagiga ega bo'lamiz

$$\varphi_2(H_0) = \left(1 - \sqrt{\lambda_{\min}(E - AA^T}\right)^{-1} = \frac{\sqrt{2}}{1 - \sqrt{3/4 - \delta^2 - \delta\sqrt{1+\delta^2}}}.$$

Bu $\delta \to 3/4$ intilsa $\varphi_3(H_0) \to \infty$ intiladi. Usha vaqtta Lyapunov tenglamasin echib

$$A^T HA - H = -E,$$

qo'ydagiga ega bo'lamiz

$$H_E = \begin{bmatrix} 4/3 & 8\delta/9 \\ 8\delta/9 & 4\left(1 + 20\delta^2/9\right)/3 \end{bmatrix}.$$

Bundan

$$\lambda_{1,2}(H_E) = \frac{4}{27}\left[9 + 10\delta^2 \pm \sqrt{2\left(27 + 66\delta^2 + 50\delta^4\right)}\right]$$

$$\varphi_2(H_E) = \sqrt{\frac{\lambda_{\max}(H_E)}{\lambda_{\min}(H_E)}}\left(1 - \frac{1}{\sqrt{\lambda_{\max}(H_E)}}\right)^{-1}.$$

Ixtiyoriy $|\delta| < 3/4$ uchun chegaralangan bo'ladi.

3. Shuni ko'rsatishimiz kerak, ya'ni agarda matrica $E - AA^T$ manfiy aniqlangan bo'lmasa, unda H_0 va C_0, eng yaxshi boshlang'ich qo'zdirish bahosin beradigan $\varphi_2(H)$ ni topishga ishlatilmaydi.

TEOREMA. Agarda $E - AA^T$ manfiy aniqlangan bo'lsa, unda H_0 va C_0, boshlang'ich qo'zdirishning eng yaxshi bahosin beradigan φ_2 ni cheksizlikga aylantiradi.

Isbotlash. Agarda $E - AA^T$ manfiy aniqlangan matrica bo'lmasa, H_0 va C_0 boshlang'ich qo'zdirishning eng yaxshi bahosin beradigan, unda $\lambda_{\min}(C_0) = 0$. Bundan funkciya

$$\varphi_2(H_0) = \sqrt{\frac{\lambda_{\max}(H_0)}{\lambda_{\min}(H_0)}}\left(1 - \sqrt{1 - \frac{\lambda_{\min}(C_0)}{\lambda_{\max}(H_0)}}\right)^{-1}$$

cheksizlikga aylanadi.

TEOREMA. Agarda A asimptotik turg'un matrica bo'lsa, unda Lyapunov funkciyasi $V_0(x) = x^T H_0 x$ integral ma'noda optimal bo'lgan har doim vaqt mavjud bo'ladi.

Isbotlash. $\varphi_2(H)$ funkciyasi nolinchi darajali bir hil. Shuning uchun uni $W_1(H)$ sohasida qarab o'tamiz. Minimumlashtirish masalasi echilayotganligdan, chegaraga yaqinlaganda $\partial w^1(H)$ sohasi $W_1(H)$ funkciya $\varphi_2(H) = \sqrt{\frac{\lambda_{\max}(H\ )}{\lambda_{\min}(H\ )}}\left(1 - \sqrt{1 - \frac{\lambda_{\min}(H - F^t HA)}{\lambda_{\max}(H)}}\right)^{-1} . \to \infty$ unda $\overset{-\ \Sigma}{W}_2(H)$ sohasi bilan cheklanamiz. $\overset{-\ \Sigma}{W}_2(H)$ sohasi kompaktli bo'lganlikdan, $\varphi_2(H)$ funkciyasi shu ko'nlikma qo'ydagi ko'rinishga ega bo'ladi.

$$\varphi_2(H) = \frac{1}{\sqrt{\lambda_{\max}(H)}1 - \sqrt{1 - \lambda_{\min}(H - A^t HA)}}$$

(2) chi tenglama foydalanganlikdan

$$\lambda_{\min}(H) = \lambda_{\min}(A^t HA + C) \geq \lambda_{\min}(A^t HA) + \lambda_{\min}(C) \geq \Sigma$$

Va $W_1^{\varepsilon}(H)$ qo'bligda $\varphi_2(H)$ funkciyasi chegaralangan.

$O \leq \varphi_2(H) \leq \left[\sqrt{\varepsilon}(1 - \sqrt{1 - \varepsilon})\right]^{-1}$ uzluksiz funkciya bo'ladi N matricaning h_{ij} elementlari bilan va Veyershtrass teoremasi bo'yicha ekstremal ma'noga ega bo'ladi.

Teorema. Agarda (1) chi tizimning A matricasi blokli diogonal tuzilishga ega bo’lsa, unda H_0 matricasi $V_0(x) = x^t H_0 X$ funkciyasining, optimal integral bahosin beradigan, blokli-diogonal tuzilishga ega bo’ladi.

Isbotlash. Agarda

$$H_0 = \begin{bmatrix} H_{11}^0 & H_{12}^0 \\ (H_{12}^0)^t & H_{22}^0 \end{bmatrix}, \qquad C_0 = \begin{bmatrix} C_{11}^0 & C_{12}^0 \\ (C_{12}^0)^t & C_{22}^0 \end{bmatrix} .$$

Shunda qo’ydagi matrica uchun

$$H_0^1 = \begin{bmatrix} H_{11}^0 & O \\ O & H_{22}^0 \end{bmatrix} \qquad C_0 = \begin{bmatrix} C_{11}^0 & O \\ O & C_{22}^0 \end{bmatrix}$$

Unda

$$\lambda_{\min}(H_0) \le \lambda_{\min}(H_0) \le \lambda_{\max}(H_0^1) \le \lambda_{\max}(H_0)$$

$$\lambda_{\min}(C_0) \le \lambda_{\min}(C_0) \le \lambda_{\max}(C_0^1) \le \lambda_{\max}(C_0)$$

Shuning uchun

$$\varphi_2(H_0') = \frac{\sqrt{\lambda_{\max}(H_0')/\lambda_{\min}(H_0')}}{1-\sqrt{1-\lambda_{\min}(C_0')/\lambda_{\max}(H_0')}} \le$$

$$\le \frac{\sqrt{\lambda_{\max}(H_0)/\lambda_{\min}(H_0)}}{1-\sqrt{1-\lambda_{\min}(C_0)/\lambda_{\max}(H_0)}} = \varphi_2(H_0).$$

Endi eli optimallashtirish algoritmin qaraymiz.

Nur bo’yicha optimallashtirish qaraymiz.

Endi $C = E$ deb olib va mos $H = H_E$ deb olib (2) matricali tenglamani echamiz. Al $\varphi_2(H)$ funkciyasi qo’ydagi ko’rinishga ega bo’ladi.

$$\varphi_2(H_E) = \frac{\sqrt{\lambda_{\max}(H_E)/\lambda_{\min}(H_E)}}{1-\sqrt{1-1/\lambda_{\max}(H_E)}}$$

Lyapunov tenglamasin qo’ydagi ko’rinishga keltiramiz

$$A^t(H_E + \delta E)A - (H_E + \delta E) = -\left[E + \delta(E - A^t A)\right]$$

Fiksirlangan H_E matricasi uchun $\varphi_2(H)$ funkciyasi δ parametrining funkciyasi bo’ladi.

$$\varphi_2(\delta) = \frac{\sqrt{\varepsilon + \lambda_{\max}(H_E)/\delta + \lambda_{\min}(H_E)}}{1-\sqrt{1-\lambda_{\min}\left[E + \delta(E - A^t A)\right]/(\delta + \lambda_{\max}(H_E)}}$$

Endi $\varphi_2(\delta)$ funkciyasining minimumin qidiramiz δ o’zgaruvchisi bo’yicha. Uch holatni qaraymiz.

1. A normal, asimptotik turg’un matrica. Shunda $H_0 = E,\ C_0 = E - A^T A$.

2. A normal emas, lekin $E - A^T A$ manfiy aniqlangan. Bu holatda $\lambda_{\min}(E - A^T A) > 0$ va

$$\lambda_{\min}\left[\delta(E - A^T A\right] = \begin{cases} \delta\lambda_{\min}(E - A^T A), & \delta > 0, \\ \delta\lambda_{\max}(E - A^T A), & \delta \le 0. \end{cases}$$

Shunda $\varphi_2(\delta)$ funkciyasi qo’ydagi ko’rinishga keladi

$$\varphi_3(\delta) = \begin{cases} \sqrt{\dfrac{\delta + \lambda_{\max}(H_E)}{\delta + \lambda_{\min}(H_E)}\left[1 - \sqrt{1 - \dfrac{1 + \delta\lambda_{\min}(E - A^T A)}{\delta + \lambda_{\max}(H_E)}}\right]}, & \delta > 0, \\ \sqrt{\dfrac{\delta + \lambda_{\max}(H_E)}{\delta + \lambda_{\min}(H_E)}\left[1 - \sqrt{1 - \dfrac{1 + \delta\lambda_{\max}(E - A^T A)}{\delta + \lambda_{\max}(H_E)}}\right]}, & \max\left\{-\lambda_{\min}(HE),\ -\dfrac{1}{\lambda_{\max}(E - A^T A}\right\} < \delta \le 0 \end{cases}$$

Funkciyaning ekstremumining zarur sharti

$$\Psi(\delta) = \sqrt{\frac{\delta + a}{\delta + b}\left[1 - \sqrt{1 - \frac{1 + \delta c}{\delta + b}}\right]}^{\,-1}$$

Bo’lib $\Psi^{/}(\delta) = 0$ hisoblanadi. δ_0 ma’nosi , $\Psi^{/}(\delta) = 0$ qo’ydagi shartdan olinadi

$$1 - \frac{1 + \delta c}{\delta + a} - \sqrt{1 - \frac{1 + \delta c}{\delta + a}} + \frac{(1 - ac)(\delta + b)}{(a - b)(\delta + a)} = 0$$

Ya’ni $\Psi(\delta)$ funkciyasi $\delta = \delta_0$ da minimal ma’noga erishish uchun

$$\Psi^{//}(\delta) = \frac{(a - b)\sqrt{\dfrac{\delta_0 + b}{\delta_0 + a}}(1 - ac)\left[4\sqrt{1 - \dfrac{1 + \delta_0 c}{\delta_0 + a}} - 1\right]}{4(\delta_0 + a)^2(\delta_0 + b)^2\left[1 - \dfrac{1 + \delta_0 c}{\delta_0 + a}\right]\left[1 - \sqrt{1 - \dfrac{1 + \delta_0 c}{\delta_0 + a}}\right]} > 0.$$

Yoki, manfiy a’zolarin tashlab ketib, qo’ydagiga ega bo’lamiz

$$(1 - ac)\left[4\sqrt{1 - \frac{1 + \delta_0 c}{\delta_0 + a}} - 1\right] > 0$$

Shunday qilib, (3) chi tenglama δ_0 echimga ega bo’lsa va (4) chi tengsizlik bajarilsa, unda $\Psi(\delta)$ funkciya $\delta = \delta_0$ da minimal ma’noga ega bo’ladi.

Belgilashlar kiritamiz

$$a = \lambda_{\max}(H_E), \quad b = \lambda_{\min}(H_E), \qquad c_1 = \lambda_{\min}(E - A^T A), \quad c_2 = \lambda_{\max}(E - A^T A).$$

Mayli (3) chi tenglama $c = c_1$ de $\delta_1 > 0$ echimga ega bo'lsin, $c = c_2$ da δ_2 echimga ega bo'lsin, qo'ydagi tengsizliklarni qanoatlantiruvchi

$$\max\left\{-b,\ -\frac{1}{c_2}\right\} < \delta_2 \le 0.$$

Shunda bunday olamiz

$$\delta_0 = \arg\min\{\varphi_2(\delta_1),\ \varphi_2(\delta_2)\}.$$

Agarda shu shartlarning hech biri bajarilmasa, unda $\delta_2 = 0.$

3. Mayli $E - A^T A$ manfiy aniqlangan matrica bo'lmasin, ya'ni $\lambda_{\min}(E - A^T A) < 0.$ Agarda A asimptotik turg'un bo'lsa, unda $\lambda_{\max}(E - A^T A) > 0.$ Shuning uchun

$$\lambda_{\min}\left[\delta(E - A^T A)\right] = \begin{cases} \delta\lambda_{\min}(E - A^T A), & \delta > 0, \\ \delta\lambda_{\max}(E - A^T A), & \delta \le 0. \end{cases}$$

Shunda $\varphi_2(\delta)$ funkciyasi qo'ydagi ko'rinishga keladi

$$\varphi_2(\delta) = \begin{cases} \sqrt{\dfrac{\delta + \lambda_{\max ч}(H_E)}{\delta + \lambda_{\min}(H_E)}}\left[1 - \sqrt{1 - \dfrac{1 + \delta\lambda_{\max}(E - A^T A)}{\delta + \lambda_{\max}(H_E)}}\right]^{-1}, \\ \qquad\qquad 0 < \delta < -\dfrac{1}{\lambda_{\min}(E - A^T A)}, \\ \sqrt{\dfrac{\delta + \lambda_{\max}(H_E)}{\delta + \lambda_{\min}(H_E)}}\left[1 - \sqrt{1 - \dfrac{1 + \delta\lambda_{\min}(E - A^T A)}{\delta + \lambda_{\max}(H_E)}}\right]^{-1}, \\ \max\left\{-\lambda_{\min}(H_E),\quad -\dfrac{1}{\lambda_{\max}(E - A^T A)}\right\} < \delta \le 0. \end{cases}$$

Agarda $\delta_1$va δ_2 lar (3) chi tenglamaning echimi bo'lsa, c_1 va c_2 ga mos keluvchi, unda

$$0 < \delta_1 < -\frac{1}{\lambda_{\min}(E - A^T A)},\quad \max\left\{-\lambda_{\min}(H_E),\ -\frac{1}{\lambda_{\max}(E - A^T A}\right\} < \delta_2 \le 0.$$

Bu holatda bunday olamiz

$$\delta_0 = \arg\min\{\varphi_2(\delta_1),\ \varphi_2(\delta_2)\}.$$

Agarda shart bajarilmasa, unda $\delta_0 = 0,$ ya'ni kvazipolinom sifatida H_E olish kerak.

O`qni cho'zish algoritmi.

Mayli U ortogonal turlandirish, H_E ni diagonal ko'rinishga keltiruvchi Lyapunov tenglamasin chapdan U^T ga, ungdan U ga ko'paytirib qo'ydagiga ega bo'lamiz

$$A_1^T \Delta A - \Delta = -E,$$

Bunda $A_1 = U^T AU$. hosil bo'lgan tenglamani turlandiramiz

$$A_1^T \begin{bmatrix} \lambda_1(H_E)+\varepsilon & 0 & \dots & 0 \\ 0 & \lambda_2(H_E) & \dots & 0 \\ . & . & \dots & . \\ 0 & 0 & \dots & \lambda_n(H_E) \end{bmatrix} A_1 - \begin{bmatrix} \lambda_1(H_E)+\varepsilon & 0 & \dots & 0 \\ 0 & \lambda_2(H_E) & \dots & 0 \\ . & . & \dots & . \\ 0 & 0 & \dots & \lambda_n(H_E) \end{bmatrix} =$$

$$= -E + \varepsilon \begin{bmatrix} (a_{11}{}^1)^2 - 1 & a_{11}{}^1 a_{12}{}^1 & \dots & a_{11}{}^1 a_{1n}{}^1 \\ a_{12}{}^1 a_{11}{}^1 & (a_{12}{}^1)^2 & \dots & a_{12}{}^1 a_{1n}{}^1 \\ . & . & \dots & . \\ a_{1n}{}^1 a_{11}{}^1 & a_{1n}{}^1 a_{12}{}^1 & \dots & (a_{1n}{}^1)^2 \end{bmatrix},$$

Bunda $a_{ij}{}^1$ bo'lsa A_1 matricasining elementi. Al $\varphi_2(H)$ funkciyasi qo'ydagi ko'rinishga ega bo'ladi

$$\varphi_2(\varepsilon) = \sqrt{\frac{\lambda_{\max}(H_E)}{\varepsilon + \lambda_{\min}(H_E)}} \left[1 - \sqrt{1 - \frac{\lambda_{\min}(E - \varepsilon\Delta)}{\lambda_{\max}(H_E)}} \right]^{-1}.$$

$E - \varepsilon\Delta$ matricasining hususiy ma'nosi bo'lib $\lambda_i = 0.$ $i = \overline{1, n-2}$ hisoblanadi

$$\lambda_{n-1,n} = 1 + \frac{\varepsilon}{2} \left[\left(\sum_{i=1}^{n} (a_{11}{}^1)^2 - 1 \right) \pm \sqrt{ \left(\sum_{i=1}^{n} (a_{11}{}^1)^2 - 1 \right)^2 + 4 \sum_{i=1}^{n} (a_{11}{}^1)^2 } \right].$$

Belgilashlar kiritamiz

$$a = \lambda_{\max}(H_E), \; b = \lambda_{\min}(H_E),$$

$$c_{1,2} = \frac{1}{2} \left[\left(\sum_{i=1}^{n} (a_{11}{}^1)^2 - 1 \right) \pm \sqrt{ \left(\sum_{i=1}^{n} (a_{11}{}^1)^2 + 4 \sum_{i=1}^{n} (a_{11}{}^1)^2 \right) } \right].$$

$$\lambda_{\min}(E - \varepsilon\Delta) = \begin{cases} 1 - \varepsilon c_1, & \varepsilon > 0 \\ 1 - \varepsilon c_2, & \varepsilon \le 0 \end{cases}$$

$\varphi_2(\varepsilon)$ funkciyasi qo'ydagi ko'rinishga keladi

$$\varphi_2(\varepsilon)=\begin{cases}\sqrt{\dfrac{a}{\varepsilon+b}}\left[1-\sqrt{\dfrac{1-\varepsilon a}{a}}\right]^{-1}, & \varepsilon>0\\ \sqrt{\dfrac{a}{\varepsilon+b}}\left[1-\sqrt{1-\dfrac{1-\varepsilon a}{a}}\right]^{-1}, & \varepsilon\le 0.\end{cases}$$

Funkciyaning minimum shartin qaraymiz

$$\psi(\varepsilon)=\sqrt{\frac{a}{\varepsilon+b}}\left[1-\sqrt{1-\frac{1-\varepsilon a}{a}}\right]^{-1}.$$

ε o’zgaruvchisi bo’yicha. Ekstremumning zarur sharti $\psi^{/}(\varepsilon)=0$ qo’ydagi ko’rinishga keladi

$$\psi^{/}(\varepsilon)=-\frac{a}{2}\frac{\sqrt{a}\sqrt{(a-1)+\varepsilon}-(a+bc-1)-2\varepsilon c}{(\varepsilon+b)^{3/2}\sqrt{(a-1)+\varepsilon c}\left[\sqrt{a}-\sqrt{(a-1)+\varepsilon c}\right]}=0$$

Shuning uchun ε_0 ni qo’ydagi tenglamadan topamiz

$$\sqrt{a}\sqrt{a-1+\varepsilon c}(a+bc-1)-2\varepsilon c=0. \qquad (5)$$

Minimumning etarliligi sharti bo’lib $\psi^{/}(\varepsilon_0)>0.,$ ya’ni

$$\psi^{//}(\varepsilon_0)=\frac{ac}{4}\frac{4\sqrt{a-1+b_0c}\sqrt{a}}{(\varepsilon_0+b)^{3/2}\left[a-1+\varepsilon_0c\right]\left[\sqrt{a}-\sqrt{a-1+\varepsilon_0c}^{\,2}\right]}>0$$

yoki

$$c(3a+4bc-5+8\varepsilon_0c)>0$$

Mayli ε_1 va ε_2 (3) chi tenglamaning echimi bo’lsin, c_1 va c_2 ga mos keluvchi va qo’ydagi tengsizlikni qanoatlantiruvchi

$$0<\varepsilon_1<1/c_1,\quad \max\{-b,\ 1/c_2\}<\varepsilon_2\le 0$$

Va

$$c_1(3a+4bc_2-5+8\varepsilon_1c_1)>0$$
$$c_2(3a+4bc_2-5+8\varepsilon_2c_2)>0$$

Shunda kvazioptimal Lyapunov funkciyasin topish uchun bunday olamiz $\varepsilon_0=\arg\min\{\varphi_2(\varepsilon_1),\ \varphi_2(\varepsilon_2)\},$

Agarda shart bajarilmasa, unda $\varepsilon_0=0,\ H_0=H_E.$

§8 Sonli masala.

Qo’ydagi matricali tenglamani echish

$$A^T HA - H = C \qquad (10)$$

Bunda:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

Agarda A, C matricalarining koefficientlarin (10)-tenglamaga olib borib quysak, unda qo’ydagi ko’rinishga ega bo’ladi:

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} 2h_{11} & 2h_{12} \\ h_{11} + 2h_{21} & h_{12} + 2h_{22} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4h_{11} & 2h_{12} \\ 2h_{11} + 4h_{21} & h_{11} + 2h_{21} + 2h_{12} + 4h_{22} \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3h_{11} & 2h_{11} + 3h_{12} \\ 2h_{11} + 3h_{21} & h_{11} + 2h_{12} + 2h_{21} + 3h_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 3h_{11} = 1 \\ 2h_{11} + 3h_{12} = 0 \\ 2h_{11} + 3h_{21} = 0 \\ h_{11} + 2h_{12} + 2h_{21} + 3h_{22} \end{cases} \Leftrightarrow \begin{cases} h_{11} = 0{,}3333 \\ h_{12} = -0{,}2222 \\ h_{21} = 0{,}2222 \\ h_{22} = 0{,}5185 \end{cases}$$

Demak (10)-matricali tenglamaning echimi bo’lgan H matricasi qo’ydagi ko’rinishga ega:

$$H = \begin{bmatrix} 0{,}3333 & -0{,}2222 \\ -0{,}2222 & 0{,}5185 \end{bmatrix}$$

$$\det H = 0{,}12344568$$

Endi (10)-matricali tenglamaning o'ng tarafi bo'lgan C matricasi Teng kelgan simmetrik matrica bo'lgan holatin qarab o'tamiz. Mayli C matricasi qo'ydagicha ko'rinishga ega bo'lsin:

$$C = \begin{bmatrix} -3 & -1 \\ -1 & -2 \end{bmatrix}$$

Unda (10) –matricali tenglama qo'ydagicha bo'ladi:

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Bu matricali tenglamani turlandirib qo'ydagi chiziqli algebrik tenglamalar tizimiga ega bo'lamiz:

$$\begin{cases} h_{11} = 1 \\ h_{12} = -0{,}3333 \\ h_{21} = -0{,}3333 \\ h_{22} = 0{,}7777 \end{cases}$$

Demak matricasi qo'ydagi ko'rinishga ega

$$H = \begin{bmatrix} 1 & -0{,}3333 \\ -0{,}3333 & 0{,}7777 \end{bmatrix}$$

$$\det\ H = 0{,}666612$$

Hulosa.

Bu ishda doimiy koefficientli chiziqli ajiralmali tizimi qaraladi. Bu chiziqli ajiralmali tizimning turg’unliligin aniqlash uchun Lyapunov funkciyasi kvadratli shakl ko’rinishida olinadi.

Odatda bunday tizimlarni aniqlash uchun Lyapunovning to’gri usuli foydalaniladi. Chiziqli ajiralmali tizim uchun Lyapunov funkciyasin foydalanib baho olinadi. Bitiruv malakaviy ishida ikki algoritm, chegaraga chiqish algoritmi va o’qni cho’zish algoritmi qaraladi.

Shunday qilib Lyapunov funkciyasi yordamida turg’unliligin aniqlash masalasi matricali Lyapunov tenglamasin echishga olib kelinadi. Sonli masala qaraladi va bu masalaning echimi elektron hisoblash mashinasida olinadi. Ko’rilayotgan ishda asosiy teoremalar to’liq isbotlanadi.

Yanada bu bitiruv malakaviy ishida chiziqli ajiralmali tizimlarni gradient usulidan foydalanib echish qaraladi. Shuningdek integral sifat kriteriyasi va uni optimallashtirish masalasi qaraladi. Matricali Lyapunov tenglamasiga sonly masala ko’riladi. Uning dasturi elektron hisoblash mashinasida olinadi.

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